Asymptotic Notations

$1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < \cdots < 2^n < 3^n < \cdots < n^n$

O	big-oh	upper bound
Ω	big-omega	Lower bound
Θ	theta	Average bound

The function $f(n)=O(g(n))$ iff $\exists$ +ve constants c and n_0

such that $f(n) \leq c * g(n) \;\forall\; n \geq n_0$

eg: $f(n)=2n+3$

The function $f(n) = O(g(n))$ iff $\exists$ +ve constants c and n_0

such that $f(n) \leq c * g(n) \quad \forall \; n \geq n_0$

eg: $f(n) = 2n+3$

$$\underbrace{2n+3}_{f(n)} \leq \underbrace{10}_{c}\,\underbrace{n}_{g(n)} \quad n \geq 1$$

$\therefore f(n) = O(n)$

$< n < n\log n < n^2 < n^3 < \cdots < 2^n < 3^n < \cdots < n^n$

avg bound

upper bound

Big-Oh

The function $f(n) = O(g(n))$ iff $\exists$ +ve constants c and n_0

such that $f(n) \leq c * g(n) \;\forall\; n \geq n_0$

eg: $f(n) = 2n+3$

$2n+3 \leq 2n^2 + 3n^2$

$\underbrace{2n+3}_{f(n)} \leq \underbrace{5}_{c}\,\underbrace{n^2}_{g(n)} \quad n \geq 1$

$\therefore f(n) = O(n)$

$f(n) = O(n^2)$

$f(n) = O(2^n)$

$f(n) = O(\log n)$

Omega

The function $f(n) = \Omega(g(n))$ iff $\exists$ +ve constants c and n_0

such that $f(n) \geq c * g(n) \ \forall \ n \geq n_0$

$< \sqrt{n} < n < n\log n < n^2 < n^3 < \cdots < 2^n < 3^n < \cdots < n^n$

ound avg bound upper bound

Omega

The function $f(n) = \Omega(g(n))$ iff $\exists$ +ve constants c and n_0

such that $f(n) \geq c * g(n) \;\; \forall\, n \geq n_0$

eg: $f(n) = 2n+3$ $\qquad \therefore f(n) = \Omega(n)$

$2n+3 \geq 1 \times n \quad \forall\, n \geq 1$

$f(n) \qquad c \qquad g(n)$

$1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < \cdots < 2^n < 3^n < \cdots < n^n$

Lower bound

avg bound

upper bound

Omega

The function $f(n) = \Omega(g(n))$ iff $\exists$ +ve constants c and n_0

such that $f(n) \geq c * g(n)\ \forall\ n \geq n_0$

eg: $f(n) = 2n + 3$

$\therefore f(n) = \Omega(n)$

$\underset{f(n)}{2n+3} \geq \underset{c}{1} \times \underset{g(n)}{\log n}\ \forall\ n \geq 1$

$f(n) = \Omega(\log n)$

$\times\ f(n) = \Omega(n^2)$

$1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < \cdots < 2^n < 3^n < \cdots < n^n$

Lower bound avg bound upper bound

Theta Notation

The function $f(n) = \Theta(g(n))$ iff $\exists$ +ve constants c_1, c_2 and n_0

such that $c_1 * g(n) \le f(n) \le c_2 * g(n)$

eg: $f(n) = 2n+3$

$$\underbrace{1}_{c_1} \times \underbrace{n}_{g(n)} \le \underbrace{2n+3}_{f(n)} \le \underbrace{5}_{c_2} \times \underbrace{n}_{g(n)}$$

$\therefore\ f(n) = \Theta(n)$

$f(n) = \cancel{\Theta}(n^2)$

$1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < \cdots < 2^n < 3^n \cdots < n^n$

$f(n) = 2n^2 + 3n + 4$ $\qquad f(n) = O(n^2)$

$2n^2 + 3n + 4 \leq 2n^2 + 3n^2 + 4n^2$

$2n^2 + 3n + 4 \leq \underset{c}{9}\,\underset{g(n)}{n^2} \quad n \geq 1$

$$1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < \cdots < 2^n < 3^n \cdots < n^n$$

$$f(n) = 2n^2 + 3n + 4$$

$$2n^2 + 3n + 4 \geq 1 \times n^2 \quad \Omega(n^2)$$

$$1 \times n^2 \leq 2n^2 + 3n + 4 \leq 9n^2 \quad \Theta(n^2)$$

$$1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < \cdots < 2^n < 3^n \cdots < n^n$$

$$f(n) = n^2 \log n + n$$

$$1 \times n^2 \log n \le n^2 \log n + n \le 10\, n^2 \log n$$

$$O(n^2 \log n) \quad \Omega(n^2 \log n)$$

$$\Theta(n^2 \log n)$$

$$1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < \cdots < 2^n < 3^n \cdots < n^n$$

$$f(n) = n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$$

$$1 \times 1 \times 1 \times \cdots \times 1 \leq 1 \times 2 \times 3 \cdots \times n \leq n \times n \times n \times \cdots \times n$$

$$1 \leq n! \leq n^n$$

$$\Omega(1) \quad O(n^n)$$

$$1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < \cdots < 2^n < 3^n \cdots < n^n$$

$$f(n) = n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$$

$$1 \times 1 \times 1 \times 1 \times \cdots \times 1 \leq 1 \times 2 \times 3 \cdots \times n \leq n \times n \times n \times \cdots \times n$$

$$1 \leq n! \leq n^n$$

$$n^{10} < n! < n^{11}$$

$$\Omega(1) \quad O(n^n)$$

$$1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < \cdots < 2^n < 3^n \cdots < n^n$$

$$f(n) = \log n!$$

$$\log(1\times1\times\cdots 1) \leq \log(1\times2\times3\cdots\times n) \leq \log(n\times n\times\cdots\times n)$$

$$1 \leq \log n! \leq \log n^n$$

$$n\log n$$

$$\Omega(1) \quad O(n\log n)$$