1. a) Let n be an even integer and $n \geq 4$. By method of induction show that there exists a 3-regular graph of order n. **7 marks**

b) Justify the statement "every finite connected vertex transitive graph has a Hamiltonian path" 7 **marks**

c) Prove by contradiction that Peterson graph has no Hamiltonian cycle. 6 **marks**

2. a) Prove the theorem “Every graph G with $\Delta(G) = r$ is an induced subgraph of an r-regular graph”. 7 **marks**

b) Prove that a graph G is bipartite if and only if it contains no odd cycle.

7 **marks**

c) Prove that an edge $e \in E(G)$ is a bridge if and only if e is not in any cycle of G.

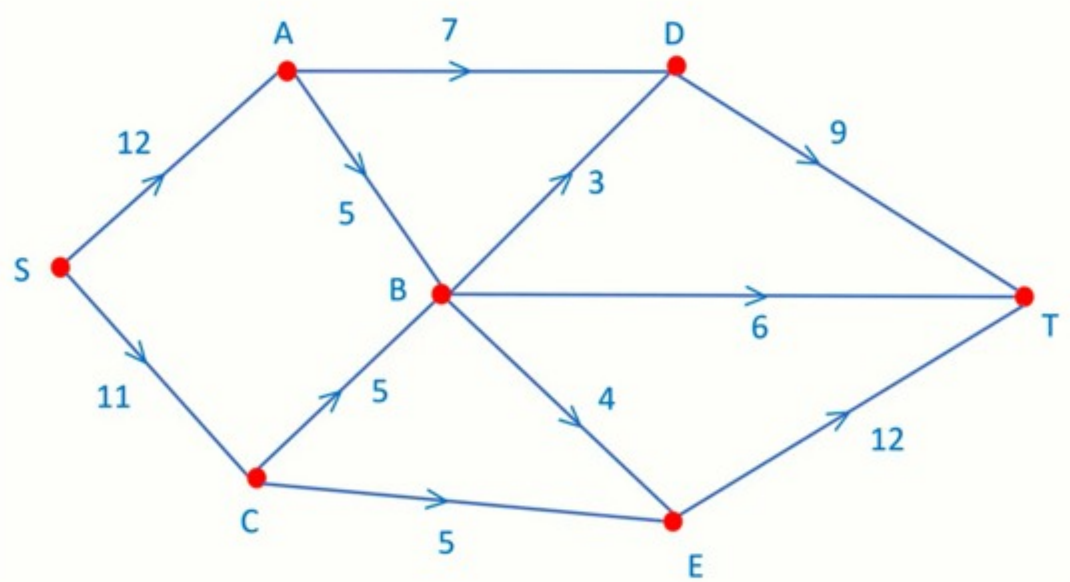
6 **marks**

3.a) Using all pairs shortest path algorithm, find the shortest path among all vertices of the following directed graph (showing all the steps of updating the matrices) . **Marks 7**

$$A^0 = \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix}$$

b) Explain with a suitable example a greedy algorithm for find the minimal connected dominating set. (show all the steps in the illustration). **Marks 7**

c) Find the maximum flow of the following network from node S to T. **Marks 6**



4. a) Consider any suitable labelled tree T and find the Prufer sequence S of the tree T mentioning all the steps required for the same. Using the same sequence, reconstruct the tree T and mention the steps to do the same. **Marks 7**

b) Describe Wagner Theorem and Kuratowski's Theorem for checking planarity of a graph and also illustrate them with Peterson Graph for each theorem.

Marks 7

c) What do you mean by isomorphic graphs? Mention the steps required to check for isomorphic graph. And also illustrate with an example. **Marks 6**

5.a) Illustrate NP-hard and NP-complete problem and Clique Decision Problem (CDP) **Marks 6**

b) Using Cook's Theorem prove that CDP is NP-Hard. **Marks 7**

c) Prove that Knapsack problem is NP-Hard. **Marks 7**