

End semester Examination (Monsoon 2023-24)
 Department of Computer Science & Engineering, IIT (ISM), Dhanbad
 Discipline: M.Tech. (CSE) & Research Scholar
 Subject: Algorithmic Graph Theory (CSC503) Time: 3 hours, Marks: 100

Instructions: Answer all questions

Q. No		Marks
1	a. Prove that for every graph G, $X(G) \leq V(G) + 1$. (Here $X(G)$ is the chromatic number of G) b. Prove that G is bipartite iff the chromatic number of G is 2.	5 5
2	a. The Prufer sequence S of a labelled tree T is $S = \{1, 7, 6, 6, 1\}$. Draw the tree T describing all the intermediate steps pictorially during the process. b. Write the algorithm of the same.	5 5
3	a. Prove by method of induction that a graph G is a tree iff G is acyclic and the number of edges m in G is equal to $n-1$, where n is the number of vertices in G. b. Deduce that clique decision problem is NP-Hard using Satisfiability problem.	5 5
4	a. Prove that for a simple graph G with n number of vertices ($n \geq 4$), and E number of edges and genus g satisfies $g \geq \lceil \frac{1}{8}(E - 3n) + 1 \rceil$ b. "A necessary and sufficient condition for a graph G to be planar is that for every circuit C of G the auxiliary graph $G^+(C)$ is bipartite", prove it.	5 5

$n \geq n-1-n$
 $n-1 \leq n-n$

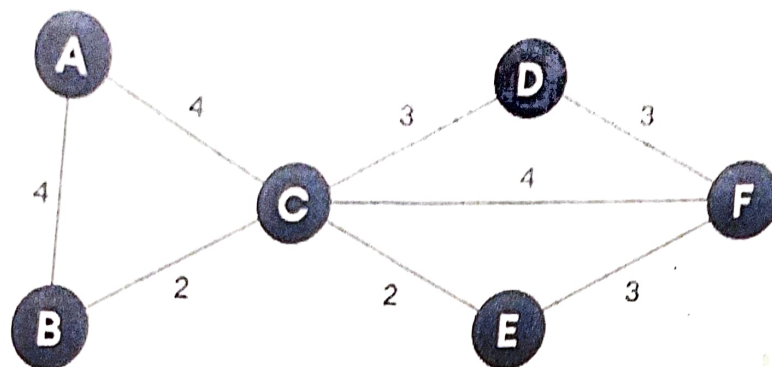
5	a. Suppose want to schedule some final exams for CS courses with following call numbers: CSE001, CSE002, CSE003, CSE004, CSE005, CSE006, CSE007, CSE008 and also suppose also that there are no common students in the following pairs of courses because of prerequisites: CSE001 - CSE002 CSE001 - CSE004 CSE001 - CSE003, CSE002 - CSE003 CSE001 - CSE007, CSE002 - CSE007 CSE001 - CSE008, CSE002 - CSE008, CSE003 - CSE008 CSE001 - CSE005, CSE002 - CSE005, CSE004 - CSE005, CSE001 - CSE006, CSE002 - CSE006, CSE004 - CSE006, CSE005 - CSE006 How many exam slots are necessary to schedule exams?	4
6	a. If a graph G is maximal planar graph with n vertices ($n \geq 3$) and m edges then show that $m = 3n - 6$.	5
	b. For a polyhedron with N number of vertices, E number of edges and F number of faces, prove by method of induction that $N - E + F = 2$.	5
7	a. Why running time measurement is important to study? Mention some of the factors affecting the running time of a program.	2+3
	b. Prove that every planar graph has a dual.	5
8	a. Discuss the merits and demits of Dijkstra's Algorithm and Bellman-Ford Algorithm with examples.	5

$$\begin{aligned}
 m - n + 2 &= 2 \\
 m - n + 2 &= 2 \\
 n - m + 2 &= 2 \\
 2 &= 2
 \end{aligned}$$

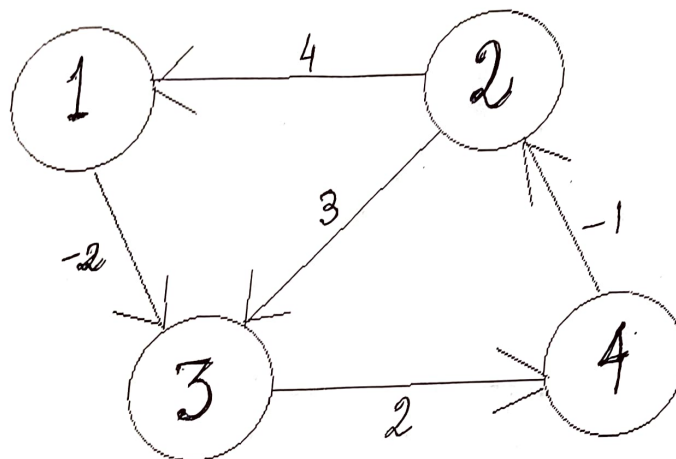
1, 2, 6, 4

5, 6

b. Use Prim's algorithm to find a minimum spanning tree that takes the following graph as input.



9 a. Find the all pairs shortest paths among all vertices for the following graph using Floyd-Warshall Algorithm (showing all intermediate steps)



a. Prove that Peterson Graph is non-planar using Kuratowski's and Wagner's Theorem as well.

Handwritten notes for the Floyd-Warshall algorithm:

Initial distance matrix (D⁽⁰⁾):

	1	2	3	4
1	0	4	-2	
2		0	3	-1
3			0	2
4				0

Intermediate steps (D⁽¹⁾, D⁽²⁾, D⁽³⁾) showing the progression of shortest paths as intermediate vertices are added.

Final shortest path matrix (D⁽³⁾):

	1	2	3	4
1	0	4	-2	1
2	1	0	3	-1
3	-2	1	0	2
4	1	0	2	0

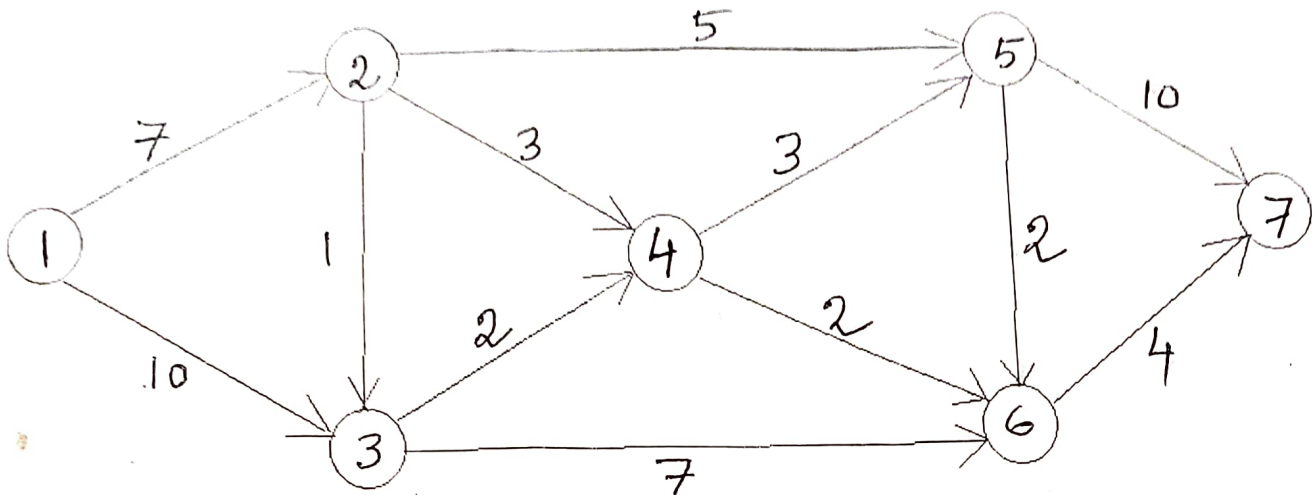
works (person_name, company_name, salary)

company (company_name, city)

manages (person_name, manager_name)

- (i) Modify the database so that Amit now lives in Dhanbad city.
- (ii) Give all employees of Facebook a 10 percent salary raise.

- 10 a. Find the maximum flow through the given network using Ford-Fulkerson algorithm.



- b. Consider the following graph to find the minimum connected dominating set showing all the intermediate steps during the constructions.

