

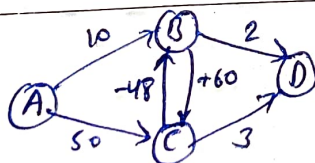
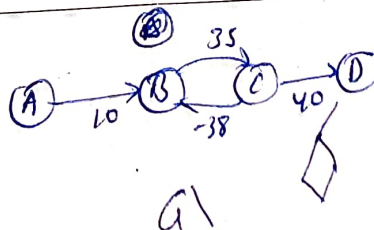
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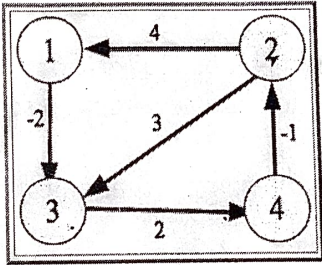
**Discipline:** M.Tech. (CSE) & Research Scholar

**Subject: Algorithmic Graph Theory (CSC503)**      **Time: 2 hours, Marks: 50**

**Instructions:** Answer all questions

|   | Q.<br>No                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | Marks       |
|---|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|
|   | <p>a. Suppose want to schedule some final exams for CS courses with following call numbers:<br/>           1007, 3137, 3157, 3203, 3261, 4115, 4118, 4156 and also suppose also that there are no common students in the following pairs of courses because of prerequisites:</p> <ul style="list-style-type: none"> <li>1007-3137 ✓</li> <li>1007-3157, 3137-3157 ✓</li> <li>1007-3203 ✓</li> <li>1007-3261, 3137-3261, 3203-3261 ✓</li> <li>1007-4115, 3137-4115, 3203-4115, 3261-4115 ✓</li> <li>1007-4118, 3137-4118 ✓</li> <br/> <li>1007-4156, 3137-4156, 3157-4156 ✓</li> </ul> <p>How many exam slots are necessary to schedule exams?</p> | 6           |
|   | <p>b. Illustrate with an example that any graph is a subgraph of a r-regular graph.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | 4           |
| 2 | <p>a. If a graph G is maximal planar graph with n vertices (<math>n \geq 3</math>) and m edges then show that <math>m = 3n - 6</math>.</p> <p>b. For a planar graph G with n number of vertices and e number of edges with r regions, prove by method of induction that <math>n - e + r = 2</math>.</p> <p>c. Illustrate the situation when Dijkstra's Algorithm may fail to find the shortest path but Bellman-Ford Algorithm may succeed. And</p>                                                                                                                                                                                                | 4<br>3<br>3 |

 $\frac{1}{2}$ 

|   |                                                                                                                                                                           |   |
|---|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|
|   | also mention the situation when Bellman-Ford Algorithm may fail (illustrate with an example).                                                                             |   |
| 3 | a. Illustrate with suitable example (considering a labelled tree $T$ with at least 9 vertices) and converting $T$ into its equivalent Prufer Sequence $S$ and vice versa. | 6 |
|   | b. Mention the important differences between BFS and DFS                                                                                                                  | 4 |
| 4 | a. Prove that Peterson Graph is non-planar using Kuratowski's and Wagner's Theorem as well.                                                                               | 6 |
|   | b. Prove that every planar graph has a dual.                                                                                                                              | 4 |
| 5 | a. Find the all pairs shortest paths among all vertices for the following graph using Floyd-Warshall Algorithm (showing all intermediate steps)                           | 6 |
|   |                                                                                        |   |
|   | b. Also describe Floyd-Warshall Algorithm to find the shortest paths among all vertices.                                                                                  |   |

Handwritten notes and calculations for the Floyd-Warshall Algorithm:

**Initial Distance Matrix (D<sup>0</sup>):**

|   | 1 | 2 | 3  | 4  |
|---|---|---|----|----|
| 1 | 0 | 4 | -2 |    |
| 2 |   | 0 | 3  | -1 |
| 3 |   |   | 0  | 2  |
| 4 |   |   |    | 0  |

**Iteration 1 (k=1):**

|   | 1 | 2 | 3  | 4  |
|---|---|---|----|----|
| 1 | 0 | 4 | -2 |    |
| 2 |   | 0 | 3  | -1 |
| 3 |   |   | 0  | 2  |
| 4 |   |   |    | 0  |

**Iteration 2 (k=2):**

|   | 1 | 2 | 3  | 4  |
|---|---|---|----|----|
| 1 | 0 | 4 | -2 |    |
| 2 |   | 0 | 3  | -1 |
| 3 |   |   | 0  | 2  |
| 4 |   |   |    | 0  |

**Iteration 3 (k=3):**

|   | 1 | 2 | 3  | 4  |
|---|---|---|----|----|
| 1 | 0 | 4 | -2 |    |
| 2 |   | 0 | 3  | -1 |
| 3 |   |   | 0  | 2  |
| 4 |   |   |    | 0  |

**Iteration 4 (k=4):**

|   | 1 | 2 | 3  | 4  |
|---|---|---|----|----|
| 1 | 0 | 4 | -2 |    |
| 2 |   | 0 | 3  | -1 |
| 3 |   |   | 0  | 2  |
| 4 |   |   |    | 0  |

**Final Shortest Path Matrix (D<sup>4</sup>):**

|   | 1 | 2 | 3  | 4  |
|---|---|---|----|----|
| 1 | 0 | 4 | -2 |    |
| 2 |   | 0 | 3  | -1 |
| 3 |   |   | 0  | 2  |
| 4 |   |   |    | 0  |

**Handwritten notes:**

- For (1,2):  $(1,2) \rightarrow (1,3) \rightarrow (3,2)$  with weight  $-2$ .
- For (1,4):  $(1,4) \rightarrow (1,3) \rightarrow (3,4)$  with weight  $-2$ .
- For (2,4):  $(2,4) \rightarrow (2,3) \rightarrow (3,4)$  with weight  $-1$ .
- For (3,4):  $(3,4) \rightarrow (3,3) \rightarrow (3,4)$  with weight  $2$ .