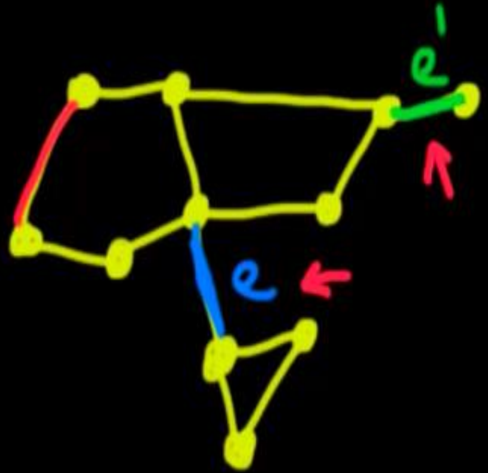

bridge

$G \setminus e$ has 2 components

$G \setminus e'$ has 2 components

G

$c(G) = 2$

$c(G \setminus f) = 3$

G

$c(G) = 2$

$c(G \backslash f) = 3$

An edge $e \in E(G)$ is a <u>bridge</u> of G if $G \backslash e$ has more connected components than G

$$c(G \backslash e) > c(G)$$

In particular, if G is connected then e is a bridge if and only if $G \backslash e$ is disconnected.

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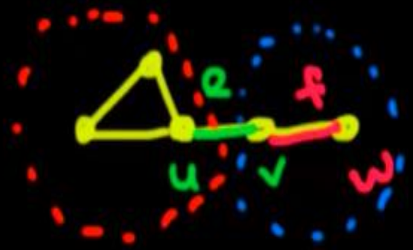

$e=uv$ is a bridge $\iff$ u, v are in diff. connected components of $G\backslash e$

Theorem: An edge $e \in E(G)$ is a bridge if and only if e is not in any cycle of G.

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$\Rightarrow$) contrapositive: e is on a cycle then e is not a bridge

Say e is on a cycle C

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$\Leftarrow$] contrapositive: e not a bridge $\Rightarrow$ e is on a cycle

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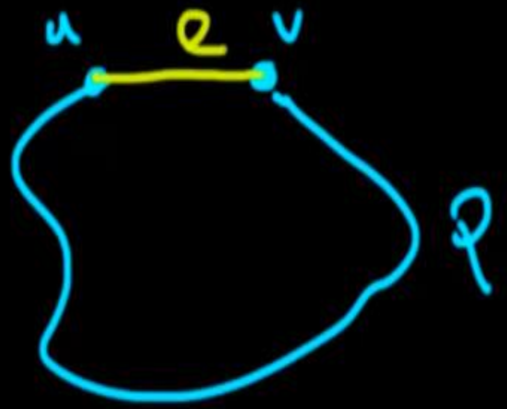
$e = uv$ is not a bridge

then $G\backslash e$ there is a path from u to v

(u & v are in the same component)

$P: u \xrightarrow{*} v$

$G\backslash e$



In G

$eP: u \underset{e}{\rightarrow} v \underset{P}{\overset{*}{\rightarrow}} u$

G

there is a cycle that contains e.

Lemma: Let e be a bridge in a connected graph G. Then $c(G\setminus e)=2$.

Proof:

$e = uv$

e is a bridge $\Rightarrow$ u and v are in different components of $G\setminus e$

$\therefore\ c(G\setminus e) \geq 2$

$G\setminus e$

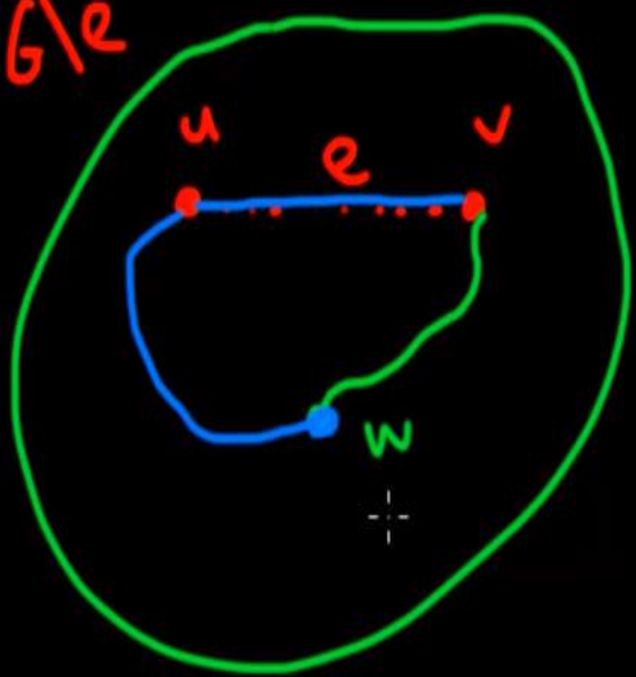
e is a bridge $\Rightarrow$ u and v are in different components of $G \backslash e$

$\therefore c(G \backslash e) \geq 2$

Let $w \in V(G)$

G connected $\Rightarrow \exists\, P: w \overset{*}{\rightarrow} v$ in G

If P does not use edge e then P is also a path from w to v in $G \backslash e$.



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If P uses edge e then $P: w \xrightarrow{*} u \xrightarrow[e]{} v$

so there is a path from w to u in $G \setminus e$

$\therefore c(G \setminus e) = 2.$

$\square$

Trees

A graph with no cycles is acyclic.

A tree is a connected acyclic graph

A forest is an acyclic graph.

Corollary: A connected graph is a tree $\iff$ all of its edges are bridges.

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isomorphic.

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$n=3$ $\qquad 3^1 = 3$

Cayley's Formula: There are n^{n-2} trees on a vertex set V of n elements.

isomorphic.

$n=3$ $\quad$ $3^{1}=3$

Cayley's Formula: There are n^{n-2} trees on a vertex set V of n elements.

n	1	2	3	4	5	6	7
# non-isom trees	1	1	1	2	3	6	11

isomorphic

$n=3$ $\quad 3^1 = 3$

Cayley's Formula: There are n^{n-2} trees on a vertex set V of n elements.

n	1	2	3	4	5	6	7	…	16
# non-isom trees	1	1	1	2	3	6	11		19320

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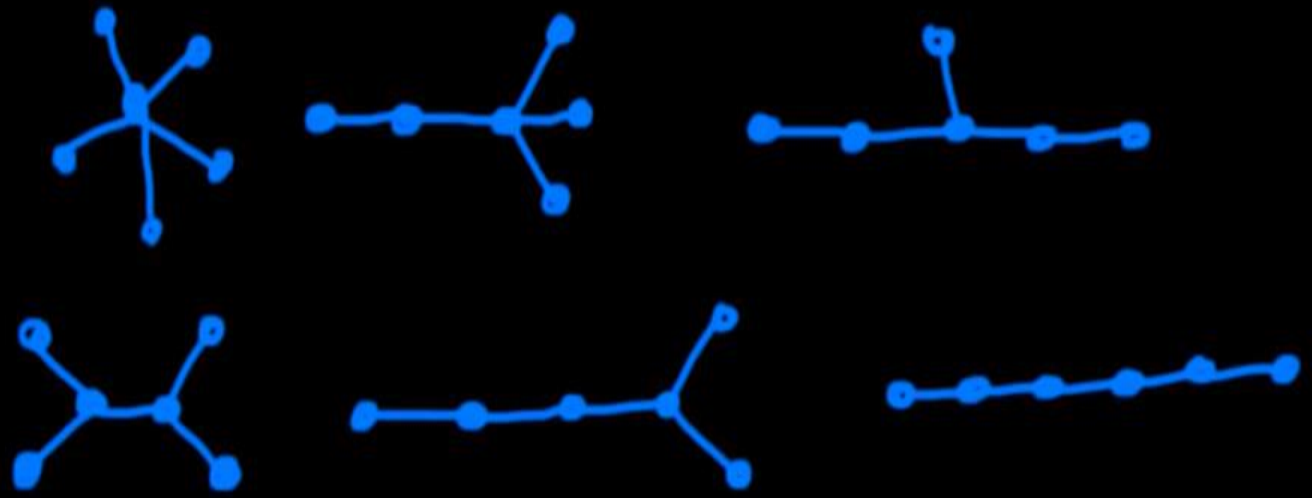
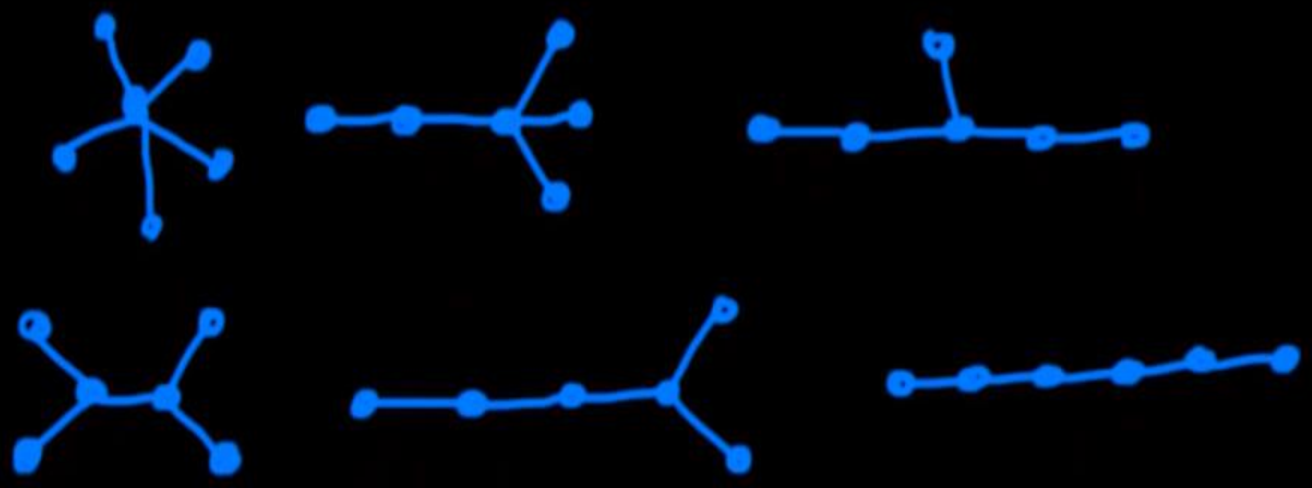
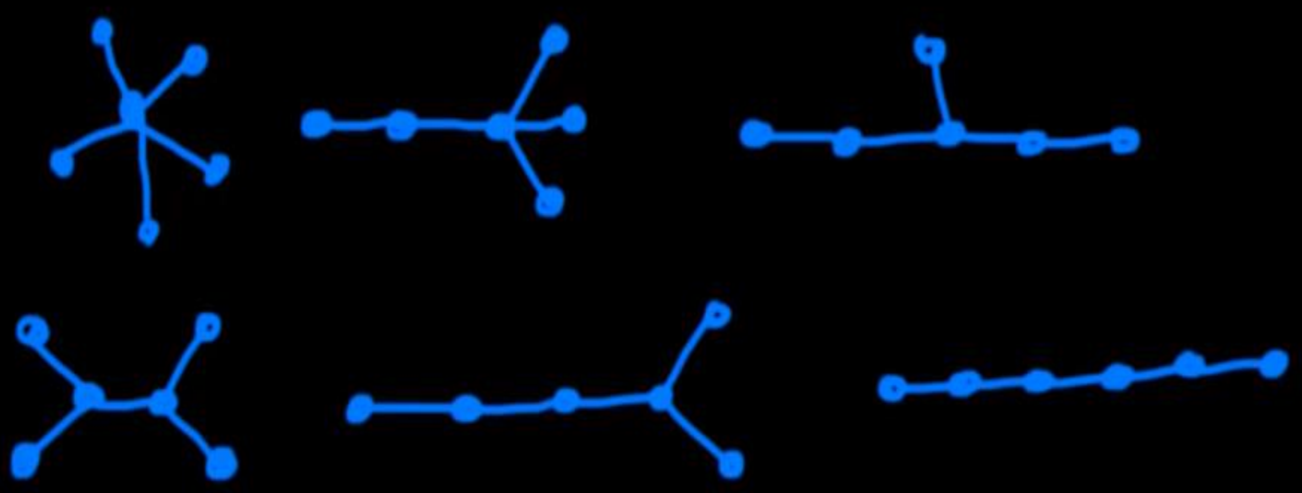




Let T be a tree. If $v \in V(T)$ and $d(v) = 1$ then v is a <u>leaf</u>



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<u>Fact</u>: Every tree T with $|V(T)| \geq 2$ has at least 2 leaves.

↳ Take a maximal path T.
Each end-vertex of this path is a leaf.

More about trees :)

acyclic
connected

Theorem: A graph G is a tree $\iff$ G is acyclic and $|E(G)| = |V(G)| - 1$

$m = n - 1$

$\Rightarrow$] G is a tree

acyclic ✓ (by def'n)

Base: $n = 1$ • $m = 0$ ✓

Ind Hyp: Suppose all trees of order n have $n-1$ edges $(n \geq 1)$

Let T be a tree of order $\boxed{n+1}$

$\exists$ at least 2 leaves!

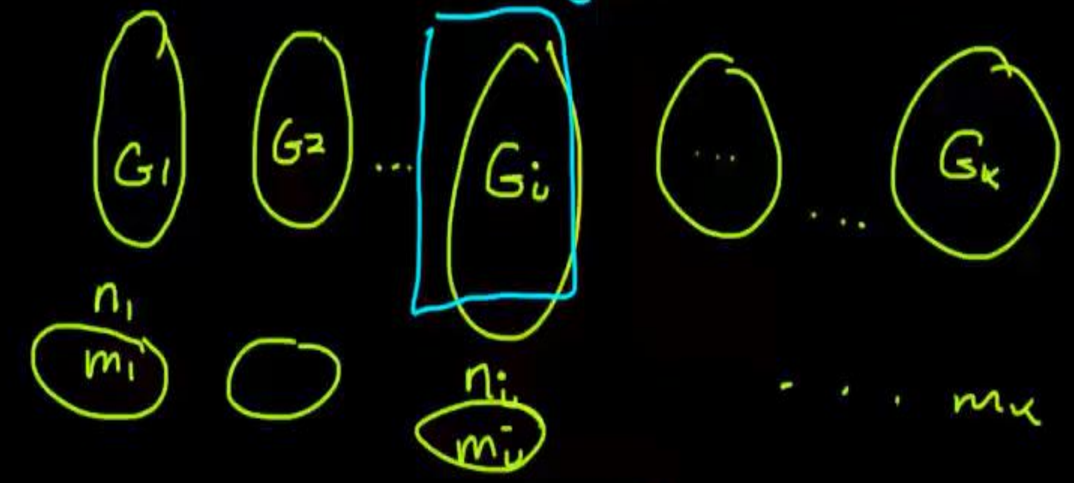
$v \in V(T)$ with degree 1 (leaf)

$\underbrace{T'}_{n} = T - v$

$\therefore T'$ has $n-1$ edges

T has $(n-1)+1$ edges $= n$ edges

⇐) Suppose G is acyclic and $\underline{m} = n-1$



connected components

$$\left.\begin{array}{l}\text{acyclic}\\ \text{connected}\end{array}\right\} \begin{array}{l} G_i \text{ is a tree} \\ m_i = n_i - 1 \end{array}$$

$$m = \sum_{i=1}^{k} m_i = \sum_{i=1}^{k} n_i - 1 = n - k$$

$n - 1 = n - k \Rightarrow k = 1 \Rightarrow$ 1 connected component

G is a tree $\square$

Thank you