

Question 1 :-

Answer A)

$$V = 20$$

degree of each vertex = 3

By using Hand-Shaking Lemma

$$\sum_{i=1}^V \deg(v_i) = 2|E|$$

so,

$$20 \times 3 = 2 \times E$$

$$30 = E$$

By using Euler Formula

$$V - E + R = 2$$

$$20 - 30 + R = 2$$

$$-10 + R = 2$$

$$R = 12$$

V = vertices

E = edges

R = regions

Answer B)

d = minimum degree of vertex

$$d \geq 3$$

given n = no. of vertices

The planar graph with minimum degree 3 for each vertex

we can say

$$3 \times n \leq 2|E| \quad \text{--- (1)}$$

From Euler Formula

$$v - e + r = 2$$

$$e = v + r - 2$$

— (i)

From (i) and (ii)

$$v + r - 2 \geq \frac{3n}{2}$$

$$[v = n]$$

$$n + r - 2 \geq \frac{3n}{2}$$

$$r \geq \frac{3n}{2} - n + 2$$

$$r \geq \frac{n}{2} + 2$$

Hence proved

Answer C)

$$v = 10$$

no. of edge on each face = 3

$$\therefore 2e = 3r$$

$$r = \frac{2e}{3}$$

From Euler Formula

$$v - e + r = 2$$

$$e = v + 2 - 2$$

$$e = v + \frac{2e}{3} - 2$$

$$e - \frac{2e}{3} = v - 2$$

$$\frac{e}{3} = v - 2$$

$$e = 3(v - 2) \quad (v = 10 \text{ given})$$

$$e = 3(10 - 2)$$

$$e = 3 \times 8$$

$$e = 24$$