1. **Dijkstra Algorithm with Examples**

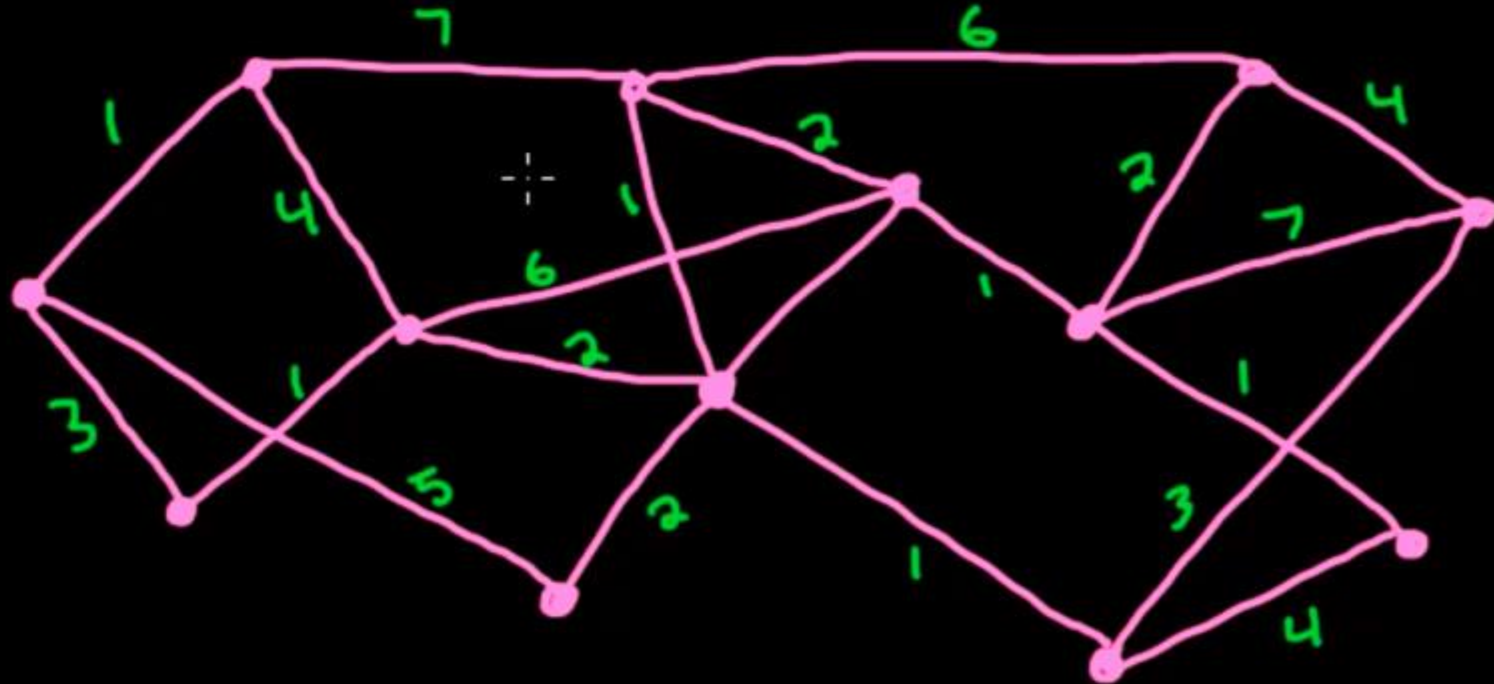
2. **Bellman Ford Algorithm with Examples**

3. **All Pairs shortest path (Warshal –Floyd) Algorithm**

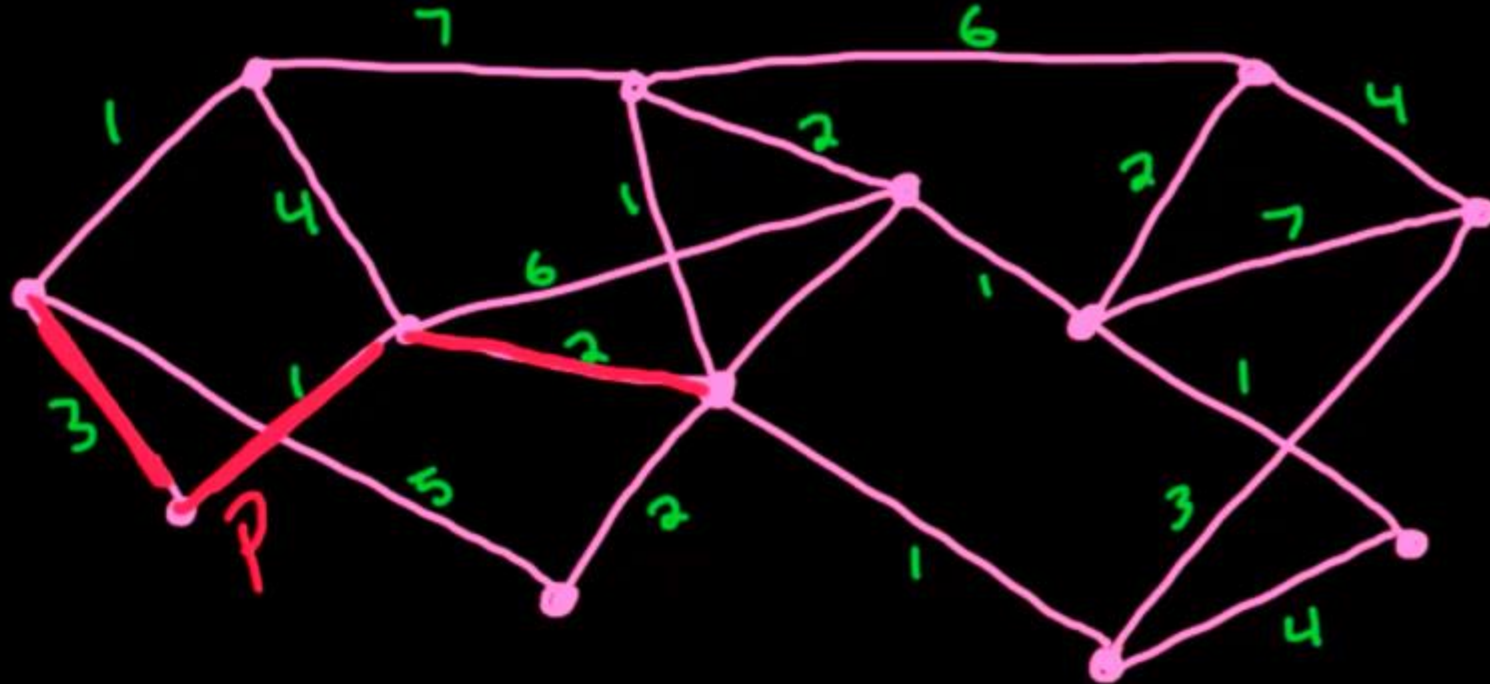
The Shortest Path Problem

Edge weighted Graph: a graph G together w

$$\alpha : E(G) \to \underbrace{\mathbb{N}}_{\{0,}$$



$\alpha : E(G) \to \underbrace{\mathbb{N}}_{\{0,1,}$



$\alpha(uv)$ called the length of uv

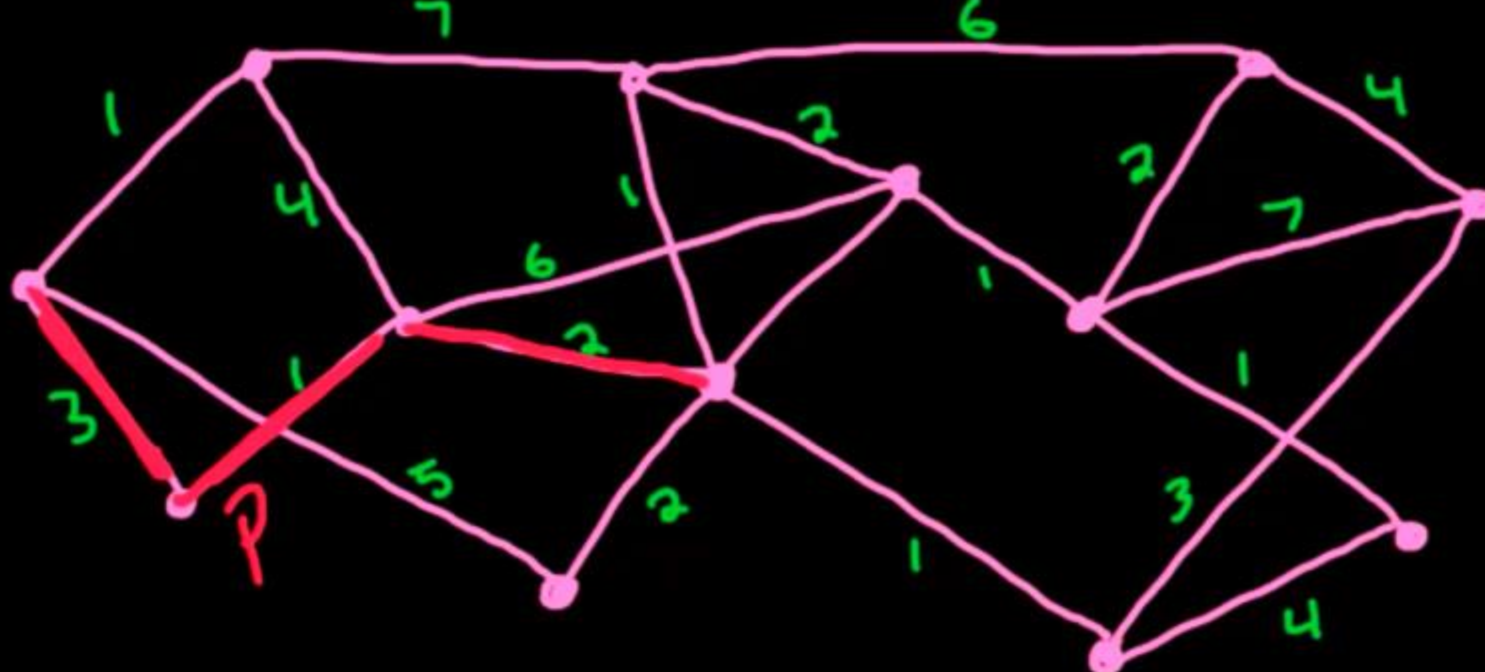
$\alpha(P) := \sum_{e \in P} \alpha(e)$

(for any

$\alpha(H) =$

$\alpha : E(G) \to \mathbb{N}$

$\{0,$



$\alpha(uv)$ called the length of uv

$\alpha(P) = \sum_{e \in P} \alpha(e)$

(for any

$\alpha(H) =$

$\alpha(uv)$ called the <u>length</u> of uv

$$\alpha(P) = \sum_{e \in P} \alpha(e)$$

(for any

$\alpha(H) =$

Shortest Path Problem: Given a conne
graph G with weight function
$\alpha : E(G) \to \mathbb{N}$, find for giv

$$d_G^{\alpha}(u,v) = \min\{\alpha(P) \mid P: u$$

For a connected edge weighted graph G and a source vertex $u \in V(G)$, find the minimum weight of a path from u to every other vertex in G. Let $\alpha : E(G) \to \mathbb{N}$ be the weight function.

Dijkstra's Algorithm:

1. Set $S = \emptyset$ which will store the vertices of G for which a shortest path has been found
2. Set $t(u) = 0$ and $t(v) = \infty$ for all $v \in V(G)$ $v \neq u$ and add u to S.
3. Let w be the newest member of S. For each $v \notin S$ $v \in N_G(w)$ set
$$t(v) = \min\{t(v),\ t(w) + \alpha(vw)\}$$
4. Pick any $w \notin S$ with minimum $t(w)$ and add w to S. Repeat step 3

function. … $\alpha: E(G) \to \mathbb{R}$ be the weight

Dijkstra's Algorithm:

① Set $S = \emptyset$ which will store the vertices of G for which a shortest path has been found

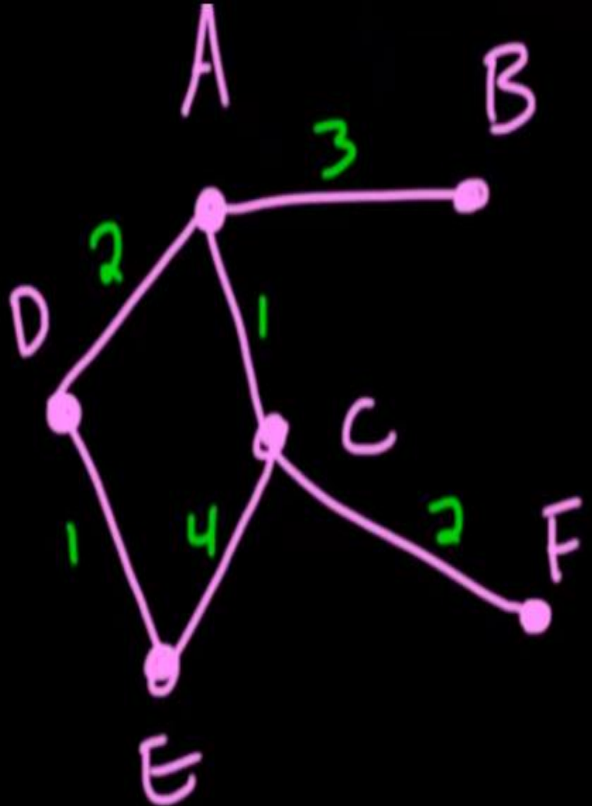
② Set $t(u) = 0$ and $t(v) = \infty$ for all $v \in V(G)$ $v \neq u$ and add u to S.

③ Let w be the newest member of S.
For each $v \notin S$ $v \in N_G(w)$ set

$$t(v) = \min\{\underline{t(v)}, \underline{t(w) + \alpha(vw)}\}$$

④ Pick any $w \notin S$ with minimum $t(w)$ and add w to S
Repeat step ③

Solution: For each $v \in S$, $t(v)$ is the minimum weight of a path in G with weight function α.



① $S = \emptyset$

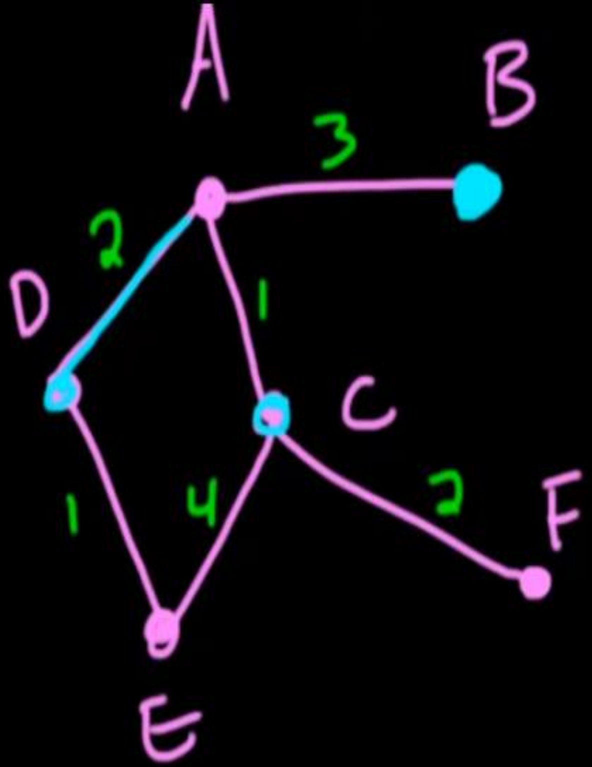
② $t(A) = 0$

$t(B) = t(C) = t(D) = t(E) = t(F) = \infty$

$S = \{A\}$

③

$$t(v) = \min\{\underline{t(v)},\ \underline{t(w) + \alpha(vw)}\}$$



① $S = \emptyset$

② $t(A) = 0$

$t(B) = t(C) = t(D) = t(E) = t(F) = \infty$

$S = \{A\}$

③ $t(B) = \min\{\infty,\ 0+3\} = 3$

$t(C) = \min\{t(C),\ t(A) + \alpha(AC)\}$

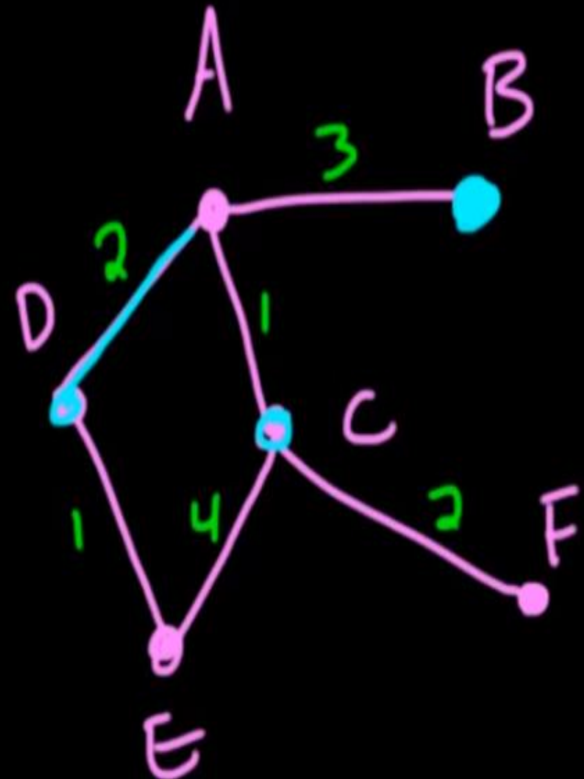
$= \min\{\infty,\ 0+1\} = 1$

$t(D) = \min\{\infty,\ 0+2\} = 2$

$$t(v) = \min\{\underbrace{t(v)},\ \underbrace{t(w) + \alpha(vw)}\}$$

$\alpha : E(G) \to \mathbb{N}$

Source vertex: A



① $S = \emptyset$

② $t(A) = 0$

$t(B) = t(C) = t(D) = t(E) = t(F) = \infty$

$S = \{A\}$

③ $t(B) = \min\{\infty,\ 0 + 3\} = 3$

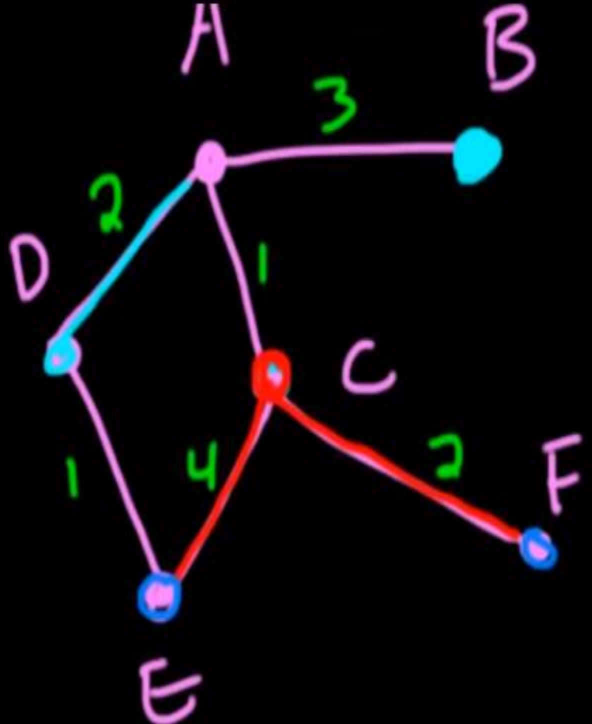
$t(C) = \min\{t(C),\ t(A) + \alpha(AC)\}$

$= \min\{\infty,\ 0 + 1\} = 1$

$t(D) = \min\{\infty,\ 0 + 2\} = 2$

④ Add C to S $\quad S = \{A, C\}$

$t(v) = \min\{\underbrace{t(v)}, \underbrace{t(w) + \alpha(vw)}\}$



① $S=\emptyset$

② $t(A)=0$

$t(B)=t(C)=t(D)=t(E)=t(F)=\infty$

$S=\{A\}$

③ $t(B)=\min\{\infty,\ 0+3\}=3$

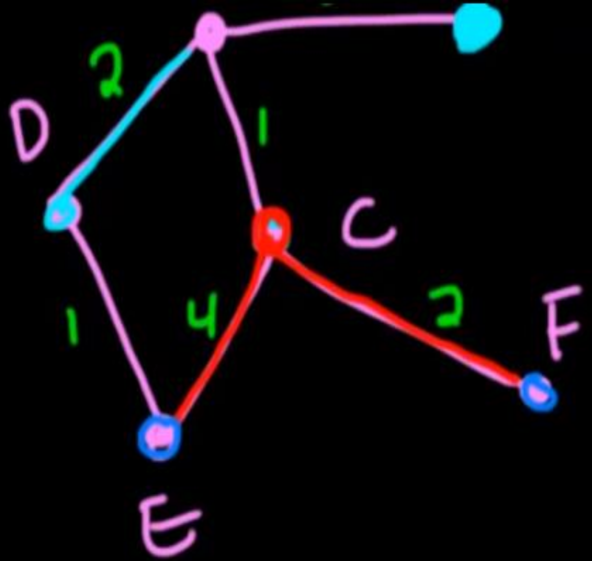
$t(C)=\min\{t(C),\ t(A)+\alpha(AC)\}$

$=\min\{\infty,\ 0+1\}=1$

$t(D)=\min\{\infty,\ 0+2\}=2$

④ Add C to S $\quad S=\{A,C\}$

③ $t(E)=\min\{\infty,\ t(C)+\alpha(CE)\}=\min\{\infty,\ 1+4\}=5$



(2) $t(A)=0$

$t(B)=t(C)=t(D)=t(E)=t(F)=\infty$

$S=\{A\}$

(3) $t(B)=\min\{\infty,\ 0+3\}=3$

$t(C)=\min\{t(C),\ t(A)+\alpha(AC)\}$

$=\min\{\infty,\ 0+1\}=1$

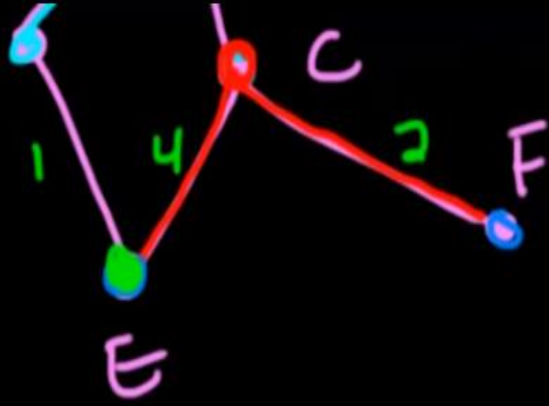
$t(D)=\min\{\infty,\ 0+2\}=2$

(4) Add C to S $\quad S=\{A,C\}$

(3) $t(E)=\min\{\infty,\ t(C)+\alpha(CE)\}=\min\{\infty,\ 1+4\}=5$

$t(F)=3$

(4) Add D to S $\quad S=\{A,C,D\}$



$S = \{A\}$

③ $t(B) = \min\{\infty, 0+3\} = 3$

$t(C) = \min\{t(C), t(A) + \alpha(AC)\}$

$= \min\{\infty, 0+1\} = 1$

$t(D) = \min\{\infty, 0+2\} = 2$

④ Add C to S $\quad S = \{A, C\}$

③ $t(E) = \min\{\infty, t(C) + \alpha(CE)\} = \min\{\infty, 1+4\} = 5$

$t(F) = 3$

④ Add D to S $\quad S = \{A, C, D\}$

③ $t(E) = \min\{t(E), t(D) + \alpha(DE)\} = \min\{5, 2+1\} = 3$

E

$t(C) = \min\{t(C), t(A) + \alpha(AC)\}$
$= \min\{\infty, 0 + 1\} = 1$

$t(D) = \min\{\infty, 0 + 2\} = 2$

④ Add C to S $\quad S = \{A, C\}$

③ $t(E) = \min\{\infty, t(C) + \alpha(CE)\} = \min\{\infty, 1 + 4\} = 5$
$t(F) = 3$

④ Add D to S $\quad S = \{A, C, D\}$

③ $t(E) = \min\{t(E), t(D) + \alpha(DE)\} = \min\{5, 2 + 1\} = 3$
Add B to S $\quad S = \{A, C, D, B\}$ no change

$= \min\{\infty,\ 0+1\} = 1$

$t(D) = \min\{\infty,\ 0+2\} = 2$

(4) Add C to S $\quad S = \{A, C\}$

(3) $t(E) = \min\{\infty, t(C) + \alpha(CE)\} = \min\{\infty, 1+4\} = 5$

$t(F) = 3$

(4) Add D to S $\quad S = \{A, C, D\}$

(3) $t(E) = \min\{t(E), t(D) + \alpha(DE)\} = \min\{5, 2+1\} = 3$

Add B to S $\quad S = \{A, C, D, B\}$ no change

Add E to S $\quad S = \{A, C, D, B, E\}$ no change

Add F to S $\quad S = \{A, C, D, B, E, F\}$ no change

(4) Add C to S

(3) $t(E) = \min\{\infty, t(C) + \alpha(CE)\} = \min\{$

$t(F) = 3$

(4) Add D to S $\quad S = \{A, C, D\}$

(3) $t(E) = \min\{t(E), t(D) + \alpha(DE)\} = \min$

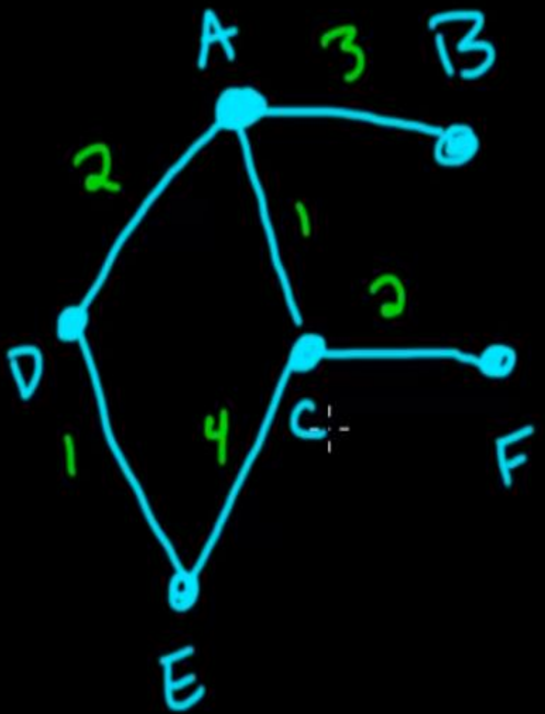
Add B to S $\quad S = \{A, C, D, B\}$ no

Add E to S $\quad S = \{A, C, D, B, E\}$ no

Add F to S $\quad S = \{A, C, D, B, E, F\}$

0 1 2 3 3 3

Use Dijkstra's Algorithm to find the shortest path weights for the following edge weighted graphs. Use A as the source vertex.



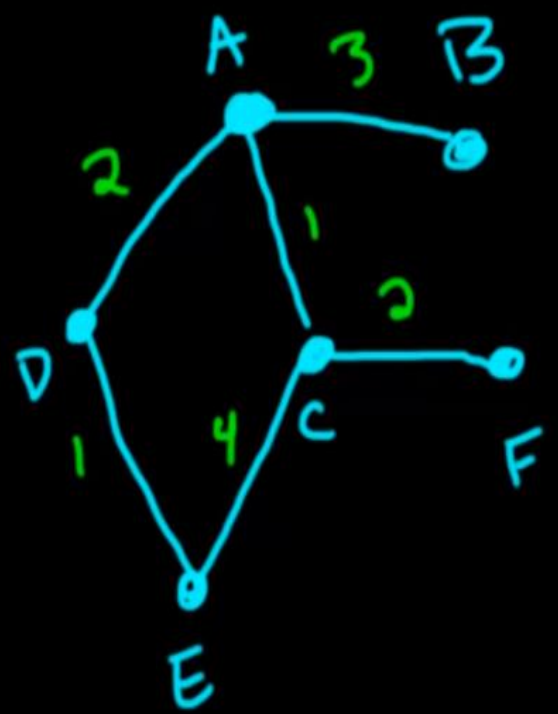
S

A	0
B	∞
C	∞
D	∞
E	∞
F	∞

Use Dijkstra's Algorithm to find the shortest path weights for the following edge weighted graphs. Use A as the source vertex.

			S
A	0		
B	∞	3	
C	∞	1	
D	∞	2	
E	∞	∞	
F	∞	∞	

Use Dijkstra's Algorithm to find the shortest path weights for the following edge weighted graphs. Use A as the source vertex.

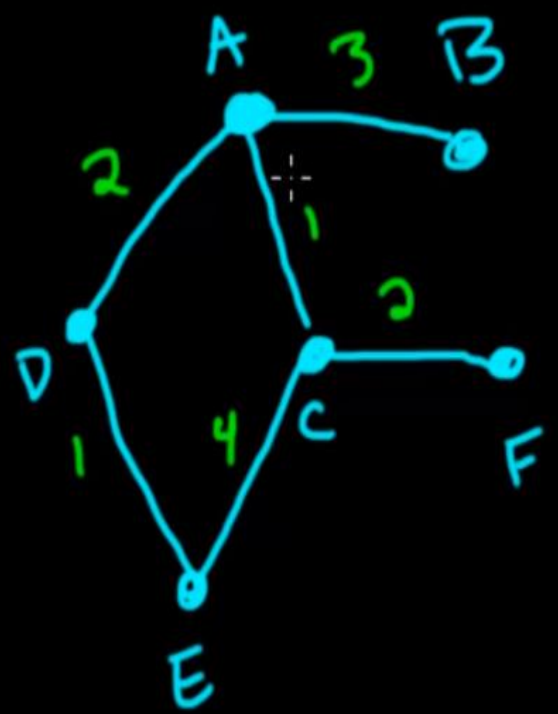


			S
A	0		
B	∞	3	
C	∞	1	
D	∞	2	
E	∞	∞	5
F	∞	∞	3

Use Dijkstra's Algorithm to find the shortest path weights for the following edge weighted graphs. Use A as the source vertex.

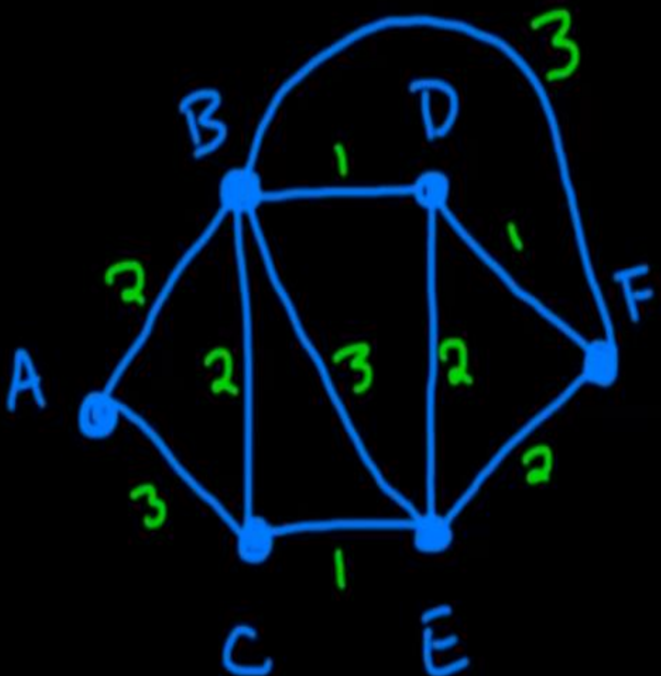
				S
A	0			
B	∞	3	3	3
C	∞	1		
D	∞	2	2	
E	∞	∞	5	3
F	∞	∞	3	3

Use Dijkstra's Algorithm to find the shortest path weights for the following edge weighted graphs. Use A as the source vertex.

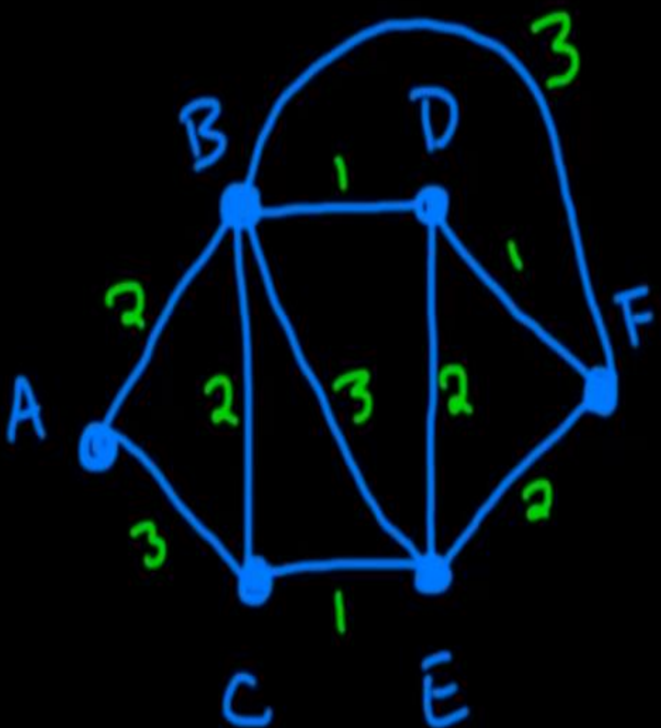


S

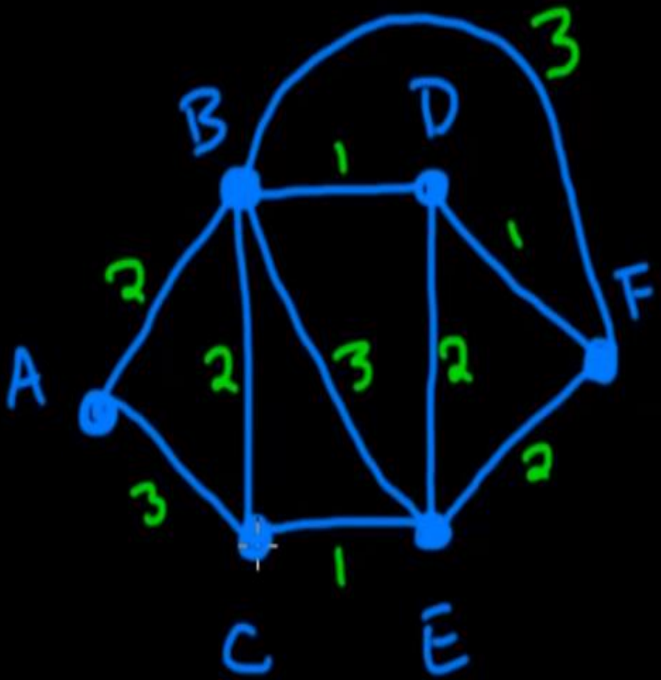
A	0					
B	∞	3	3	3		
C	∞	1				
D	∞	2	2			
E	∞	∞	5	3	3	
F	∞	∞	3	3	3	3



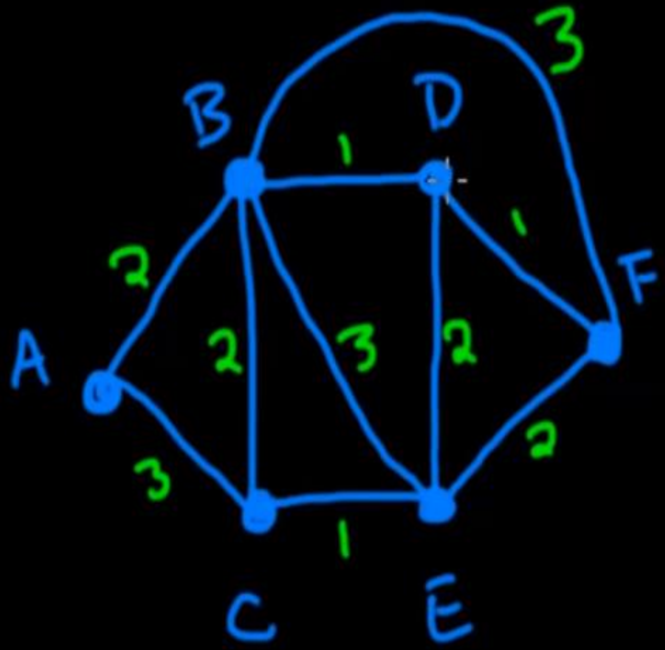
A	0
B	∞
C	∞
D	∞
E	∞
F	∞



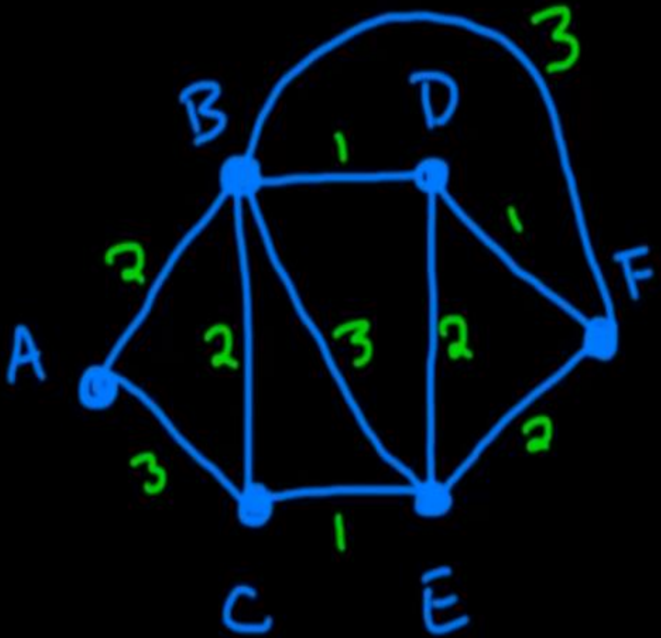
A	0	
B	∞	2
C	∞	3
D	∞	∞
E	∞	∞
F	∞	∞



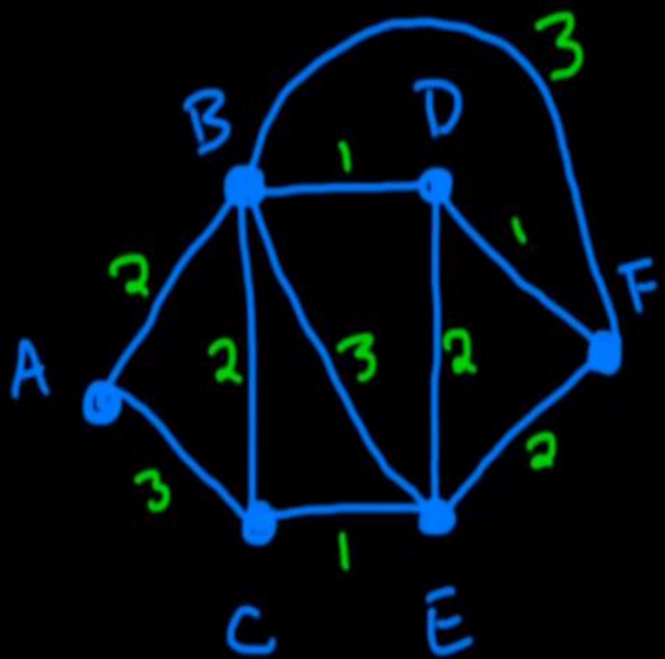
A	0		
B	∞	2	
C	∞	3	3
D	∞	∞	3
E	∞	∞	5
F	∞	∞	5



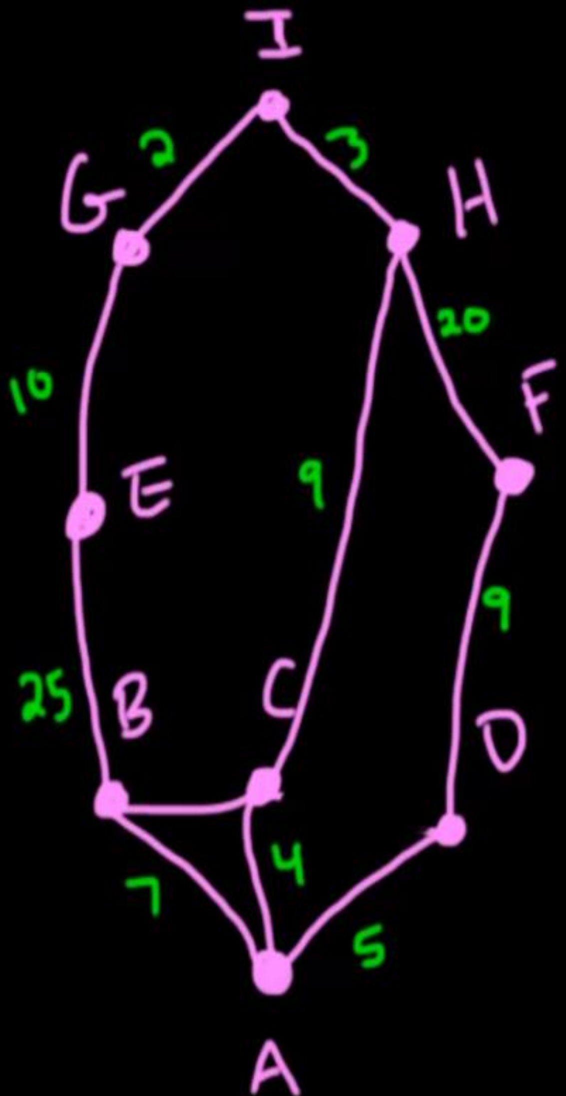
A	0			
B	∞	2		
C	∞	3	3	
D	∞	∞	3	3
E	∞	∞	5	4
F	∞	∞	5	5



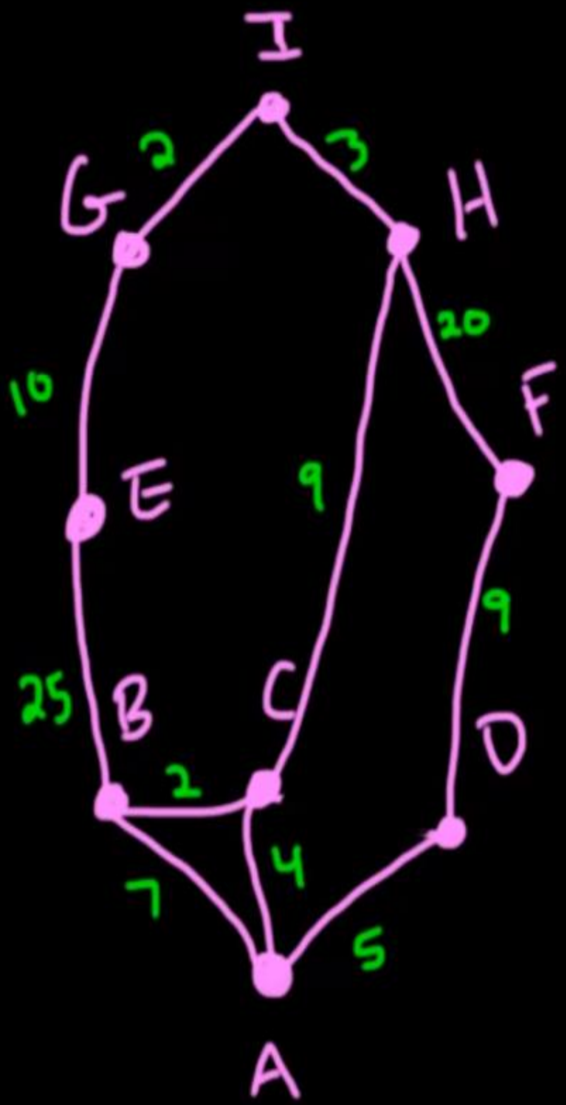
A	0				
B	∞	2			
C	∞	3	3		
D	∞	∞	3	3	
E	∞	∞	5	4	4
F	∞	∞	5	5	4



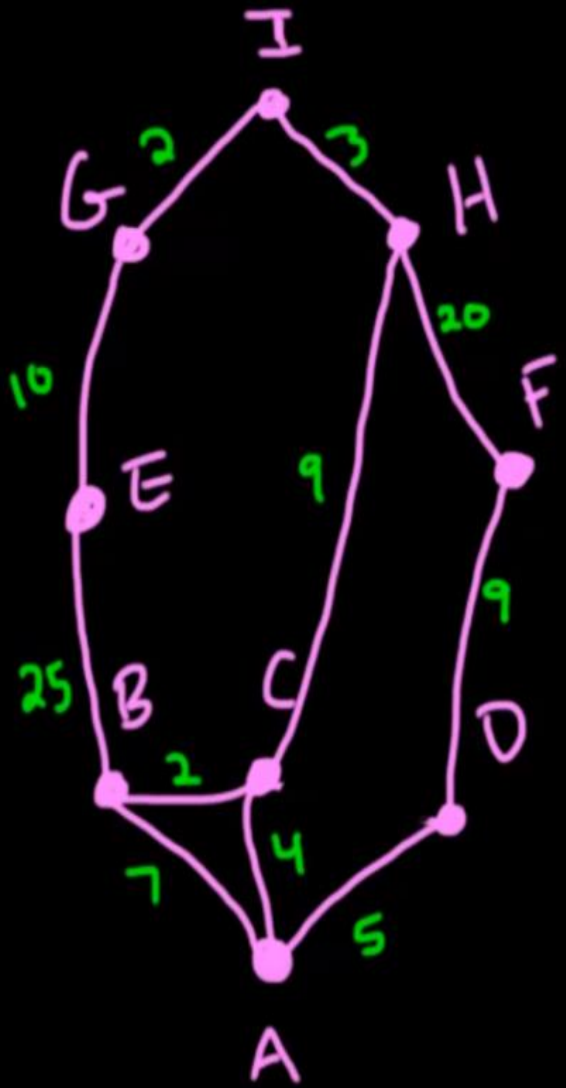
A	0					
B	∞	2				
C	∞	3	3			
D	∞	∞	3	3		
E	∞	∞	5	4	4	
F	∞	∞	5	5	4	4



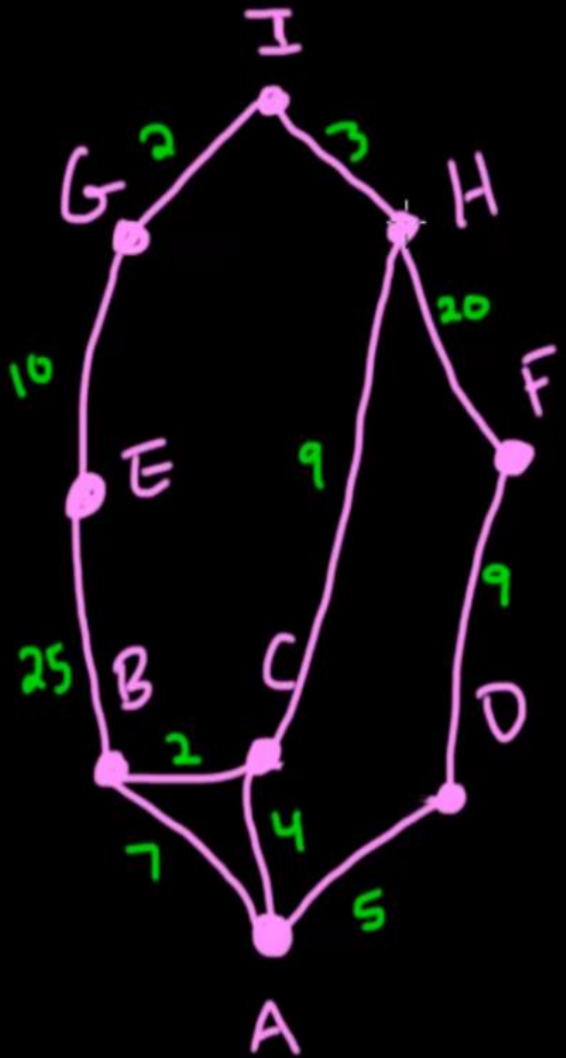
A	0
B	∞
C	∞
D	∞
E	∞
F	∞
G	∞
H	∞
I	∞



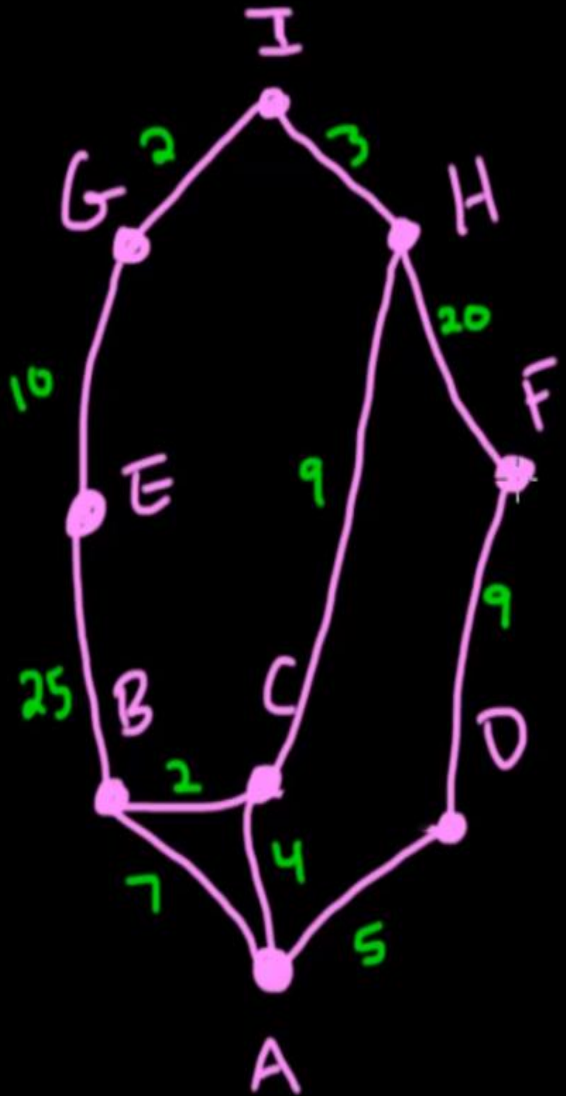
A	0	
B	∞	7
C	∞	4
D	∞	5
E	∞	∞
F	∞	∞
G	∞	∞
H	∞	∞
I	∞	∞



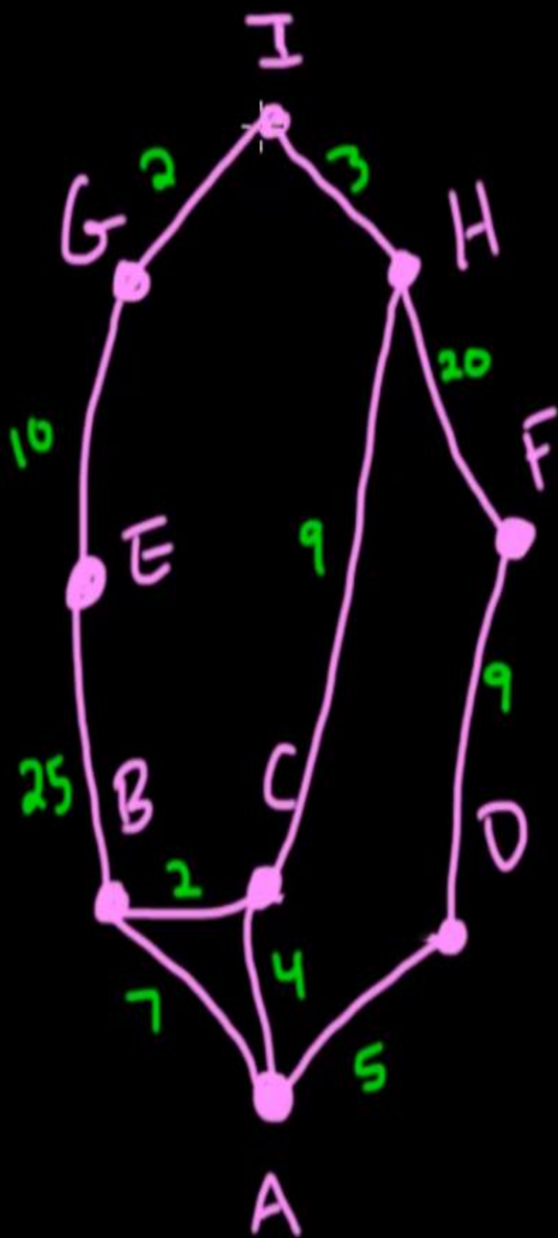
A	0		
B	∞	7	6
C	∞	4	
D	∞	5	5
E	∞	∞	∞
F	∞	∞	∞
G	∞	∞	∞
H	∞	∞	13
I	∞	∞	∞



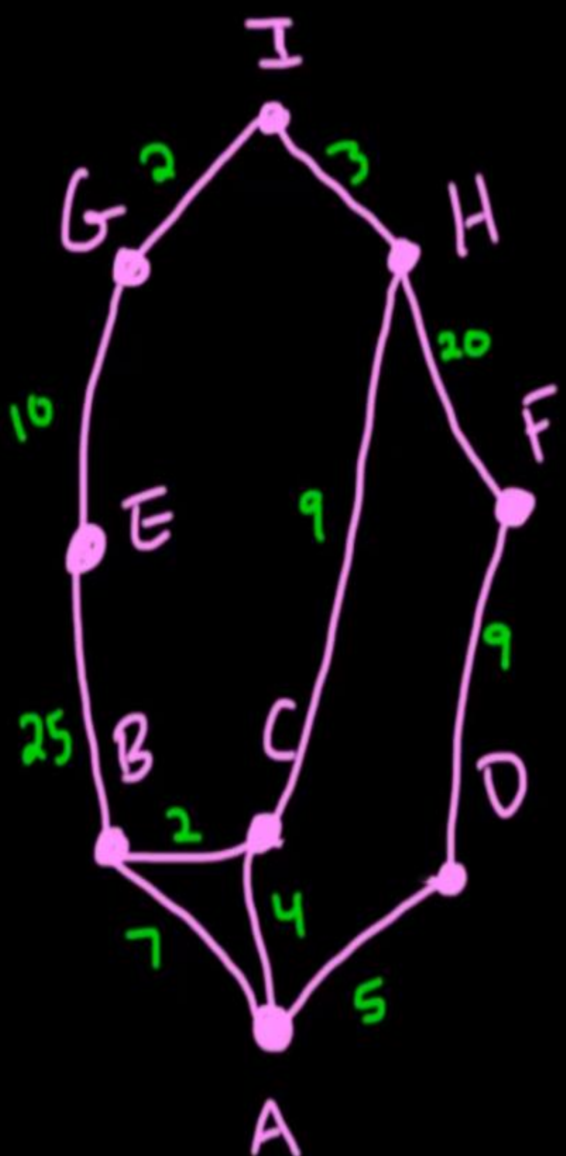
A	0				
B	∞	7	6	6	
C	∞	4			
D	∞	5	5		
E	∞	∞	∞	∞	31
F	∞	∞	∞	14	14
G	∞	∞	∞	∞	∞
H	∞	∞	13	13	13
I	∞	∞	∞	∞	∞



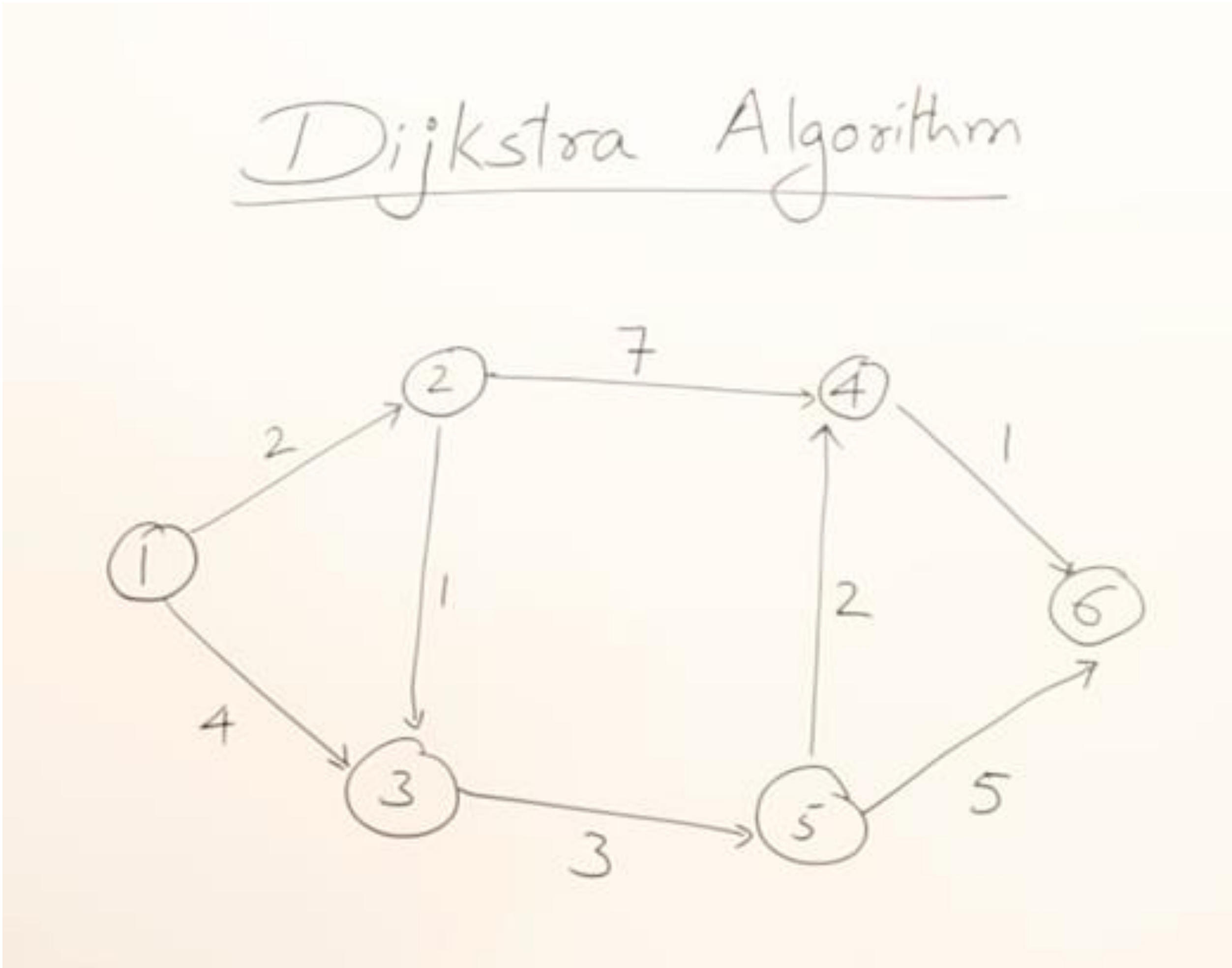
A	0					
B	∞	7	6	6		
C	∞	4				
D	∞	5	5			
E	∞	∞	∞	∞	31	31
F	∞	∞	∞	14	14	14
G	∞	∞	∞	∞	∞	∞
H	∞	∞	13	13	13	
I	∞	∞	∞	∞	∞	16



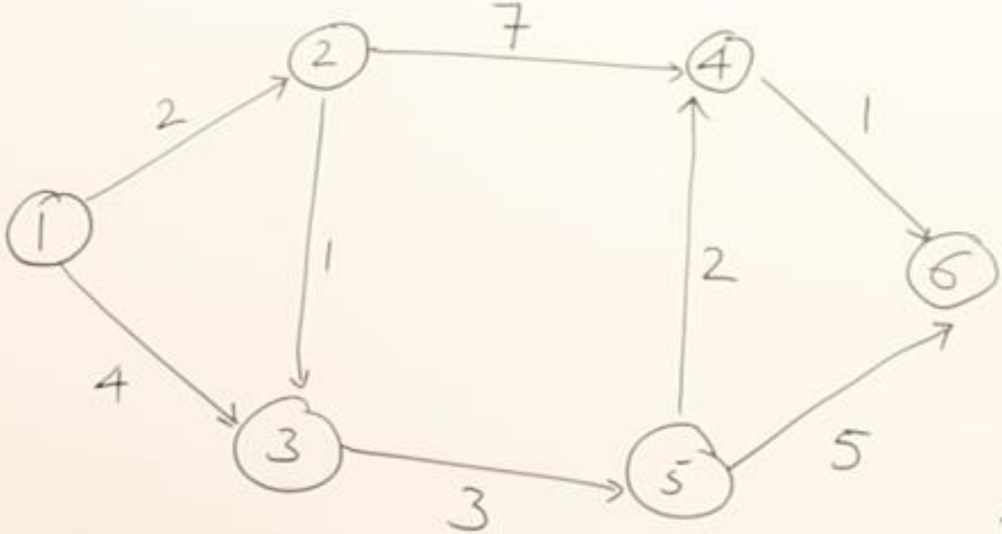
A	0						
B	∞	7	6	6			
C	∞	4					
D	∞	5	5				
E	∞	∞	∞	∞	31	31	31
F	∞	∞	∞	14	14	14	
G	∞	∞	∞	∞	∞	∞	∞
H	∞	∞	13	13	13		
I	∞	∞	∞	∞	∞	16	16

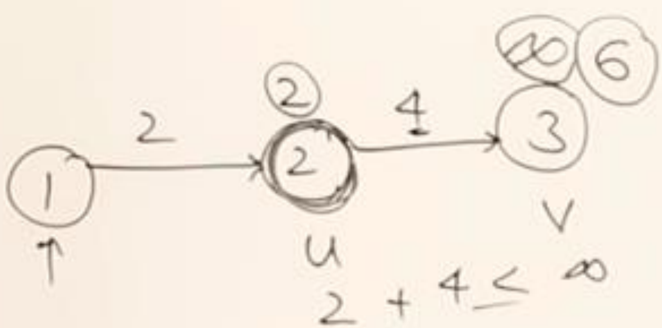


A	0								
B	∞	7	6	6					
C	∞	4							
D	∞	5	5						
E	∞	∞	∞	∞	31	31	31	31	28
F	∞	∞	∞	14	14	14			
G	∞	∞	∞	∞	∞	∞	∞	18	
H	∞	∞	13	13	13				
I	∞	∞	∞	∞	∞	16	16		


Dijkstra Algorithm
1
2
3
4
5
6
2
4
7
1
3
2
1
5

Dijkstra Algorithm

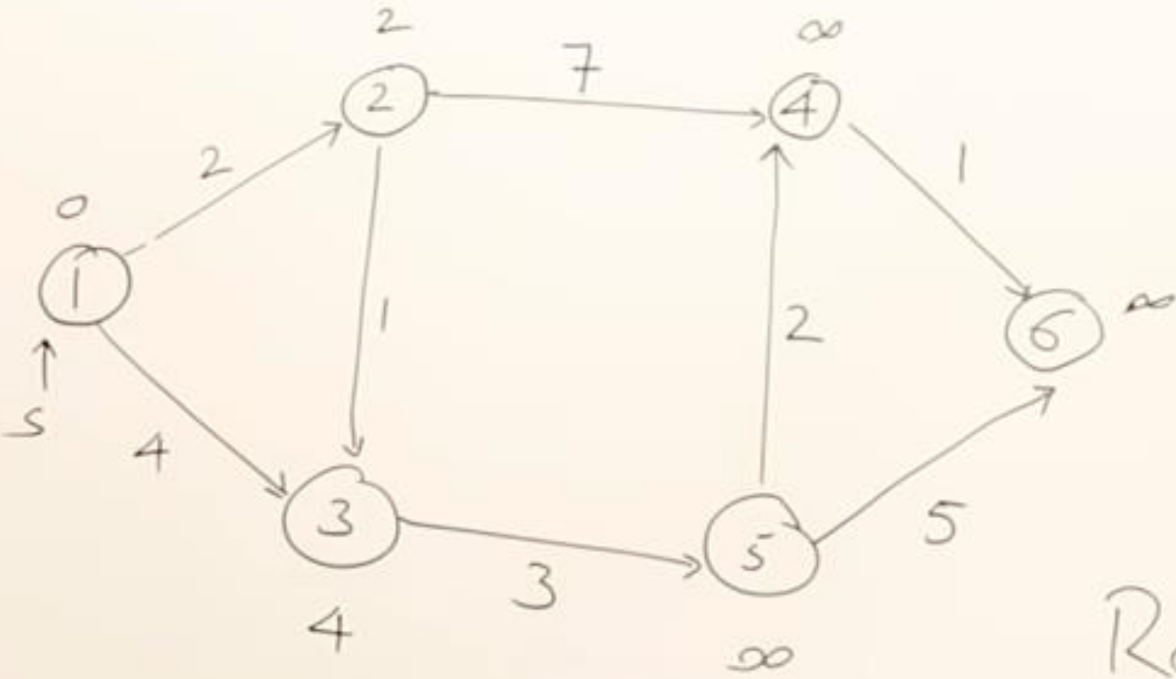




Relaxation

$$\text{if}\,(d[u] + c(u,v) < d[v])$$
$$d[v] = d[u] + c(u,v)$$

Dijkstra Algorithm

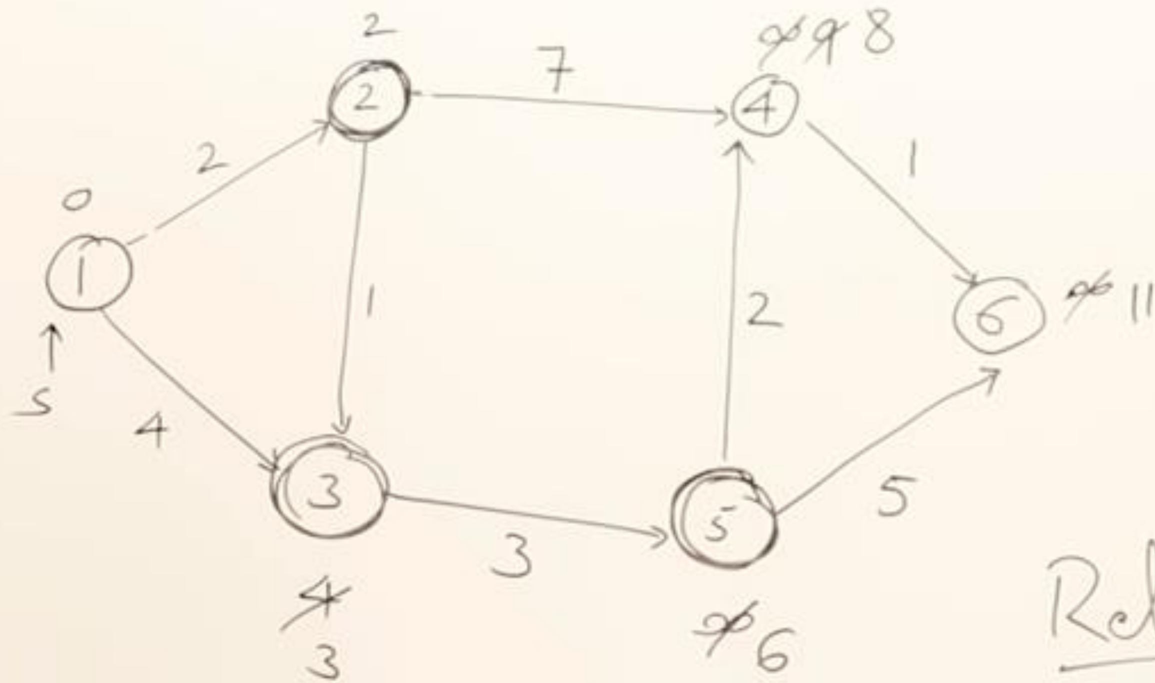


Relaxation

if $(d[u] + c(u,v) < d[v])$

$d[v] = d[u] + c(u,v)$

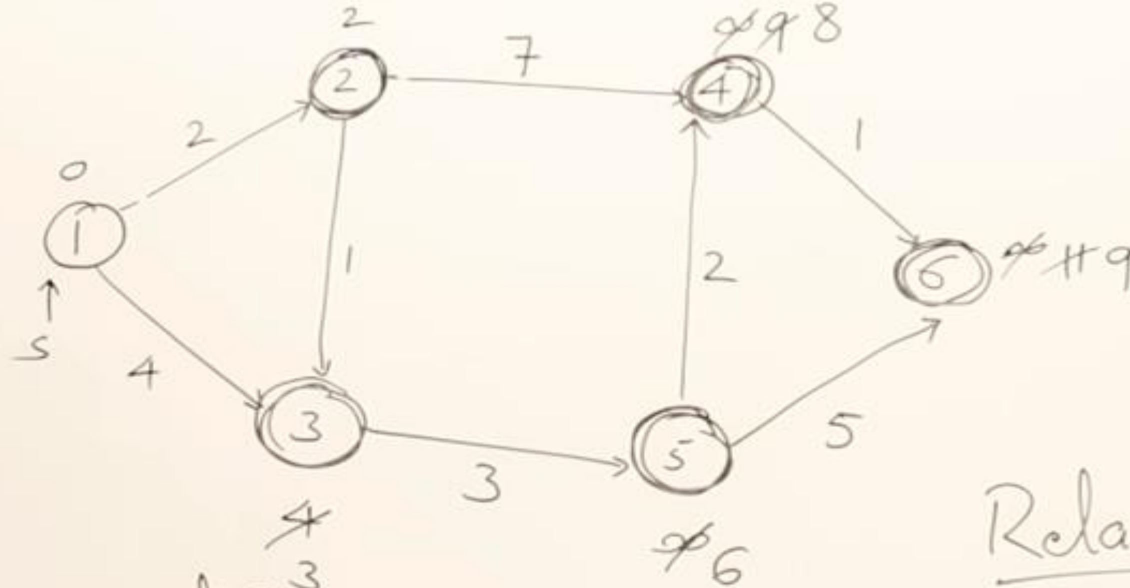
Dijkstra Algorithm



Relaxation

$$\text{if}\,(d[u]+c(u,v) < d[v])$$
$$d[v]=d[u]+c(u,v)$$

Dijkstra Algorithm

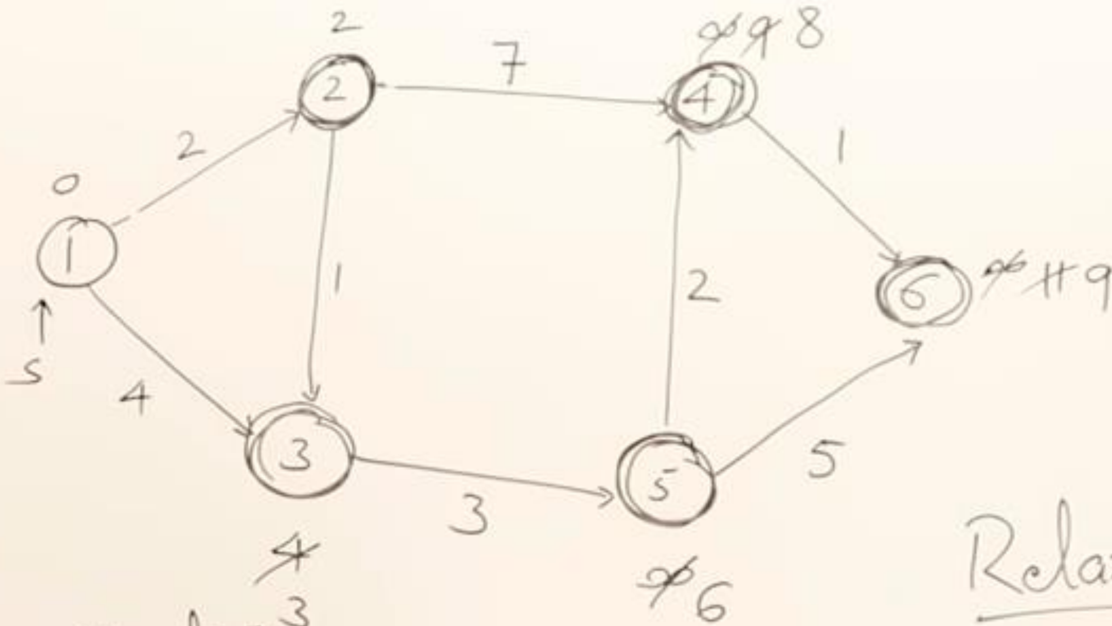


v	d[v]
2	2
3	3
4	8
5	6
6	9

Relaxation

$$\text{if}\,(d[u]+c(u,v) < d[v])$$
$$d[v]=d[u]+c(u,v)$$

Dijkstra Algorithm



v	d[v]
2	2
3	3
4	8
5	6
6	9

$n = |V| \quad |V|$

$n \times n = n^2 \quad O(|V|^2)$

$O(n^2)$

Relaxation

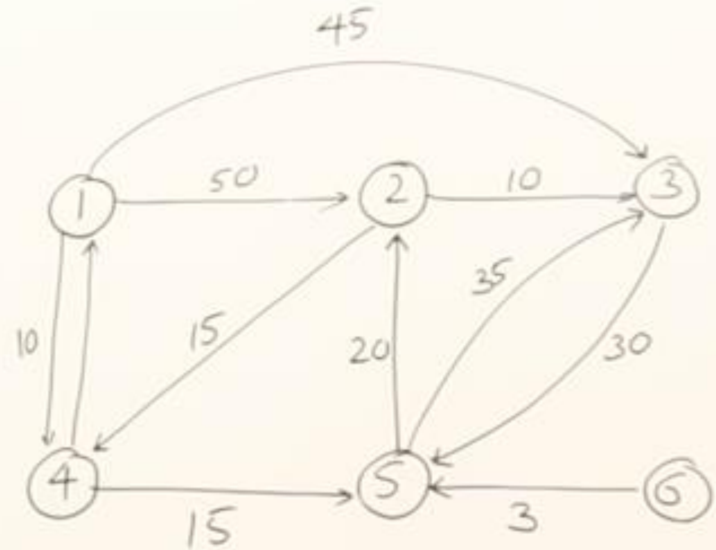
$\text{if}(d[u] + c(u,v) < d[v])$

$\quad d[v] = d[u] + c(u,v)$

Dijkstra Algorithm

starting vertex 1

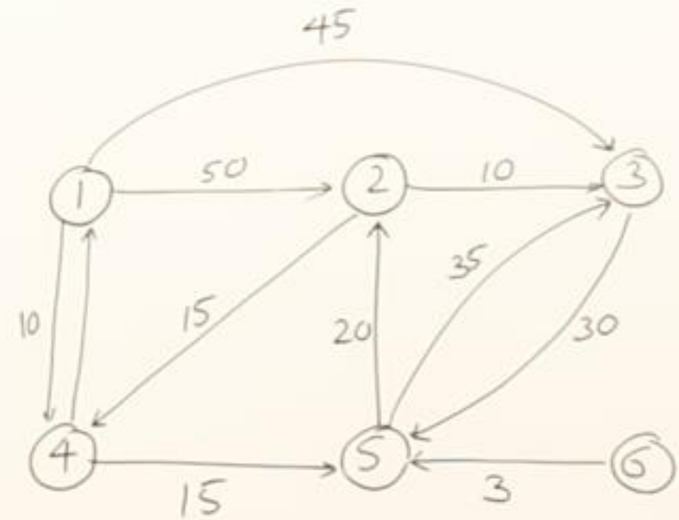
selected vertex	2	3	4	5	6
4	50	45	10	∞	∞
5	50	45	10	25	∞
	45	45	10	25	∞

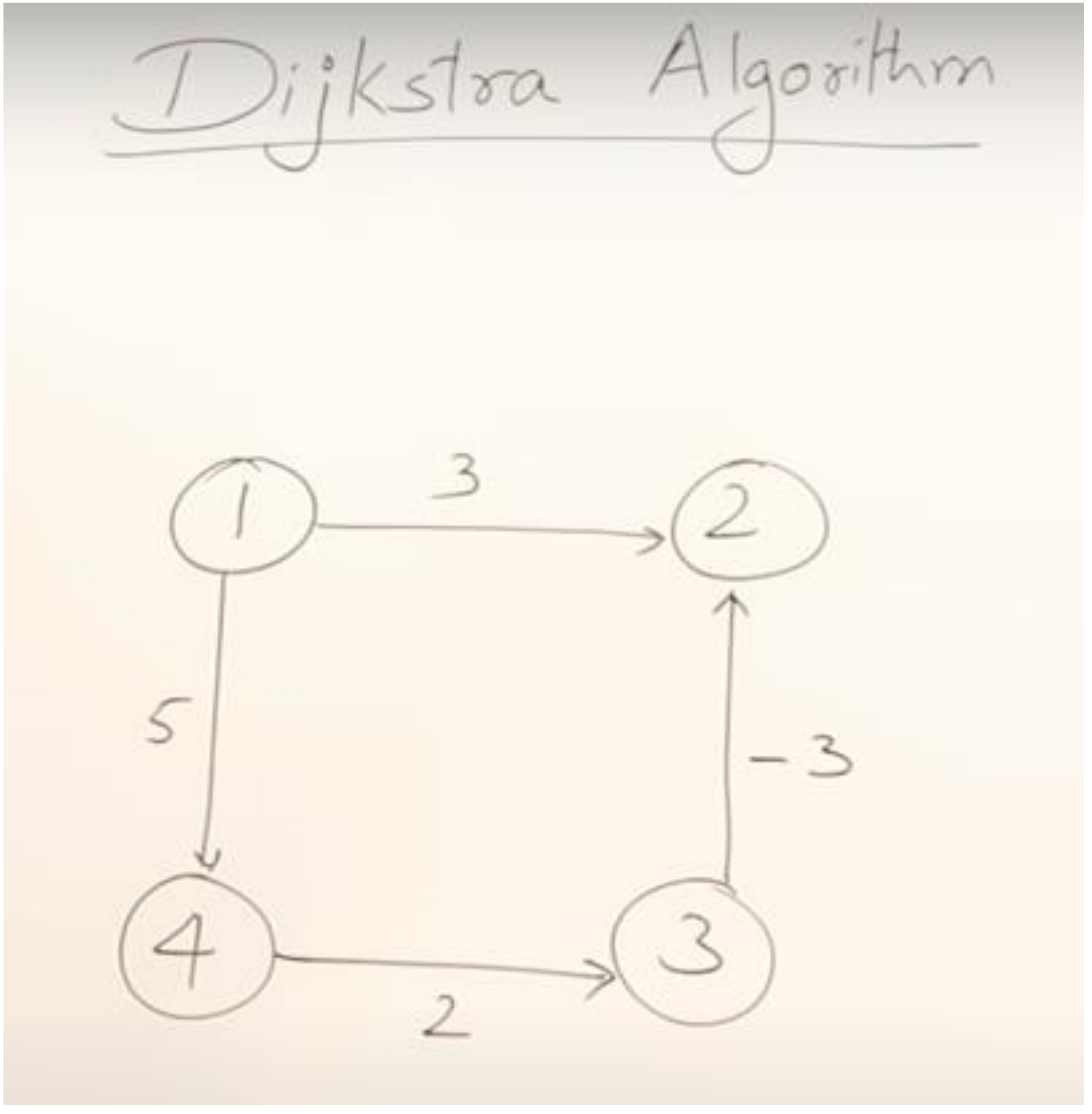


Dijkstra Algorithm

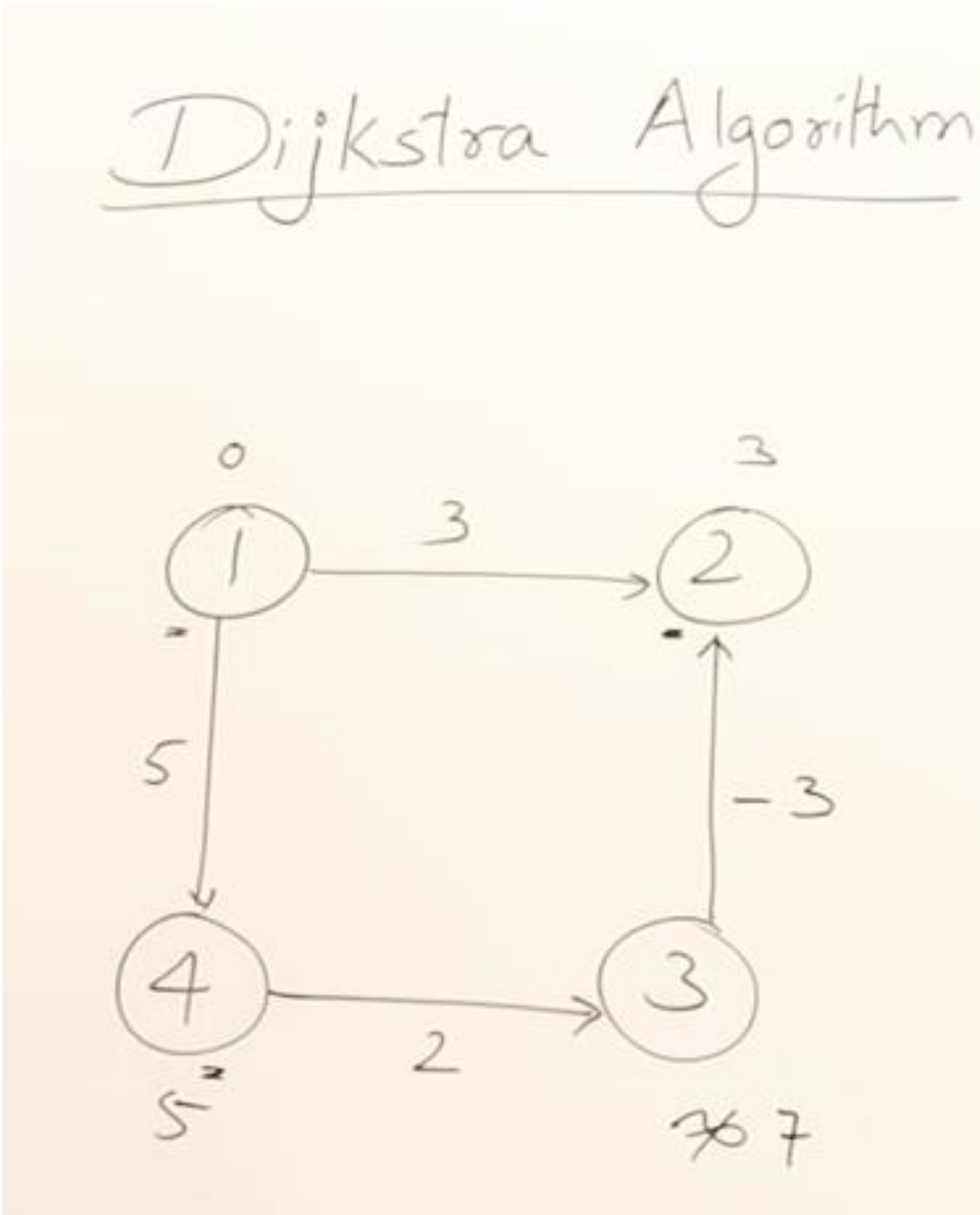
starting vertex 1

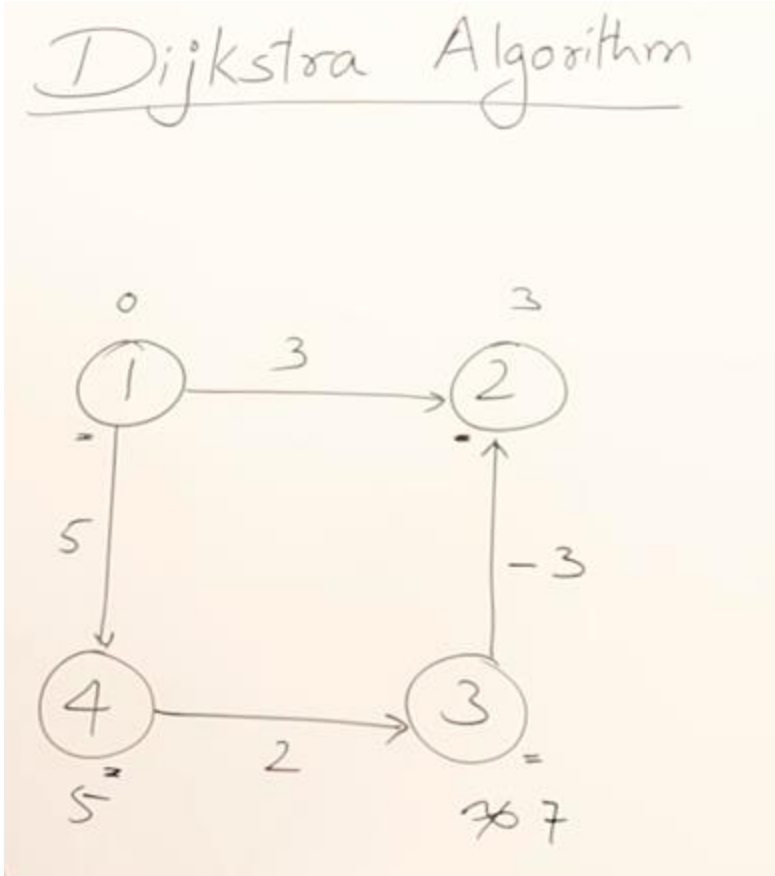
selected vertex	2	3	4	5	6
4	50	45	10	∞	∞
5	50	45	10	25	∞
2	45	45	10	25	∞
3	45	45	10	25	∞
6	45	45	10	25	∞



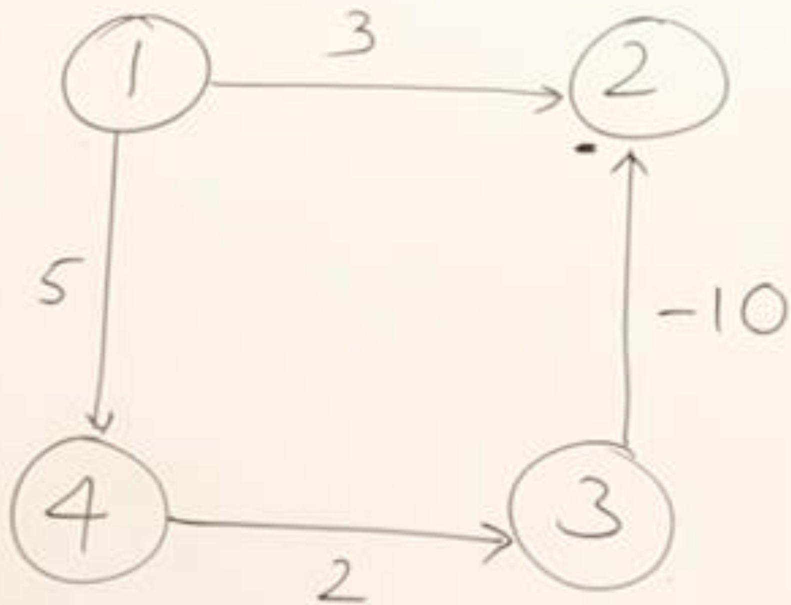


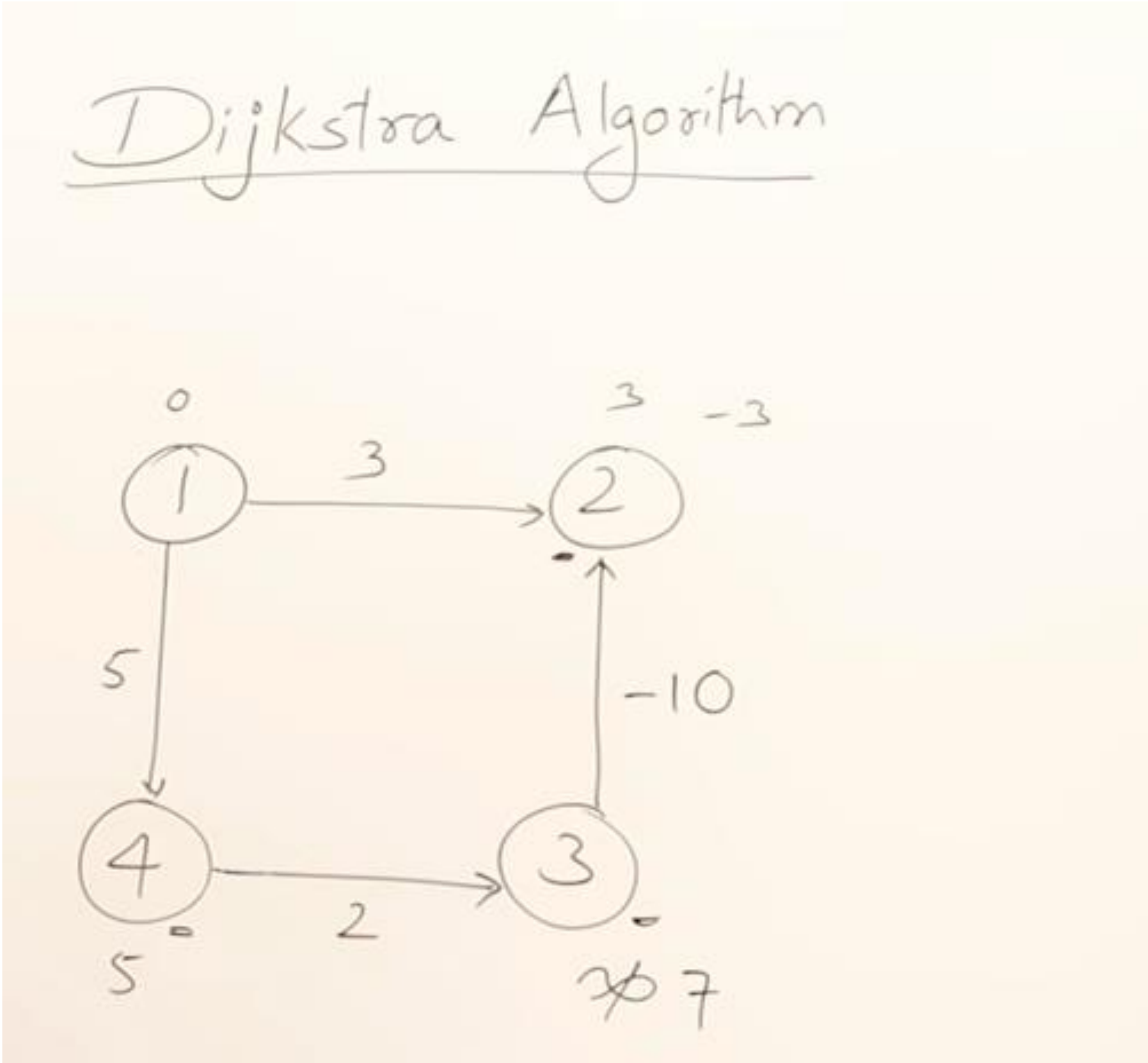
Negative weight (demerit)


Dijkstra Algorithm
0
3
1
3
2
5
-3
4
3
2
5
7


Dijkstra Algorithm
0
3
1
3
2
5
-3
4
3
2
5
7

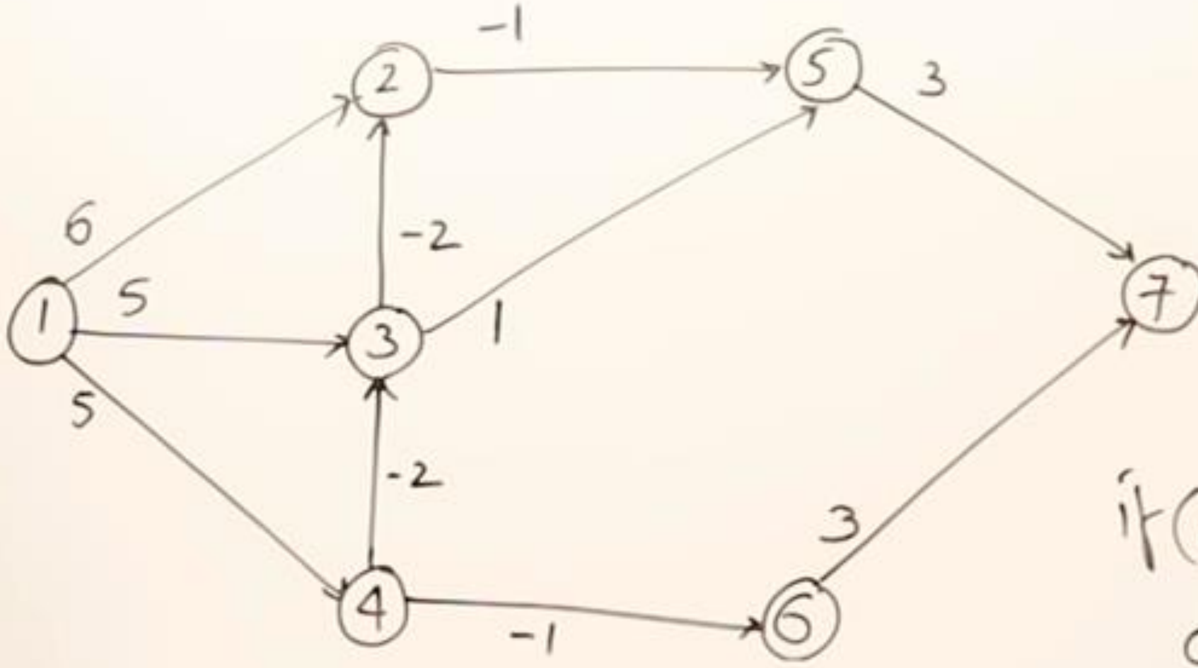
Dijkstra Algorithm




Dijkstra Algorithm
0
3
-3
1
3
2
5
-10
4
3
2
5
7

Single Source Shortest Path

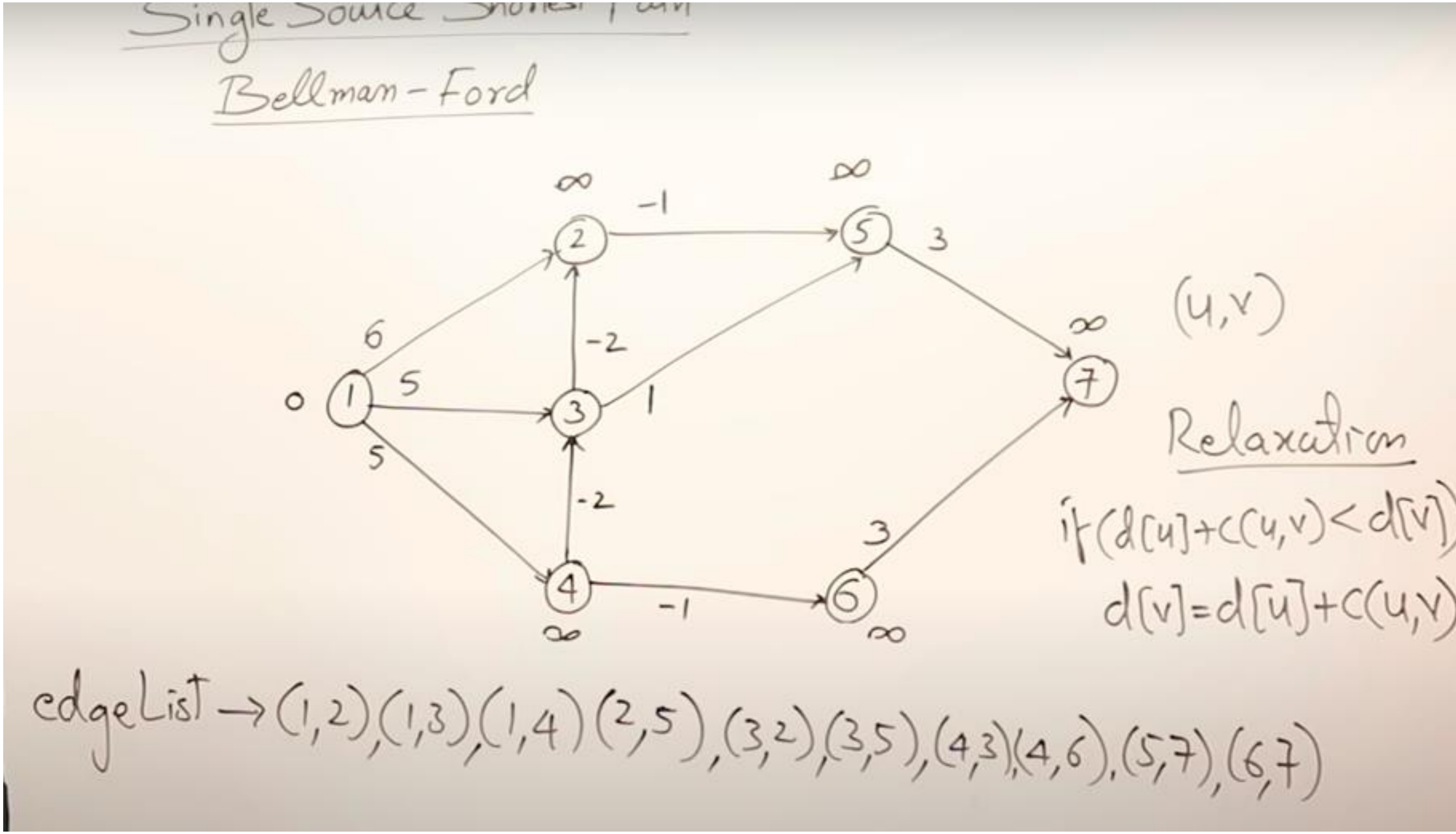
Bellman-Ford



(u,v)

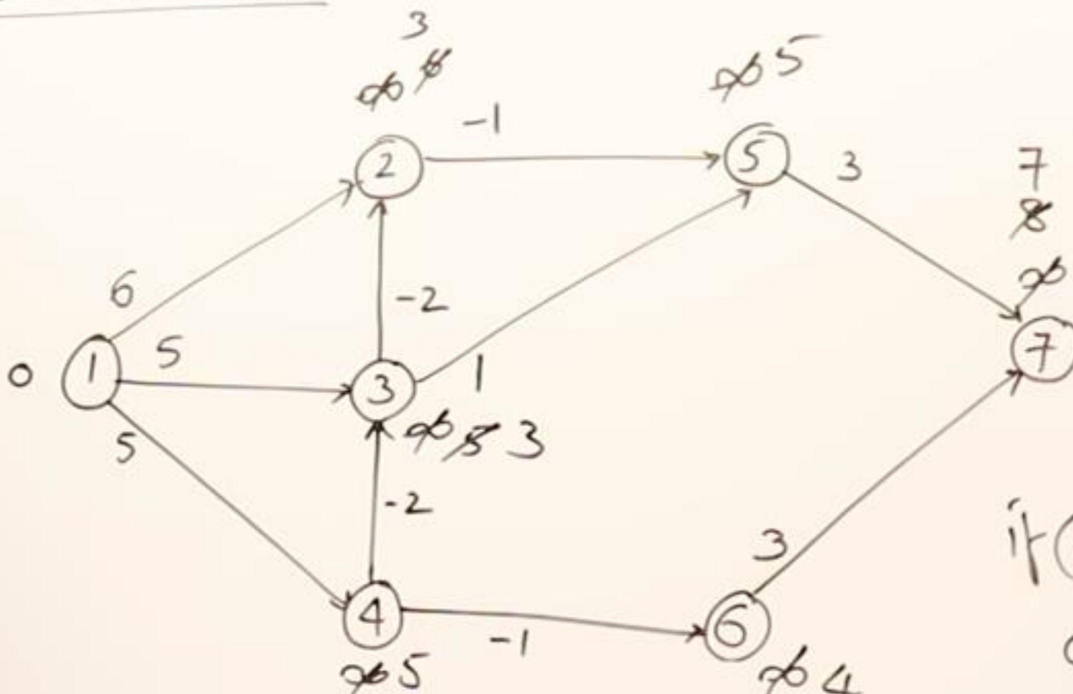
Relaxation

$$\text{if}(d[u]+c(u,v)<d[v])$$
$$d[v]=d[u]+c(u,v)$$


Single Source
Bellman-Ford
∞
∞
-1
2
5
3
6
-2
∞
(u,v)
0
1
5
3
1
7
5
Relaxation
-2
if (d[u]+c(u,v)<d[v])
3
4
-1
6
d[v]=d[u]+c(u,v)
∞
∞
edgeList → (1,2),(1,3),(1,4) (2,5),(3,2),(3,5),(4,3),(4,6),(5,7),(6,7)

Single Source Shortest Path

Bellman-Ford



(u,v)

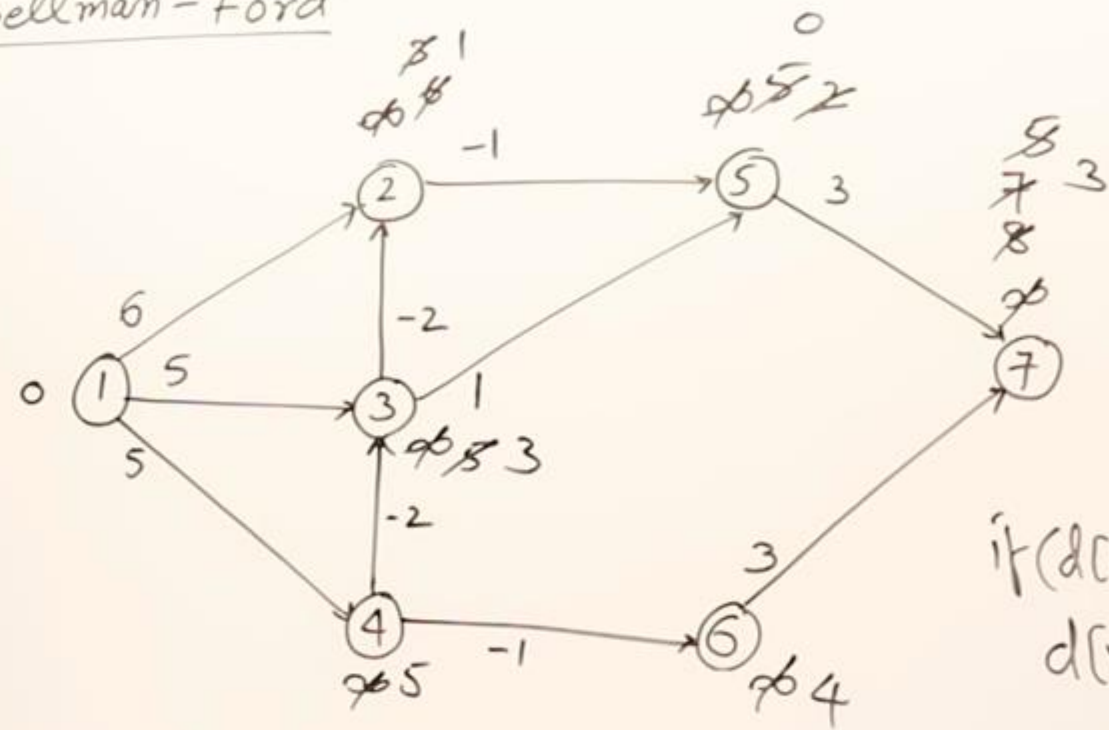
Relaxation

$$\text{if } (d[u]+c(u,v) < d[v])$$
$$d[v]=d[u]+c(u,v)$$

edgeList → (1 2), (1,3), (1,4) (2,5), (3,2), (3,5), (4,3), (4,6), (5,7), (6,7)

Single Source Shortest Path

Bellman-Ford



(u,v)

Relaxation

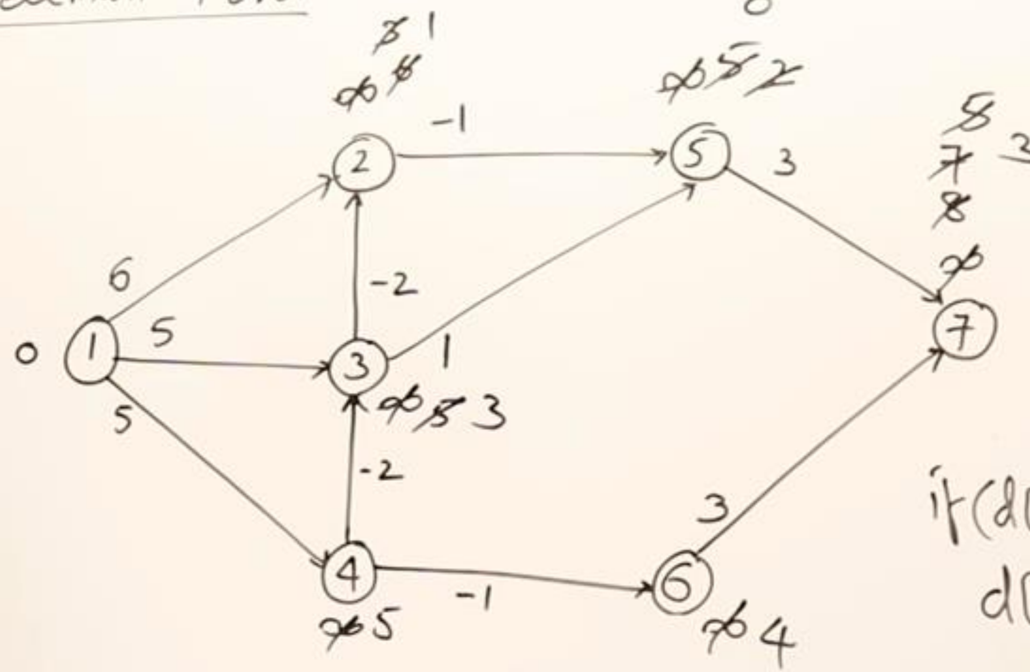
$$\text{if } (d[u] + c(u,v) < d[v])$$
$$d[v] = d[u] + c(u,v)$$

edgeList → (1,2),(1,3),(1,4)(2,5),(3,2),(3,5),(4,3)(4,6),(5,7),(6,7)

Single Source Shortest Path

Bellman-Ford

1 – 0
2 – 1
3 – 3
4 – 5
5 – 0
6 – 4
7 – 3



(u,v)

Relaxation

if $(d[u] + c(u,v) < d[v])$

$d[v] = d[u] + c(u,v)$

edgeList → (1,2),(1,3),(1,4)(2,5),(3,2),(3,5),(4,3)(4,6),(5,7),(6,7)

Single Source Shortest Path

Bellman-Ford

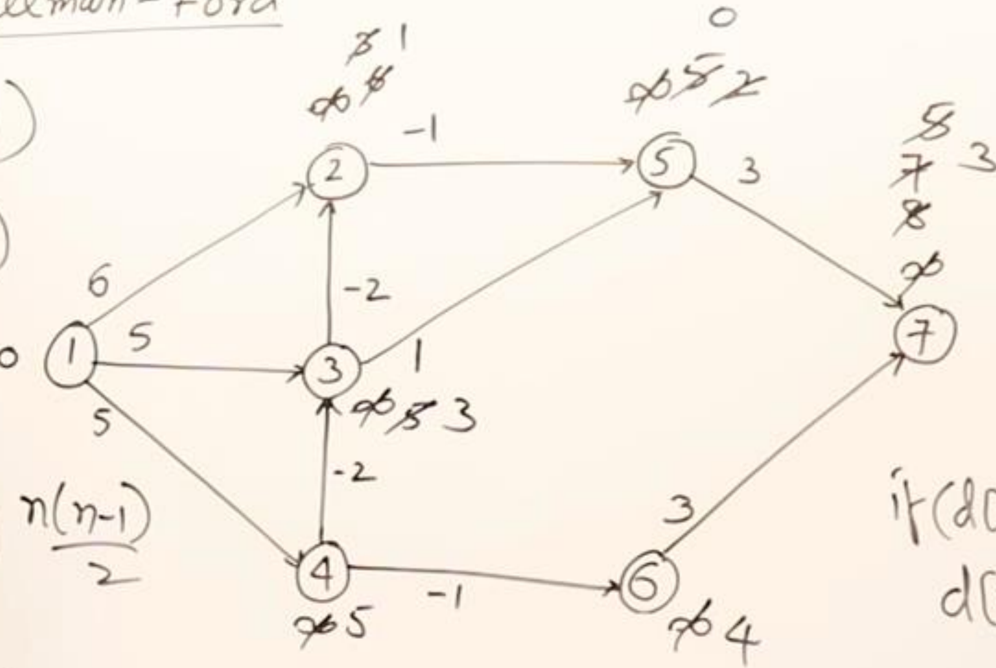
1 − 0
2 − 1
3 − 3
4 − 5
5 − 0
6 − 4
7 − 3

$O(|E|\,(|V|-1))$

$O(\underset{n}{|V|}\,\underset{n}{|E|})$

$O(n^2)$

$|E| \quad \frac{n(n-1)}{2}$



(u,v)

Relaxation

$\text{if}(d[u]+c(u,v)<d[v])$

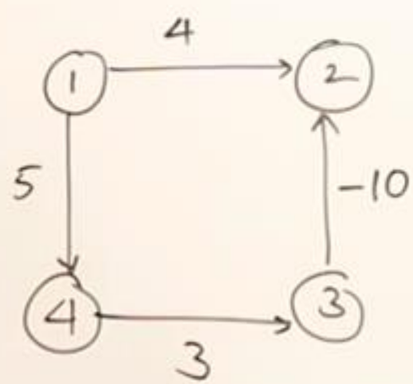
$\quad d[v]=d[u]+c(u,v)$

edgeList → (1,2),(1,3),(1,4) (2,5),(3,2),(3,5),(4,3),(4,6),(5,7),(6,7)

$$\frac{n(n-1)}{2} \times (n-1) = O(n^3)$$

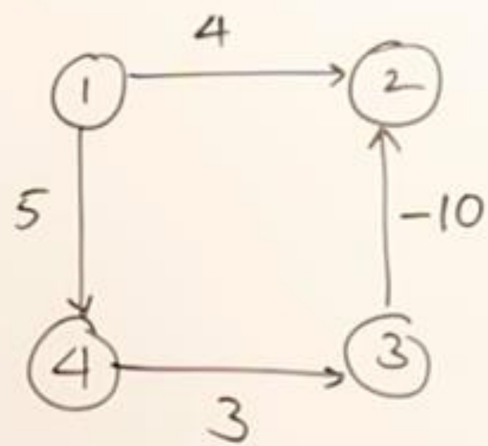
Single Source Shortest Path

Bellman-Ford



Single Source Shortest Path

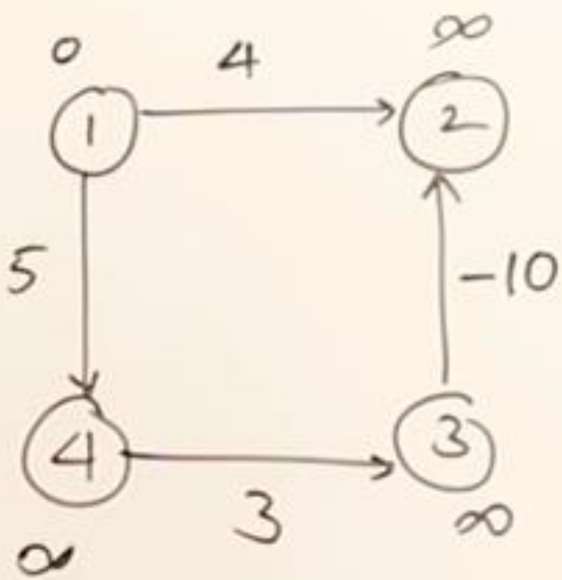
Bellman-Ford



edges → (3,2) (4,3) (1,4) (1,2)

Single Source Shortest Path

Bellman-Ford

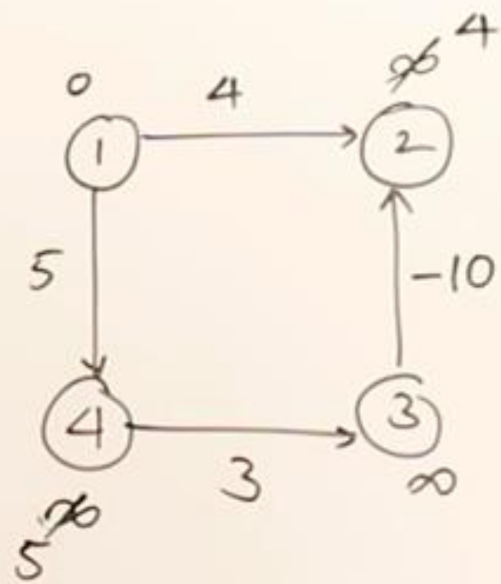


edges $\rightarrow$ (3,2) (4,3) (1,4) (1,2)

$n = 4 - 1 = 3$

Single Source Shortest Path

Bellman-Ford

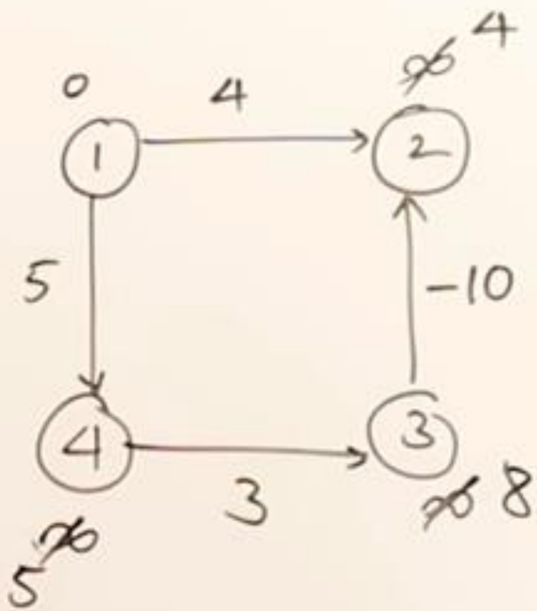


edges → (3,2) (4,3) (1,4) (1,2)

$n = 4 - 1 = 3$

Single Source Shortest Path

Bellman-Ford

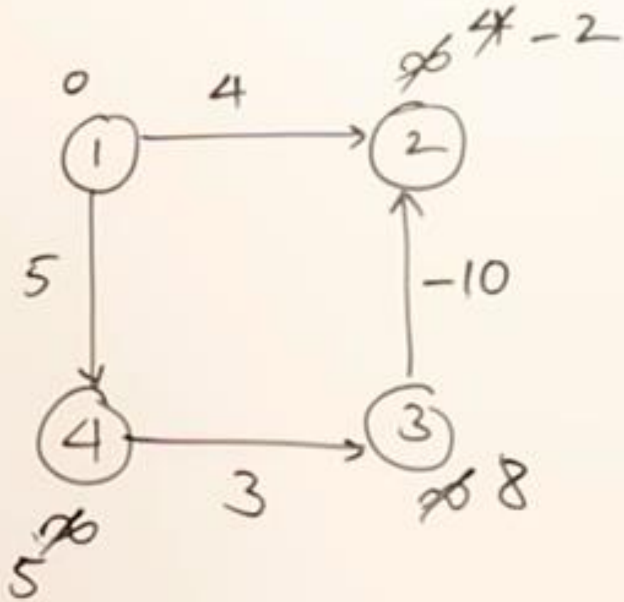


edges → (3,2) (4,3) (1,4) (1,2)

$n = 4 - 1 = 3$

Single Source Shortest Path

Bellman-Ford



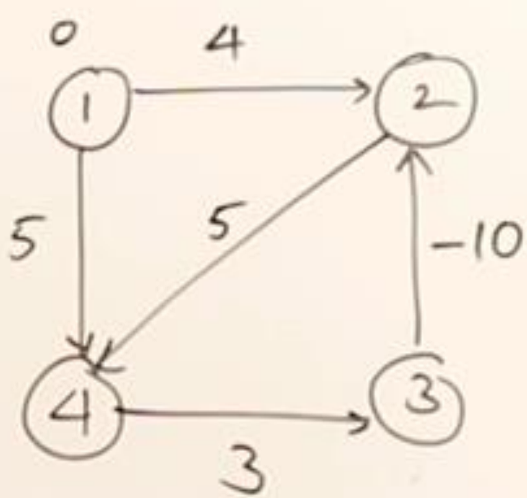
1 — 0
2 — (-2)
3 — 8
4 — 5

edges → (3,2) (4,3) (1,4) (1,2)

$n = 4 - 1 = 3$

Single Source Shortest Path

Bellman-Ford

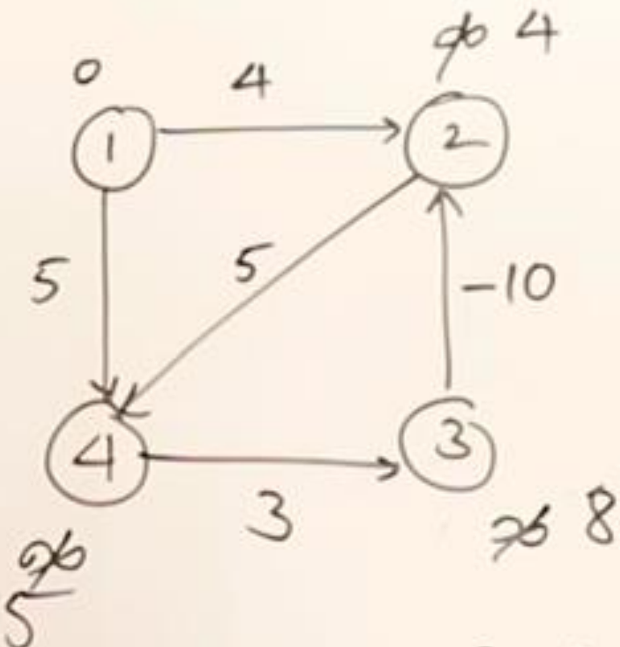


Demerit

edges → (3,2) (4,3) (1,4) (1,2) (2,4)

Single Source Shortest Path

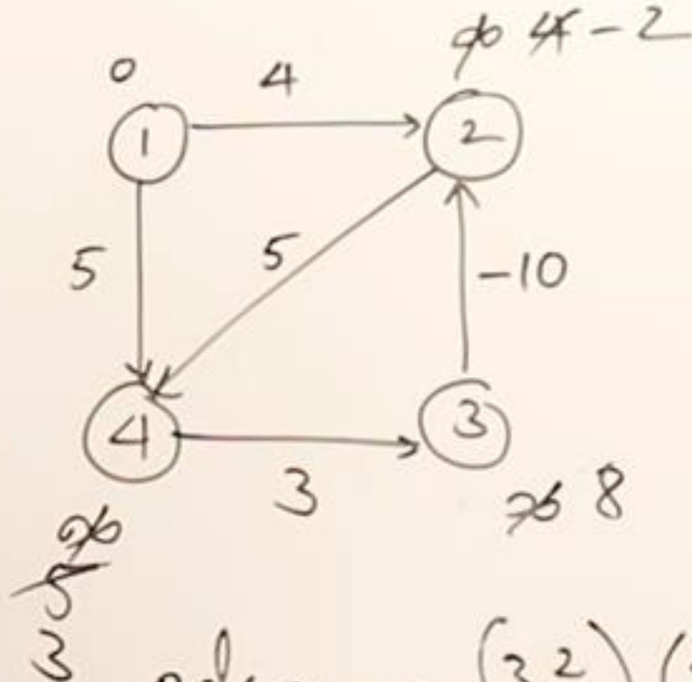
Bellman-Ford



edges → (3,2) (4,3) (1,4) (1,2) (2,4)

Single Source Shortest Path

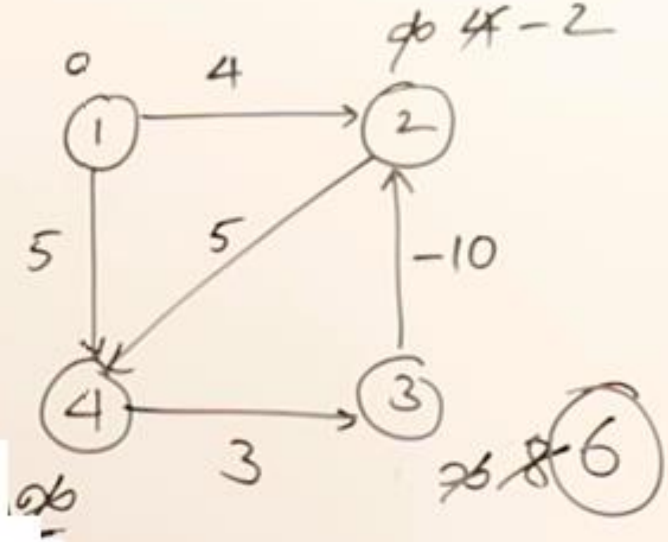
Bellman-Ford

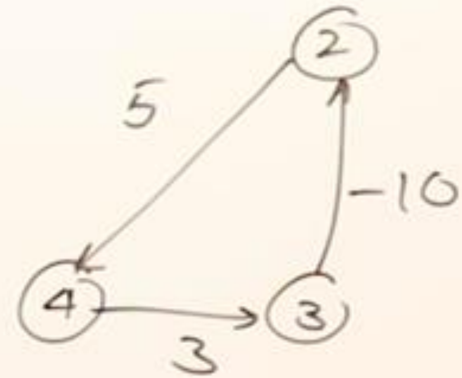


edges → (3,2) (4,3) (1,4) (1,2) (2,4)

Single Source Shortest Path

Bellman-Ford



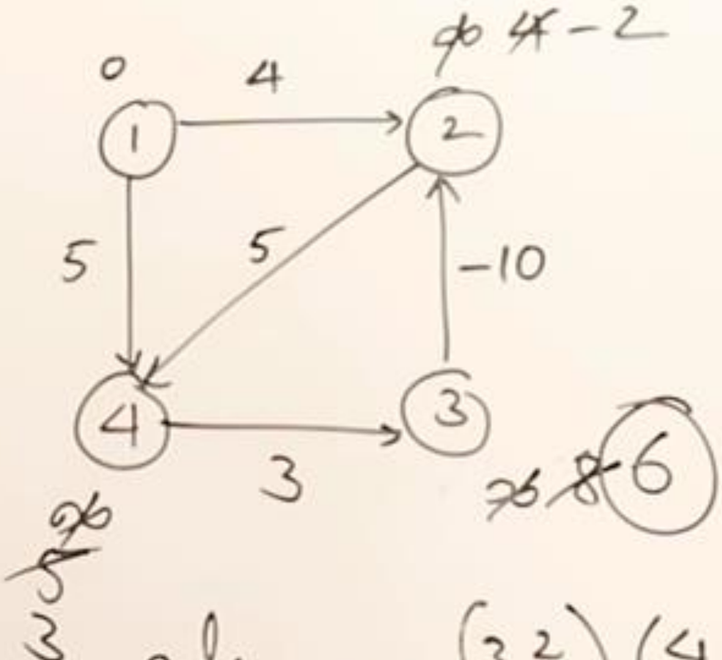


$5+3-10 = -2$

edges → (3,2) (4,3) (1,4) (1,2) (2,4)

Single Source Shortest Path

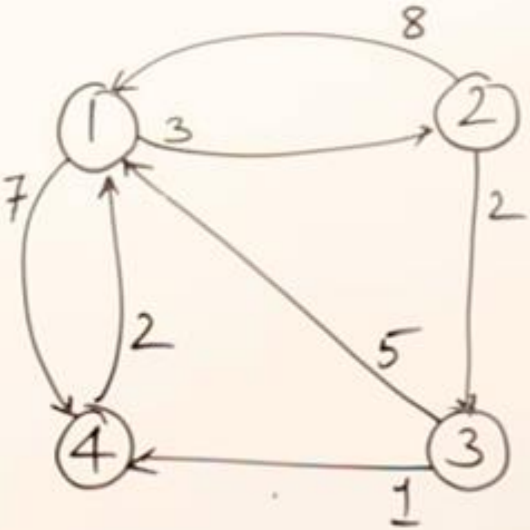
Bellman-Ford



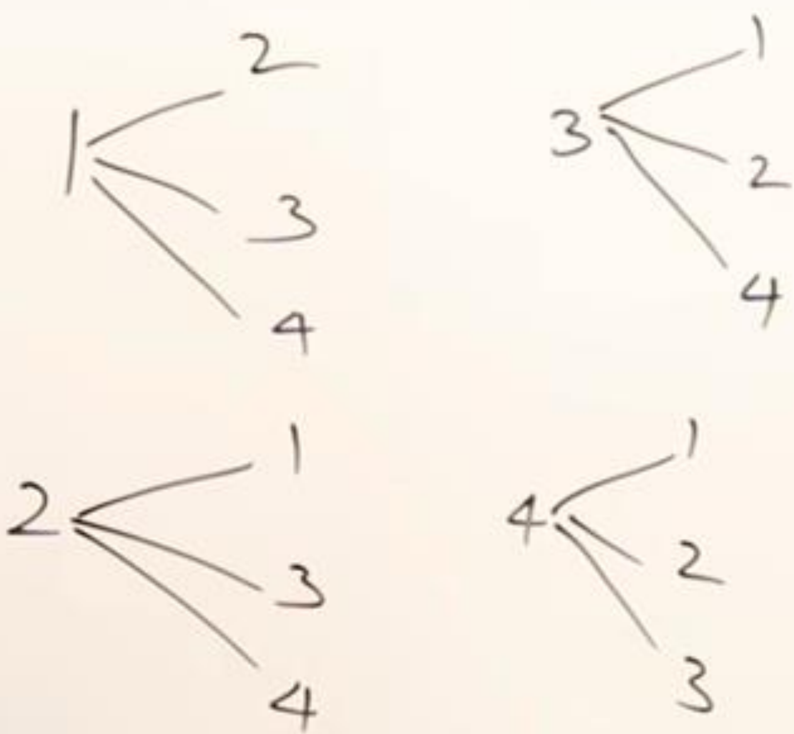
edges → (3,2) (4,3) (1,4) (1,2) (2,4)

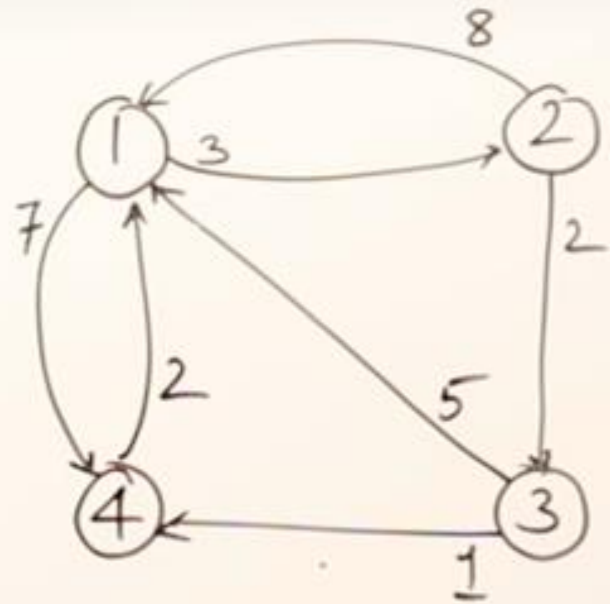
Thank you

All Pairs Shortest Path (Floyd-Warshall Algorithm)

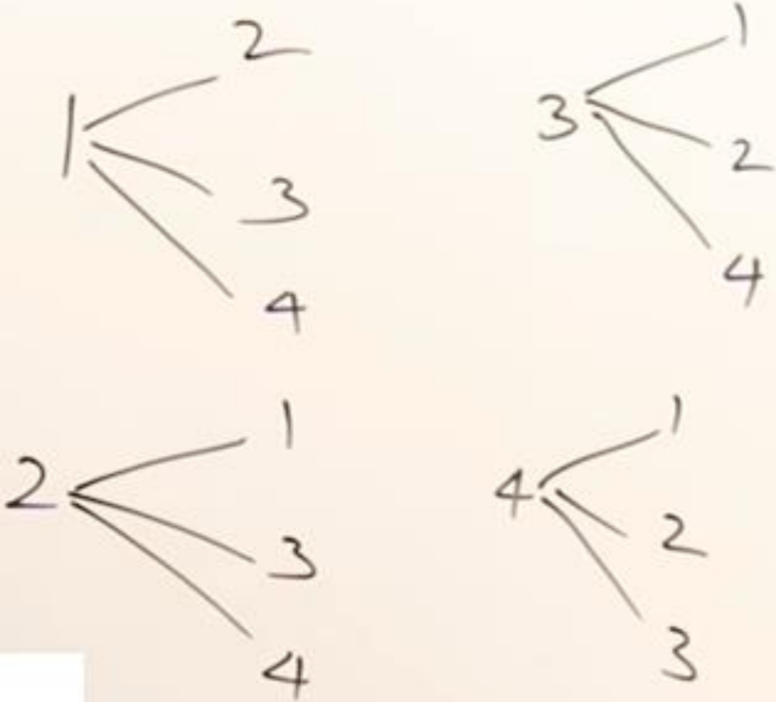


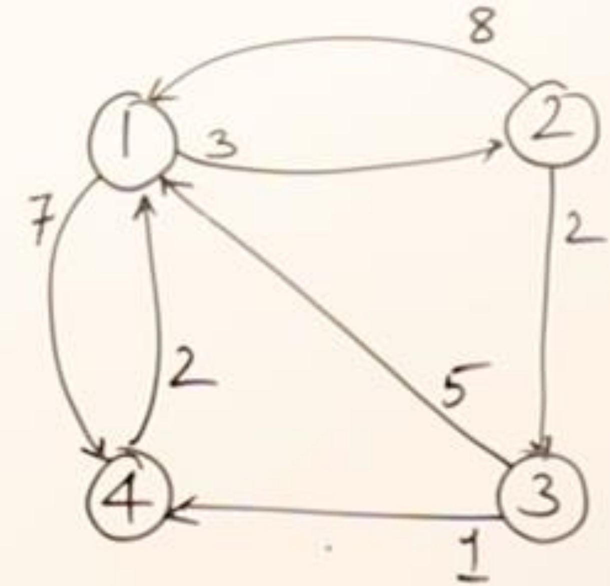
$$A^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$





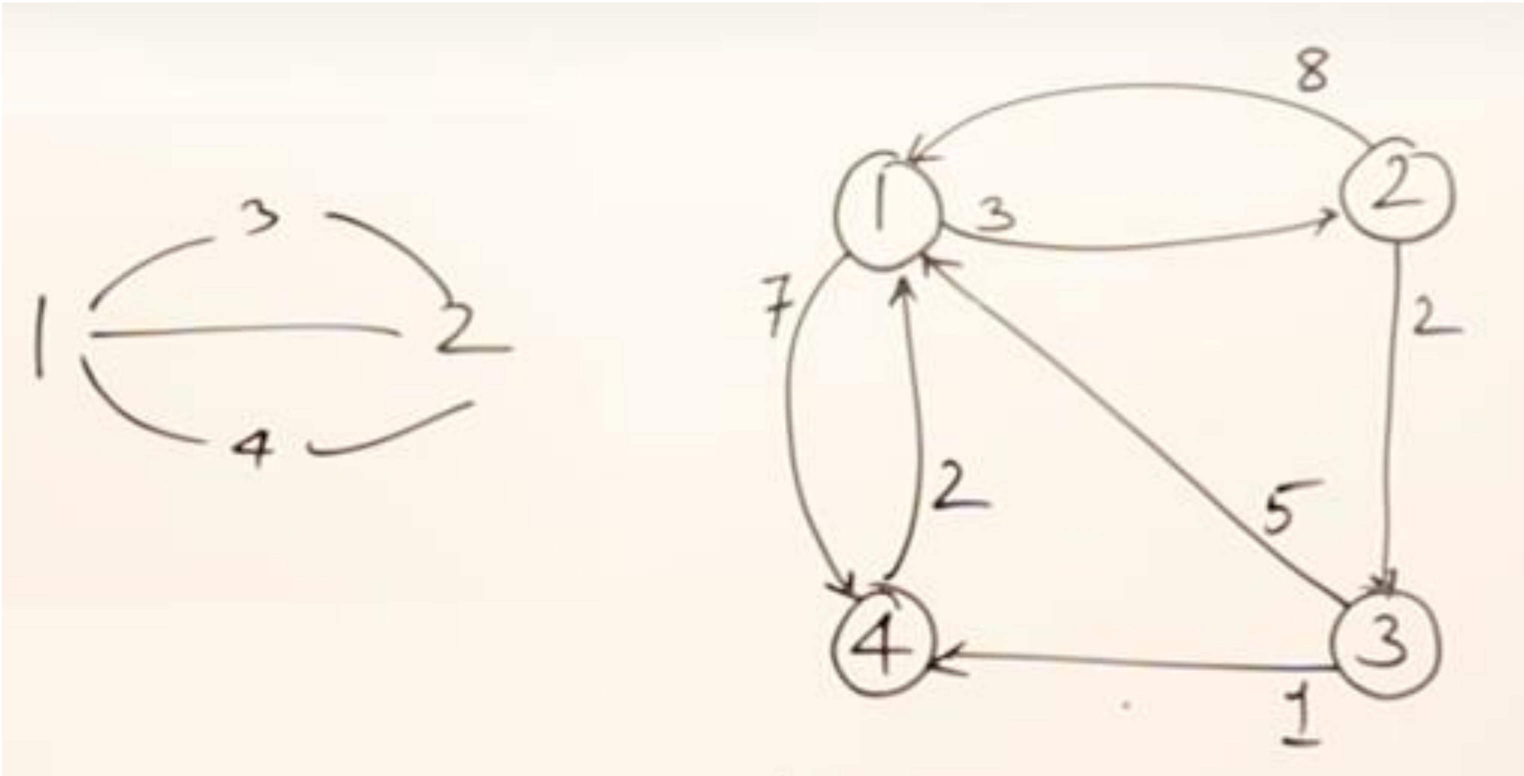
$$A^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & \infty \\ 3 & 5 & \infty & 0 & 1 \\ 4 & 2 & \infty & \infty & 0 \end{array}$$





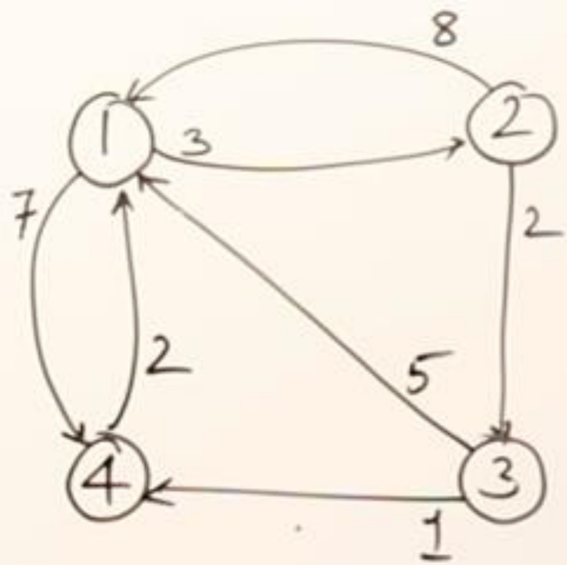
$$n^2 \times n = O(n^3)$$

$$A^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$



3
1
2
4
8
1
3
2
7
2
2
5
4
3
1

$$A^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & & \\ 5 & & 0 & \\ 2 & & & 0 \end{bmatrix} \end{array}$$

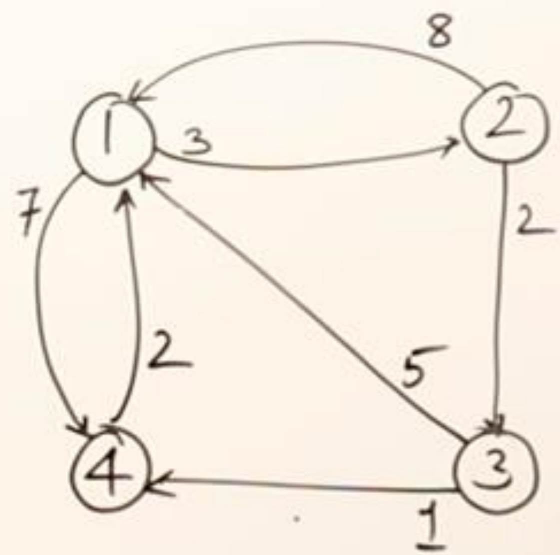


$$A^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

$$A^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \\ 5 & & 0 & \\ 2 & & & 0 \end{bmatrix} \end{array}$$

$$A^0[2,3] \quad A^0[2,1] + A^0[1,3]$$

$$2 < 8 + \infty$$



$$A^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

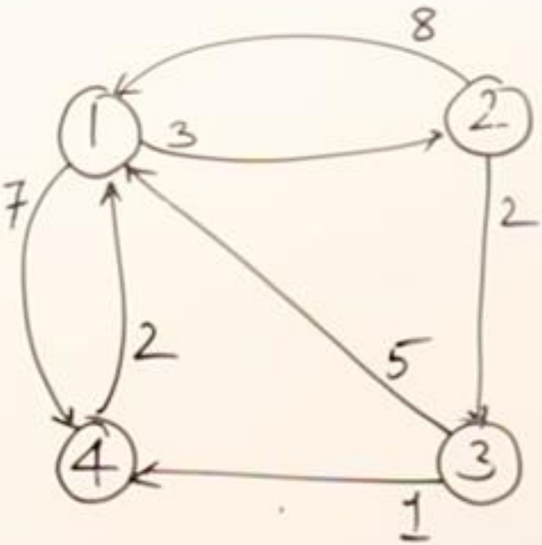
$$A^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & & 0 & \\ 2 & & & 0 \end{bmatrix} \end{array}$$

$$A^0[2,3] \quad A^0[2,1] + A^0[1,3]$$

$$2 < 8 + \infty$$

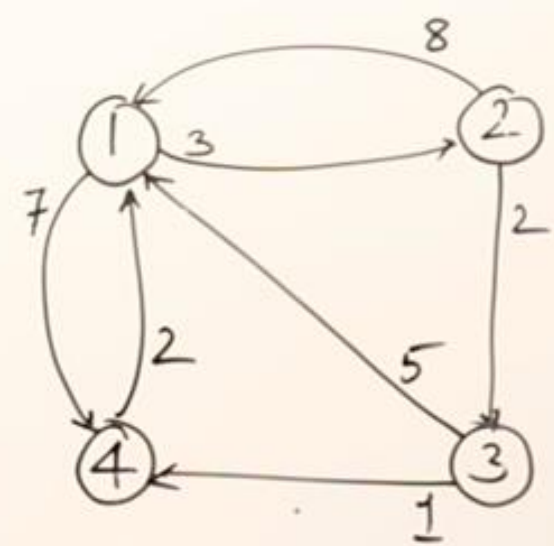
$$A^0[2,4] \quad A^0[2,1] + A^0[1,4]$$

$$\infty > 8 + 7$$



$$A^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

$$A^1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & 5 & - & 0 & \\ 4 & 2 & & & 0 \end{array}$$



$$A^0[2,3] \qquad A^0[2,1] + A^0[1,3]$$
$$2 < 8 + \infty$$

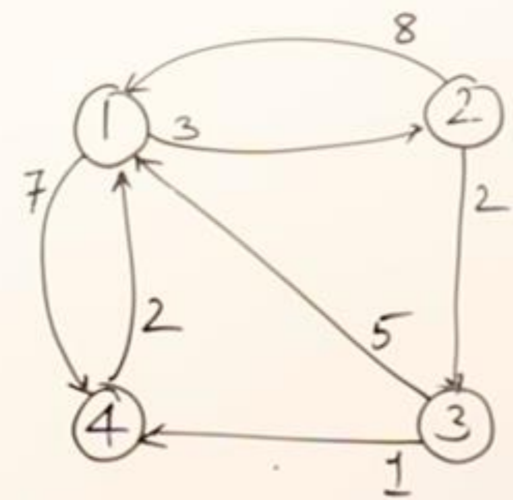
$$A^0[2,4] \qquad A^0[2,1] + A^0[1,4]$$
$$\infty > 8 + 7$$

$$A^0[3,2] \qquad A^0[3,1] + A^0[1,2]$$
$$\infty \qquad 5 + 3$$

$$A^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & \infty \\ 3 & 5 & \infty & 0 & 1 \\ 4 & 2 & \infty & \infty & 0 \end{array}$$

$$A^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 8 & \infty & 0 \end{bmatrix} \end{array}$$

$$A^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & & \\ 8 & 0 & 2 & 15 \\ & 8 & 0 & \\ & 8 & & 0 \end{bmatrix} \end{array}$$



$A^1[1,3] \qquad A^1[1,2] + A^1[2,3]$

∞

$$A^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 8 & \infty & 0 \end{bmatrix} \end{array}$$

$$A^2 = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 3 & 5 & 7 \\ 8 & 0 & 2 & 15 \\ & 8 & 0 & \\ & 8 & & 0 \end{bmatrix} \end{array}$$

$$A^1[1,3] \qquad A^1[1,2] + A^1[2,3]$$

$$\infty > 3 + 2$$

$$A^1[1,4] \qquad A^1[1,2] + A^1[2,4]$$

$$7 < 3 + 15$$

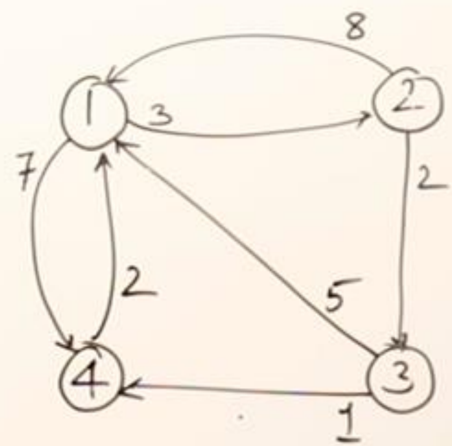
$$A^1 = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & \infty & 0 \end{bmatrix} \end{array}$$

$$A^2 = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$

$$A^3 = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & \\ & 0 & 2 & \\ 5 & 8 & 0 & 1 \\ & & 7 & 0 \end{bmatrix} \end{array}$$

$$A^2[1,2] \quad A^2[1,3] + A^2[3,2]$$

$$3 < 5 + 8$$

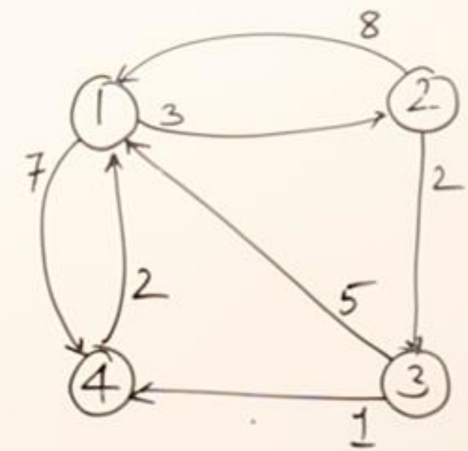


$$A^0 = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

All Pairs Shortest Path

$$A^1 = \begin{array}{c c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & \infty & 0 \end{bmatrix} \end{array}$$

$$A^2 = \begin{array}{c c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$



$$A^3 = \begin{array}{c c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 6 \\ 7 & 0 & 2 & 3 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$

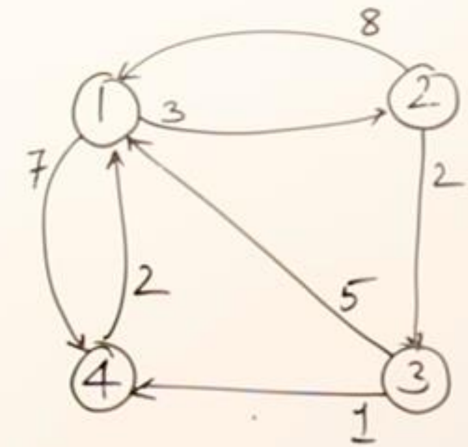
$$A^4 = \begin{array}{c c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 6 \\ 5 & 0 & 2 & 3 \\ 3 & 6 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$

$$A^0 = \begin{array}{c c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

All Pairs Shortest Path

$$A^1 = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & \infty & 0 \end{bmatrix} \end{array}$$

$$A^2 = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$



$$A^3 = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 6 \\ 7 & 0 & 2 & 3 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$

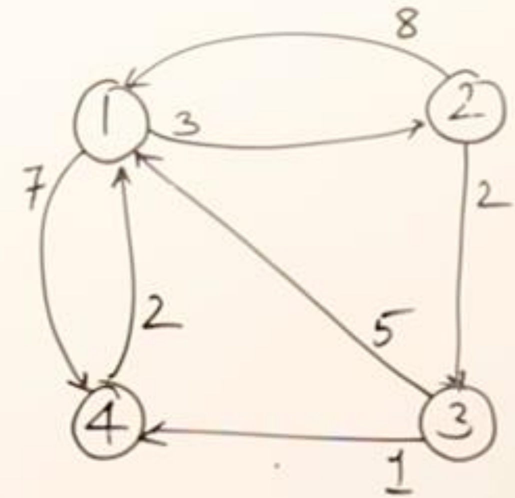
$$A^4 = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 6 \\ 5 & 0 & 2 & 3 \\ 3 & 6 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$

$$A^0 = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

$$A^k[i,j] = \min\left\{ \underline{A^{k-1}[i,j]},\ \underbrace{A^{k-1}[i,k] + A^{k-1}[k,j]} \right\}$$

2

```
for(k=1; k<=n; k++)
{
    for(i=1; i<=n; i++)
    {
        for(j=1; j<=n; j++)
        {
            A[i,j]=min(A[i,j], A[i,k]+A[k,j]);
        }
    }
}
```



$$A^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & \infty \\ 3 & 5 & \infty & 0 & 1 \\ 4 & 2 & \infty & \infty & 0 \end{array}$$

$n \times n$

Thank you