FUNDAMENTAL CONCEPTS OF DOMINATING SETS ARE DISCUSSED

MINIMUM CONNECTED DOMINATING SETS ARE EXPLAINED

APPLICATIONS OF MCDS ARE PRESENTED

- Introduction
- Minimum Connected Dominating Sets
- Greedy Algorithm
- Example
- Applications
- Conclusions

Dominating Sets

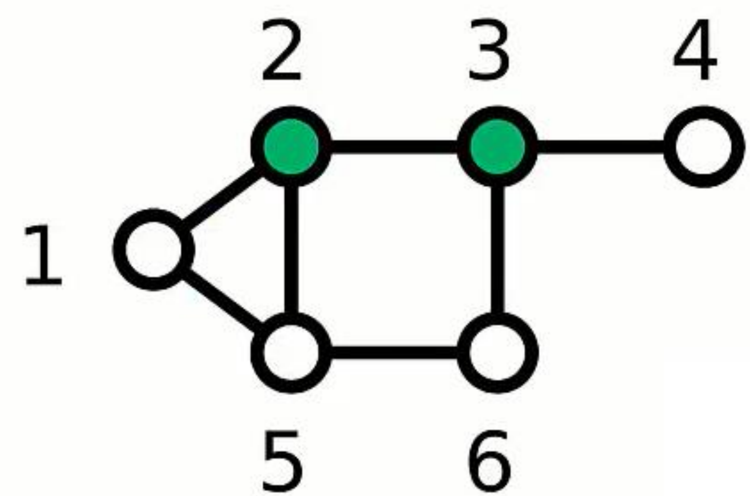
A dominating set D for a graph G is a set in which each vertex of G is either in D or adjacent to some vertex in D.

The minimum size of a dominating set of vertices in G is called domination number of the graph G.

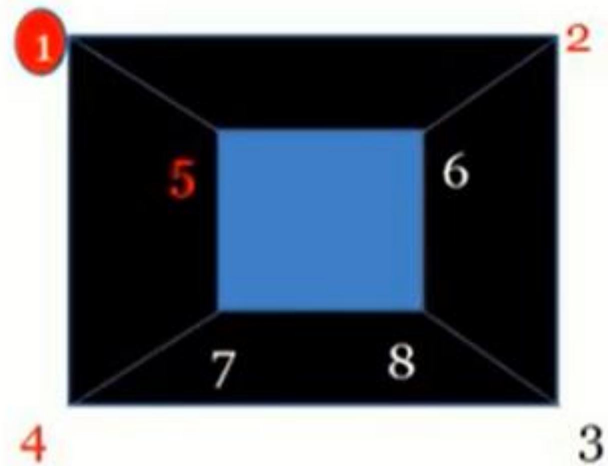
The dominating set problem concerns finding a minimum dominating set.

Dominating Set: Example

In figure 1, the green vertices 2 and 3 are clearly members of a dominating set as every vertex that is not in the dominating set $\{2,3\}$ is adjacent to either 2 or 3.

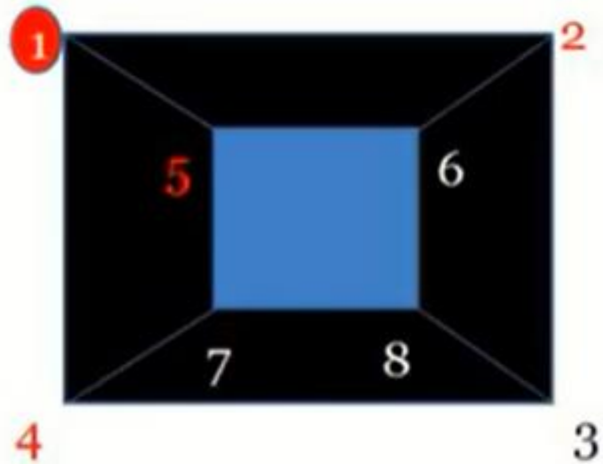


Example

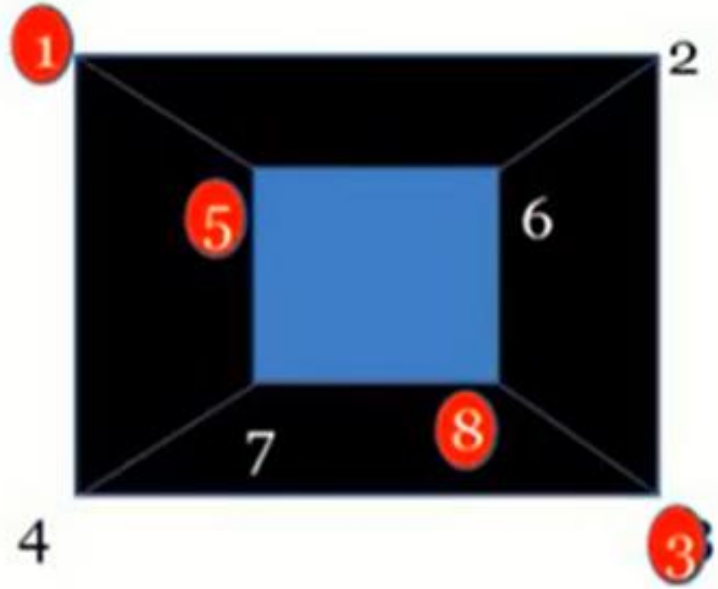


- The vertex 1 dominates the vertices 2,4 and 5.
- The vertex 2 dominates the vertices 1,3 and 6.
- The vertex 3 dominates the vertices 2,4 and 8.

Example

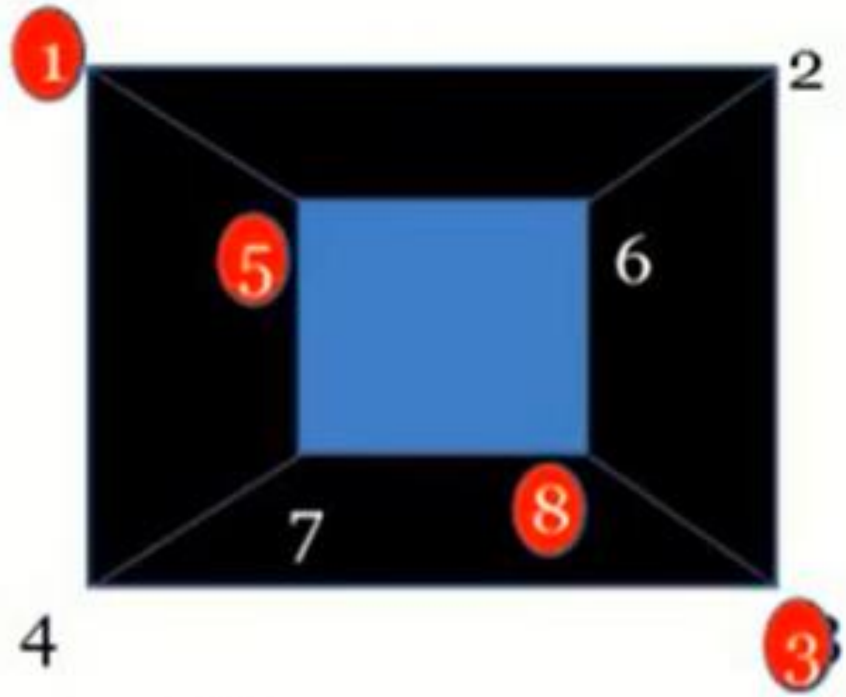


- The vertex 1 is dominated by the vertices 2,4 and 5.
- The vertex 2 is dominated by the vertices 1,3 and 6.
- The vertex 3 is dominated by the vertices 2,4 and 8.



- The vertex 4 is dominated by the vertices 1,3 and 7.
- The vertex 5 is dominated the vertices 1,4 and 6.
- The vertex 6 is dominated by the vertices 2,5 and 8.
- The vertex 7 is dominated by the vertices 4,5 and 8.
- The vertex 8 is dominated by the vertices 3,6 and 7.

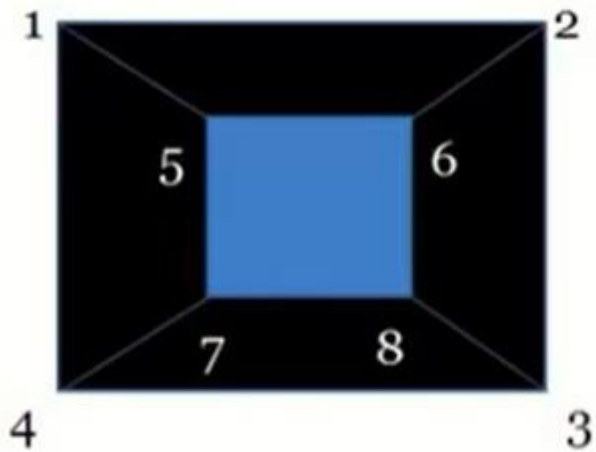
Example



Let $D_1 = \{1,3,5,8\}$

Is it a dominating set?

Example

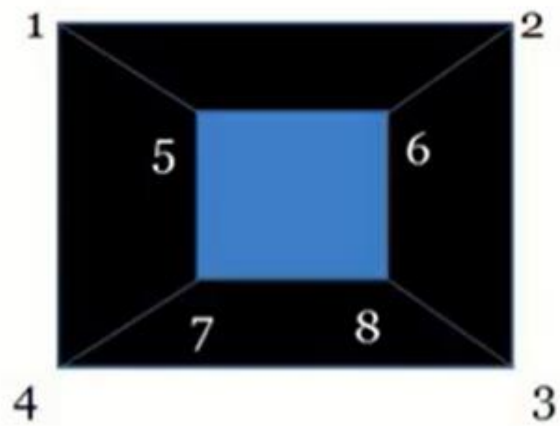


Let $D_1 = \{1,3,5,8\}$

All the vertices $\{2,4,6,7\}$ which are not in D_1 is dominated by atleast one of the vertices in D_1

Hence D_1 is a dominating set.

Example



Let $D_2 = \{1,2\}$

The vertices 7 as well as 8 are not dominated by any of the vertices in D_2

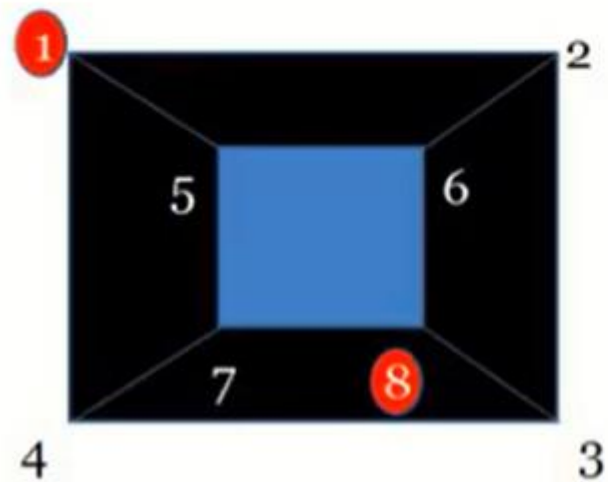
Hence D_2 is not a dominating set

Is dominating set exists for every graph?

Domination Number

The domination number of a graph G is the number of vertices in a smallest dominating set for G. It is denoted by $\gamma(G)$.

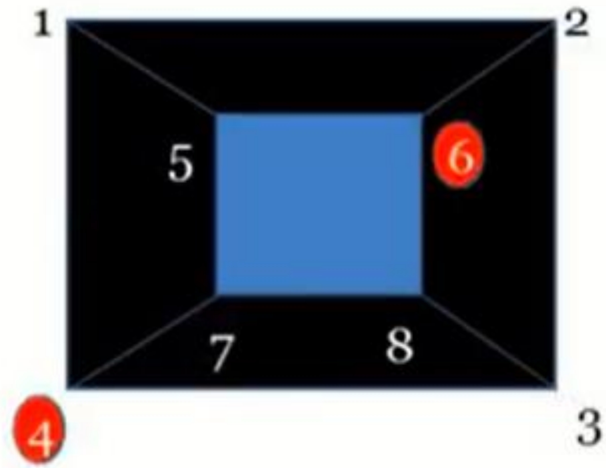
Example



- Vertices 1 and 8 represent a minimum dominating set (MDS) for G.

  Hence the domination number is 2.

Example



The other possible minimum dominating sets (MDS) are {2, 7}, {3, 5}, {4, 6}

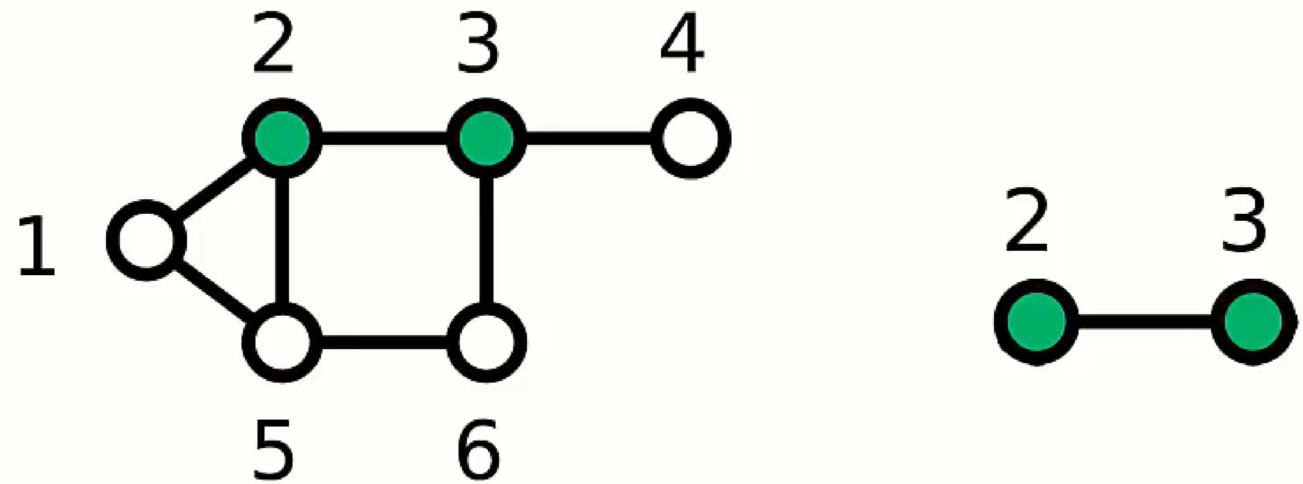
Can we find a graph having the domination number as 1?

Connected Dominating Sets

- A connected dominating set of a graph G is a dominating set D whose induced subgraph is also connected.
- A minimum connected dominating set of a graph G is a connected dominating set with the smallest possible cardinality among all connected dominating sets of G.
- The connected domination number of G is the number of vertices in the minimum connected dominating set.

Connected Dominating Set: Example

In the same example, the induced graph by the dominating set {2,3} is shown in Figure 1b, which is clearly a connected dominating set.



Bipartite Graph & Set Covering Problem

- The Connected Dominating Set (CDS) problem is to find a minimum connected dominating set.
- Finding a minimum connected dominating set is known to be an NP-complete problem.
- This essentially means that these class of problems cannot be solved quickly (in polynomial time).
- Several authors have proposed algorithms for obtaining approximate minimal connected dominating sets.

Connected Dominating Sets

- The problem of finding a minimum connected dominating set can be mapped into a Set Covering Problem.
- The Set Covering Problem is essentially a problem concerning bipartite graphs that can be stated as follows.

Bipartite Graph & Set Covering Problem

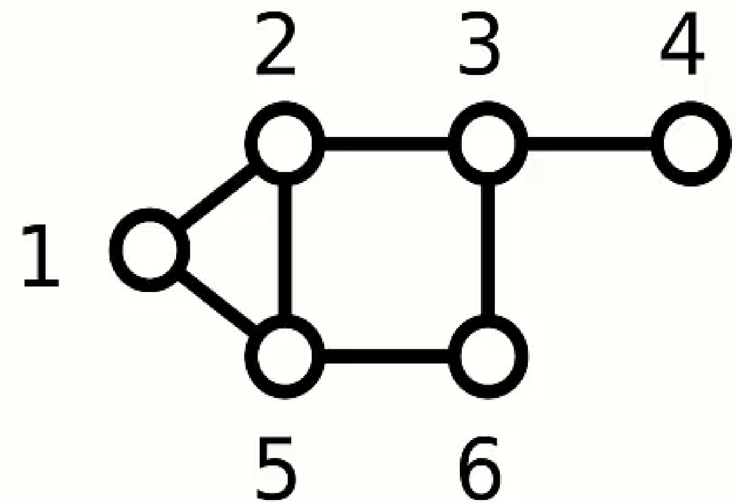
- Suppose that H is a bipartite graph, consisting of two sets of nodes A and B, where edges only make connections between A and B.
- Also assume that for each node in *B*, there is at least one edge connecting it to a node in *A*.
- The goal is to find a minimal subset C of A such that every node in B is covered by (i.e., adjacent to) some node in C.

Bipartite Graph Vertex Covering Problem

- The bipartite graph vertex covering problem can be addressed using a greedy algorithm, which will be explained in the next lecture.
- At each stage, a vertex from A is elected that covers as many vertices from B as possible that are not yet covered by an elected vertex.
- Let us look at the following example.

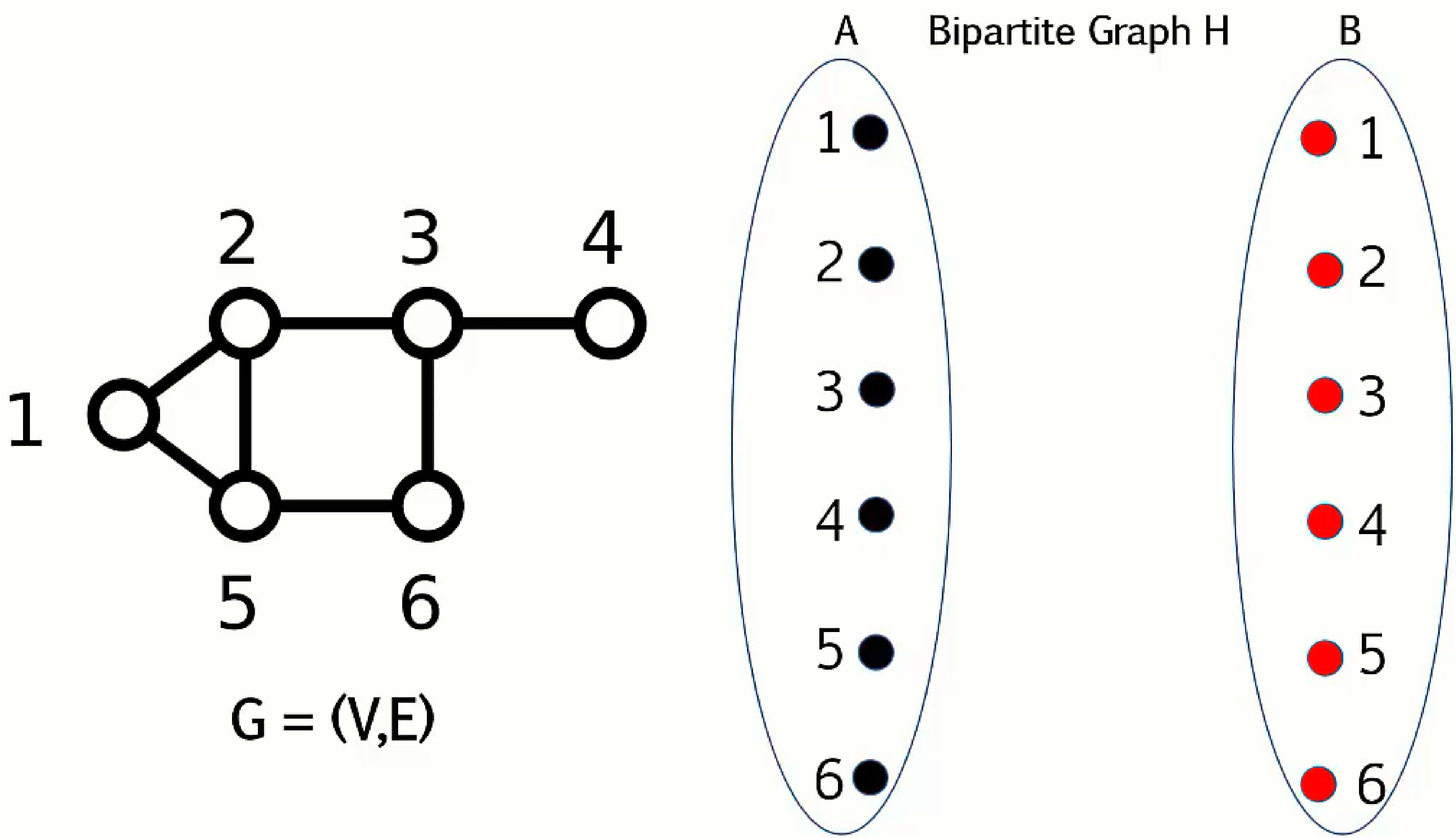
Bipartite Graph Example

- Let G be a connected graph (V,E).
- Let A and B are copies of vertices of E.
- Construct a bipartite graph H by putting an edge between vertices v of A and u of B if they are adjacent to each other.

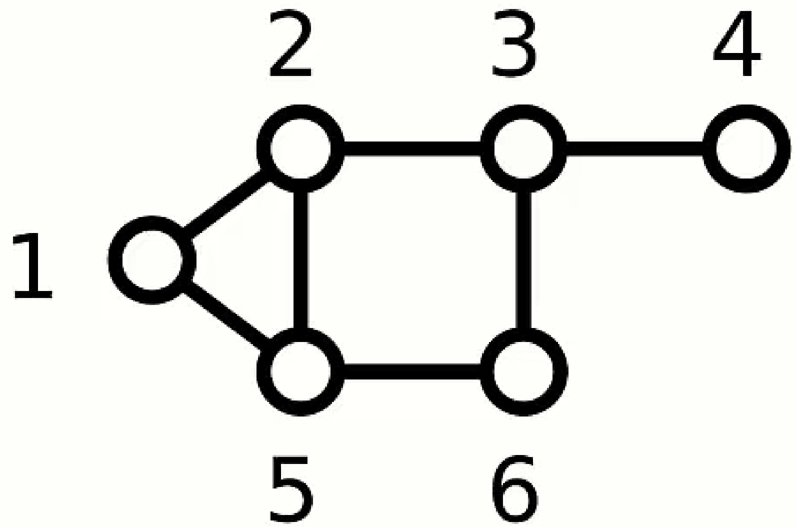


G = (V,E)

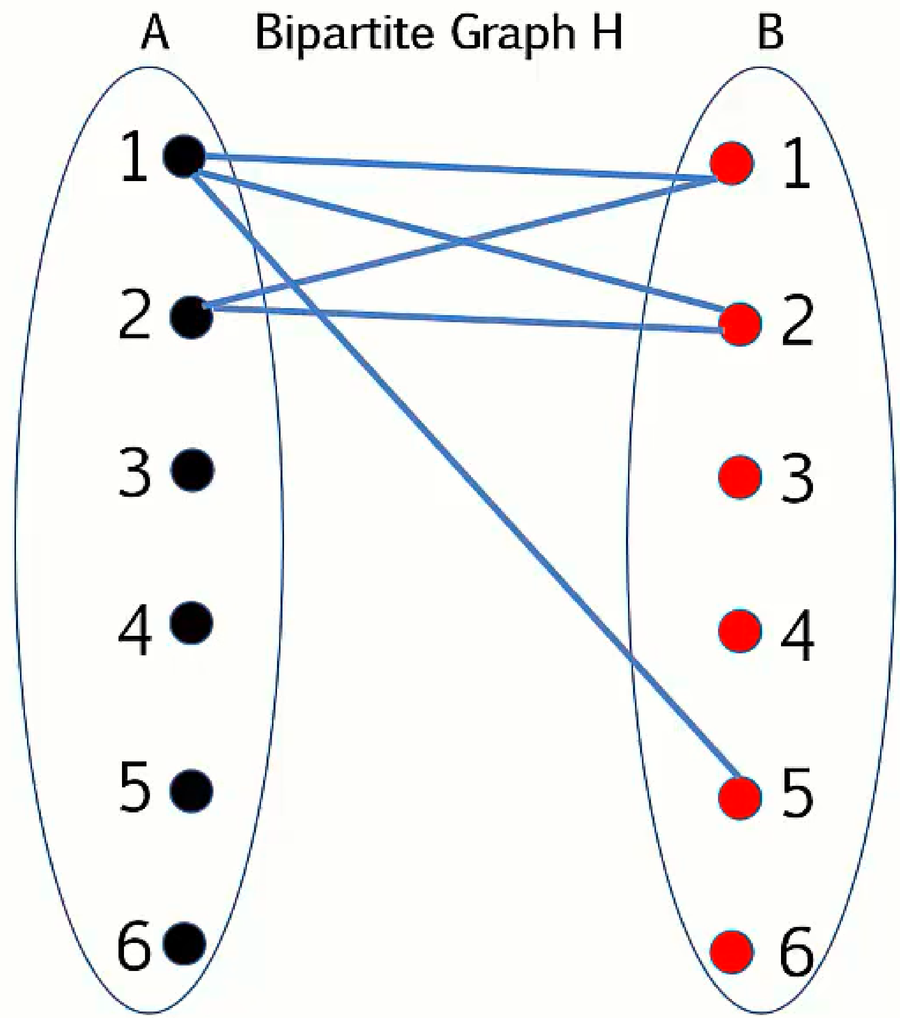
Bipartite Graph Example (Contd.)



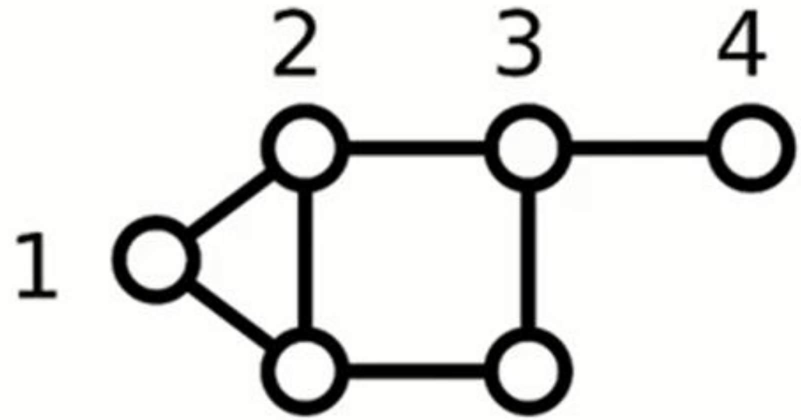
Bipartite Graph Example (Contd.)



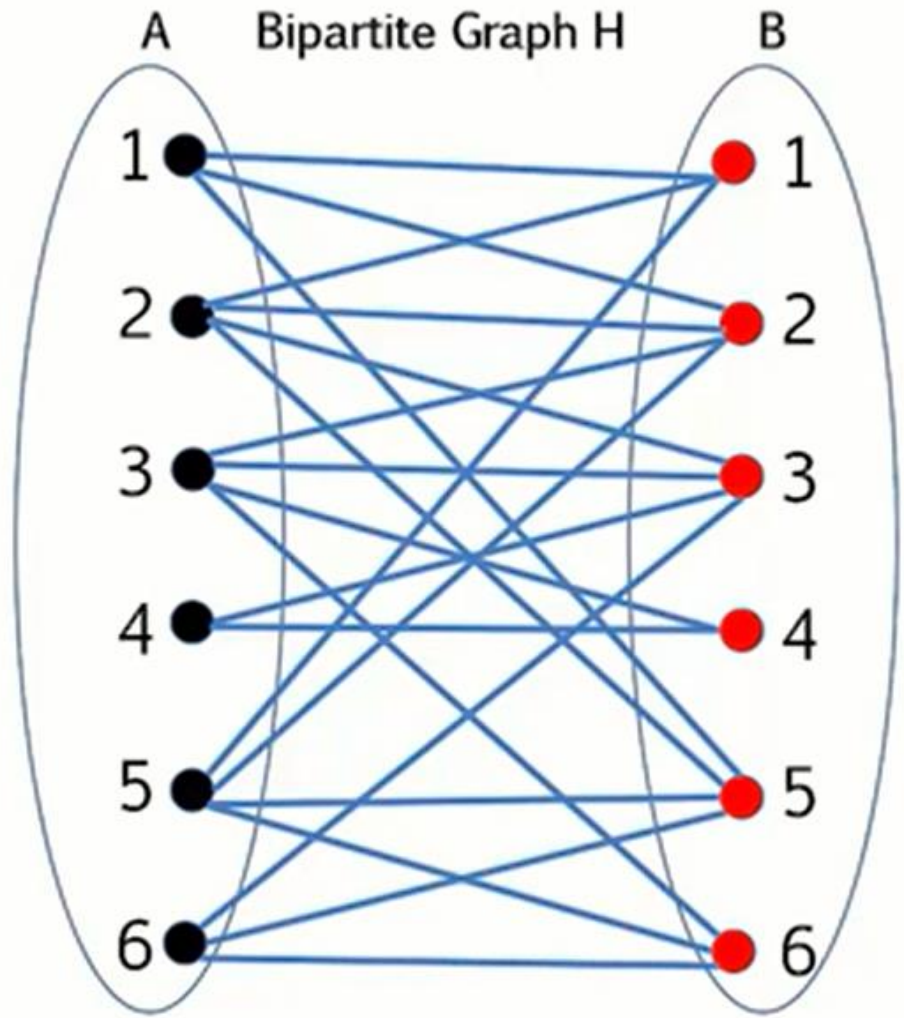
G = (V,E)



Bipartite Graph Example (Contd.)

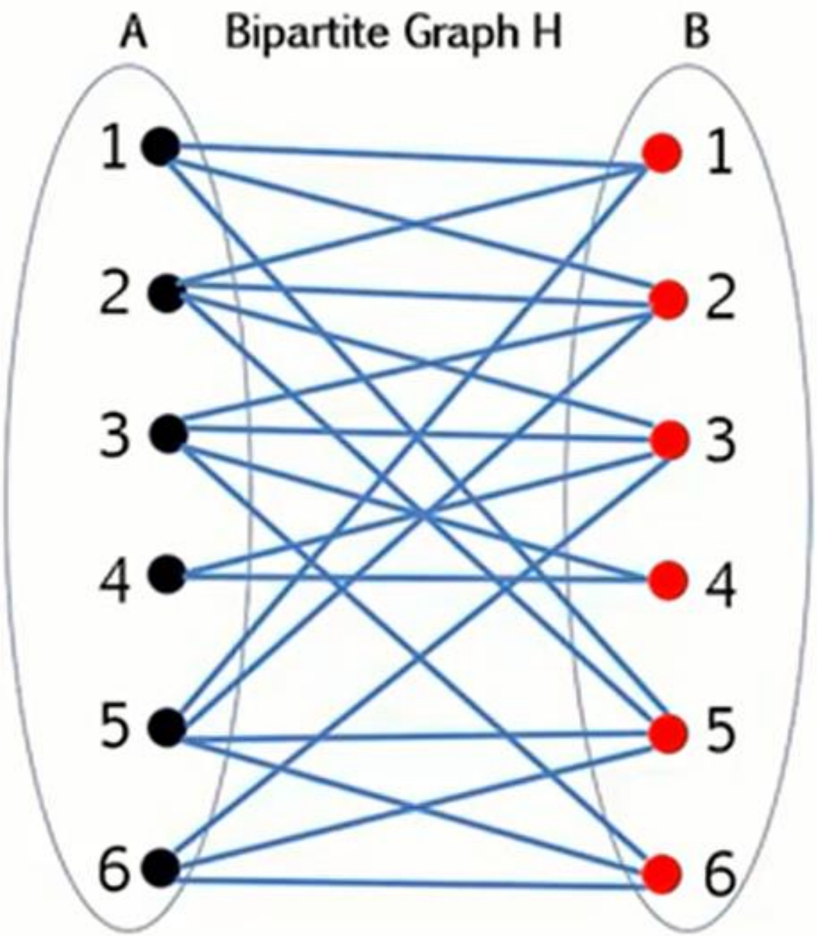


G = (V,E)



A greedy algorithm will be discussed to find a minimum connected dominating set.

Greedy Algorithm Example (Contd.)

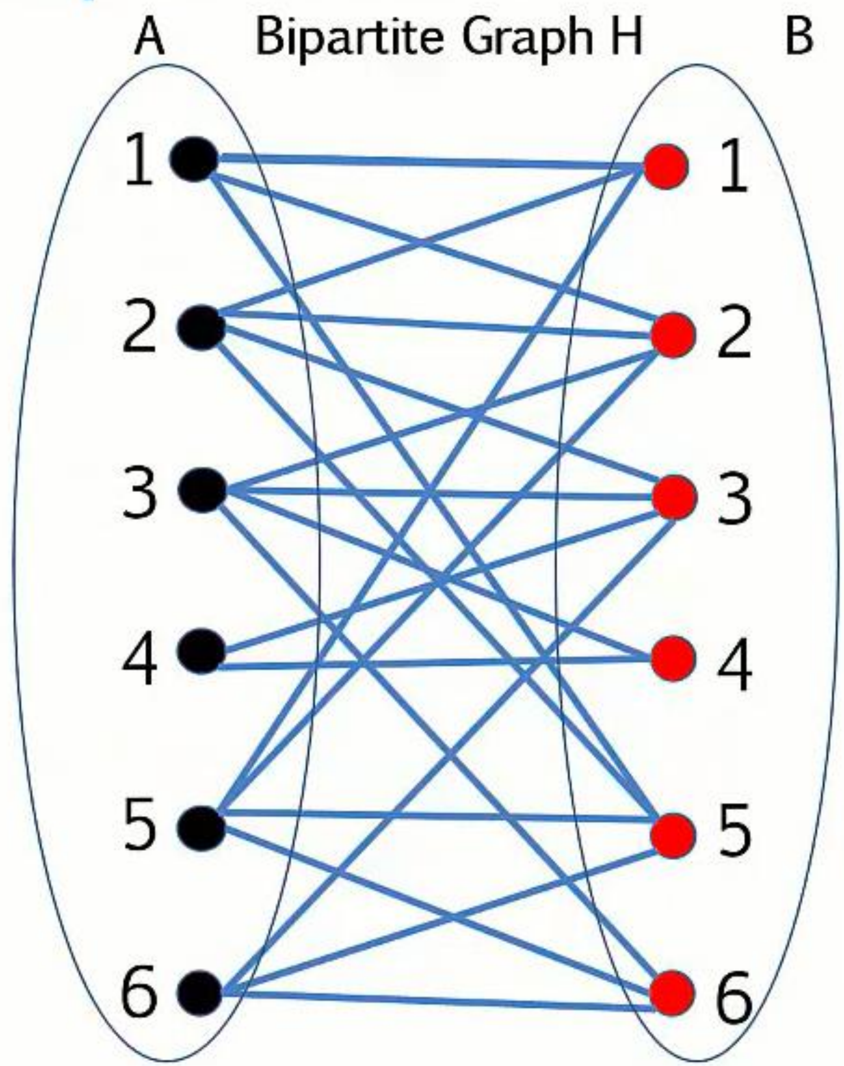


Compute the degree of all nodes in the set A in the bipartite graph H, i.e. compute the *covering numbers*.

Covering no. of vertex 1 (in A) = 3
Covering no. of vertex 2 (in A) = 4
Covering no. of vertex 3 (in A) = 4
Covering no. of vertex 4 (in A) = 2
Covering no. of vertex 5 (in A) = 4
Covering no. of vertex 6 (in A) = 3

Greedy Algorithm Example (Contd.)

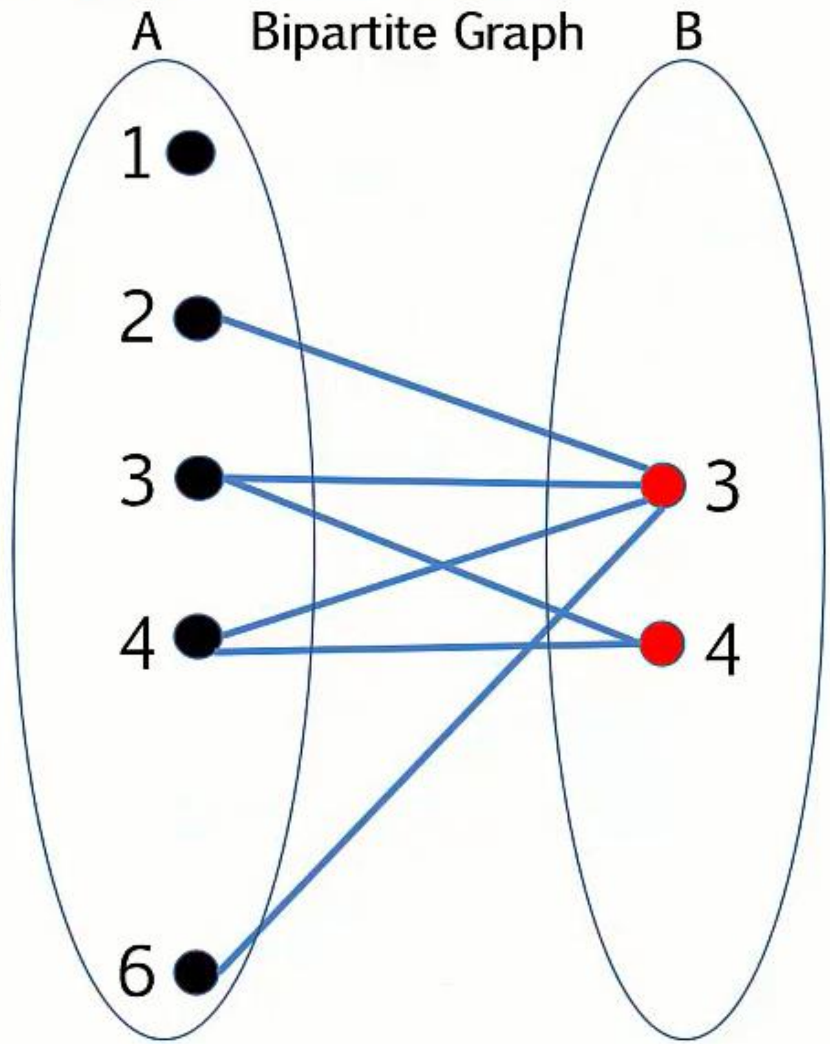
- Elect a node v from set A such that v has the highest covering number and add it to the output set C.
- If there is a tie, then the highest ID is used to break the tie.
- Remove all vertices in the set B that are covered by the node v.
- Also remove v from A.
- After the first round, C = {5} as vertex 5 from A has the highest covering no. as well as the highest ID.



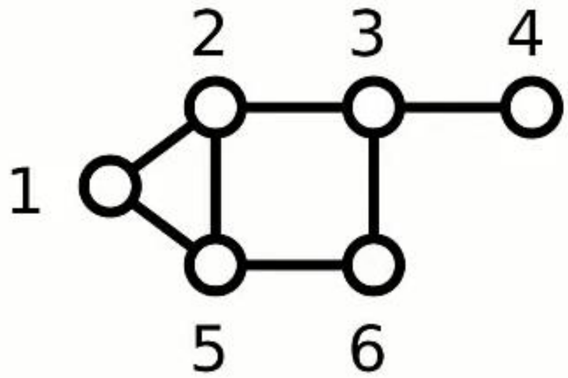
Greedy Algorithm Example (Contd.)

- During the second round, re-compute the covering no. of all remaining vertices in A.
- Although vertices 3 & 4 in A have the same covering no., vertex 4 is selected because it has the highest id.
- After the second round, all vertices in B are exhausted and the algorithm converges.
- The final output set C = {4,5} is a minimal dominating set.

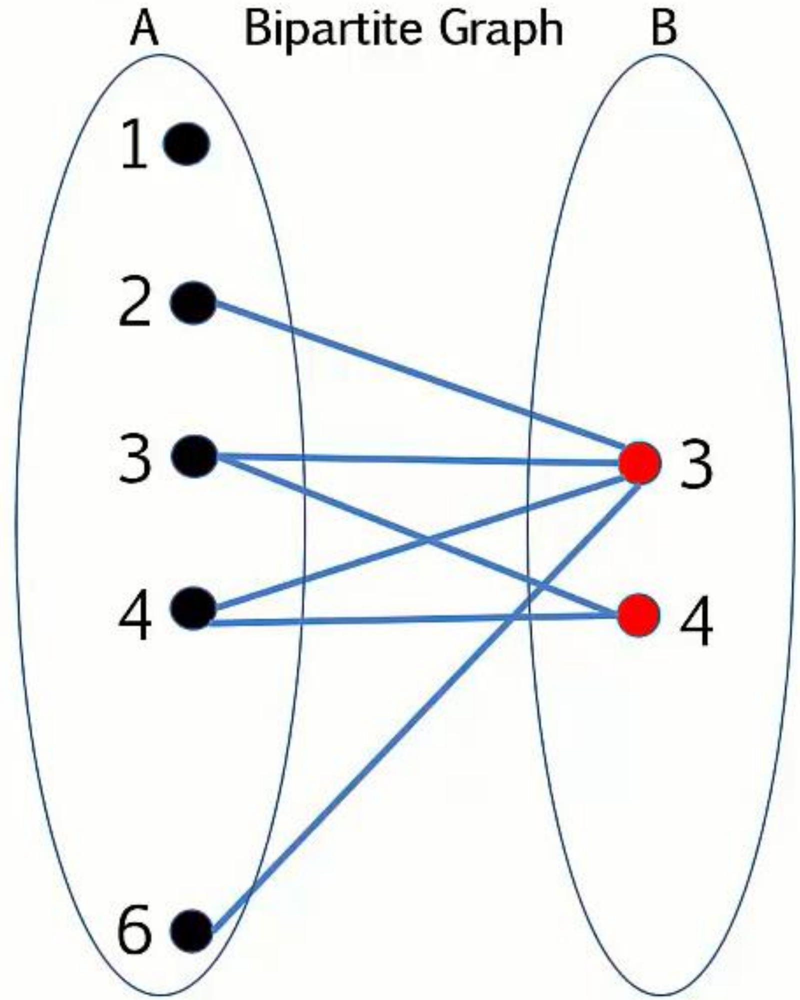
Covering no. of vertex 1 (in A) = 0
Covering no. of vertex 2 (in A) = 1
Covering no. of vertex 3 (in A) = 2
Covering no. of vertex 4 (in A) = 2
Covering no. of vertex 6 (in A) = 1



Greedy Algorithm For MCDS

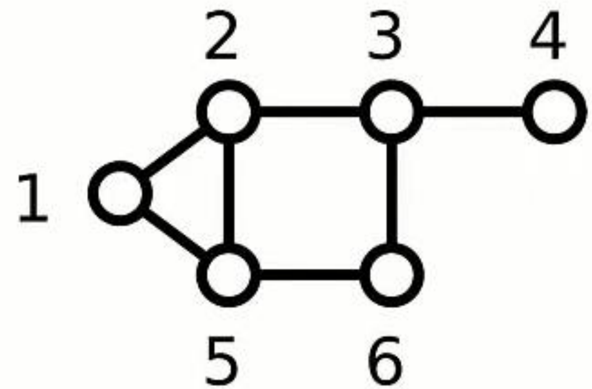


G = (V,E)

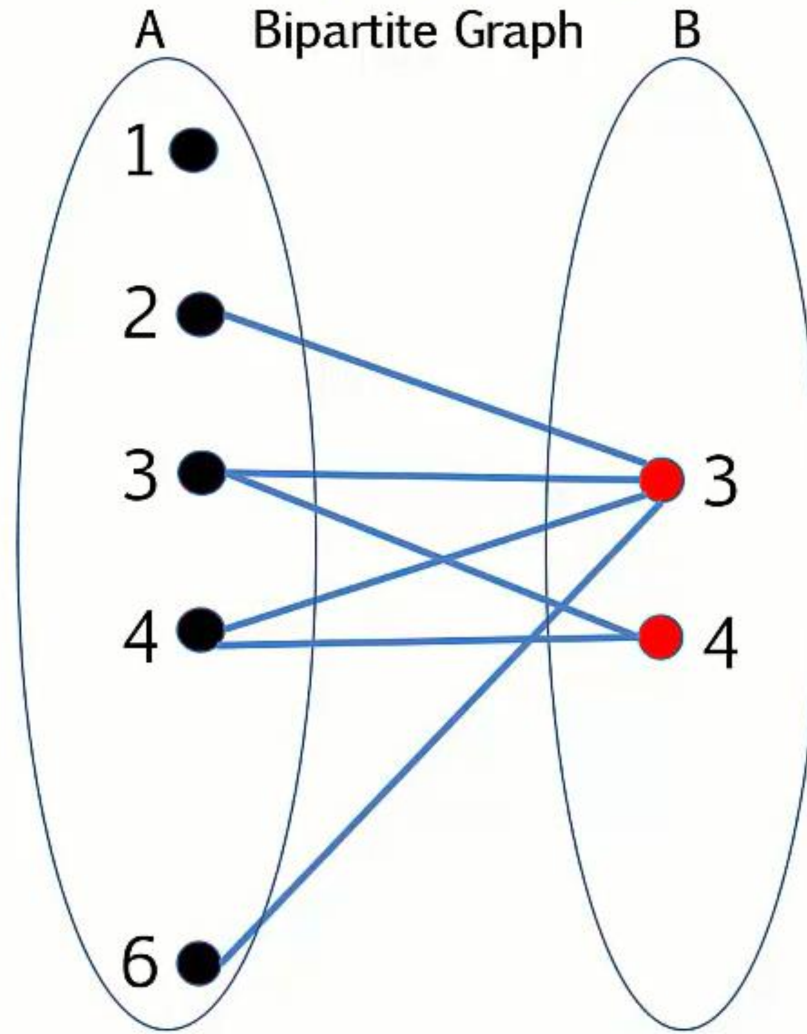


- We can adapt the same greedy algorithm to find a minimal connected dominating set.
- It will be helpful to identify the vertices of A with vertices of G.
- After we select vertex 5 from A, only a vertex adjacent to 5 in G can be selected next.

Greedy Algorithm For MCDS (Contd.)



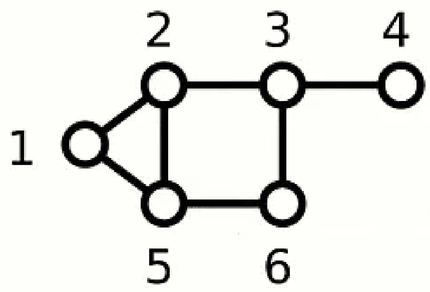
G = (V,E)



- 1, 2 and 6 are adjacent to 5 in G.
- 2 and 6 have the same covering no.
- We select 6 from A as it has the highest id.
- Then we remove vertex 6 from A and vertex 3 from B.
- Now the output set C = {5,6}

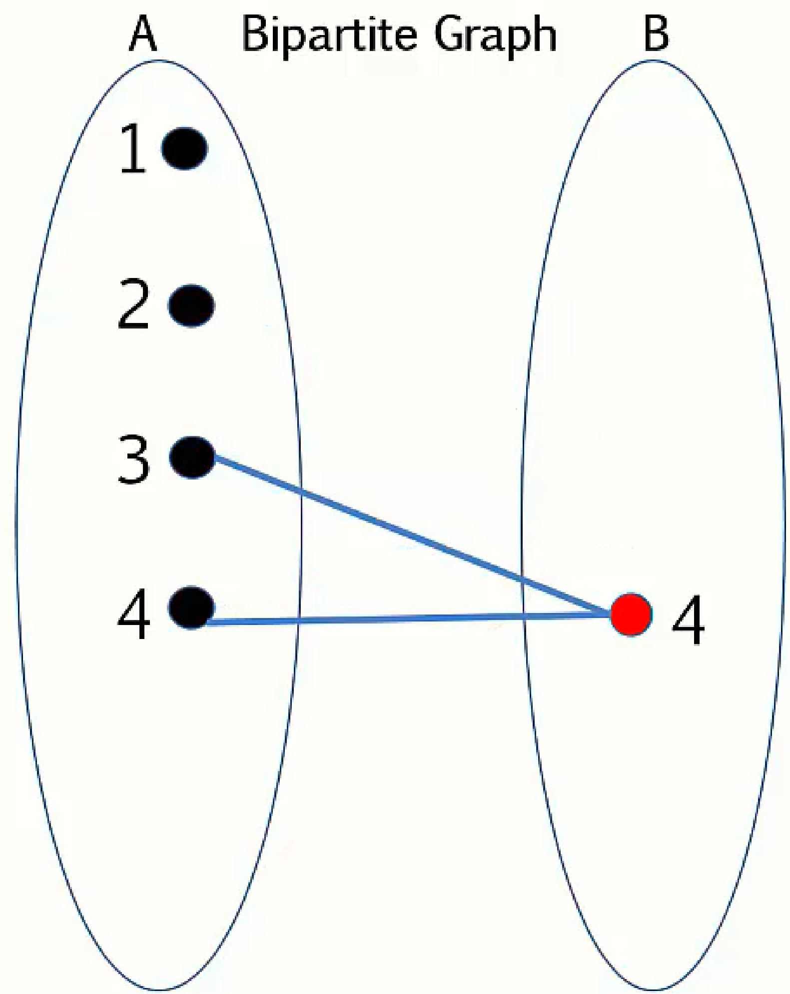
Covering no. of vertex 1 = 0
Covering no. of vertex 2 = 1
Covering no. of vertex 6 = 1

Greedy Algorithm For MCDS (Contd.)

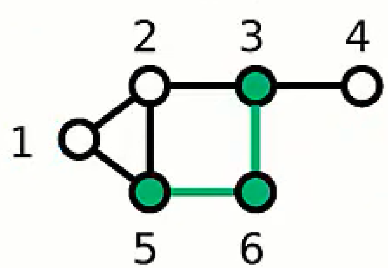


G = (V,E)

Covering no. of vertex 1 = 0
Covering no. of vertex 2 = 0
Covering no. of vertex 3 = 1

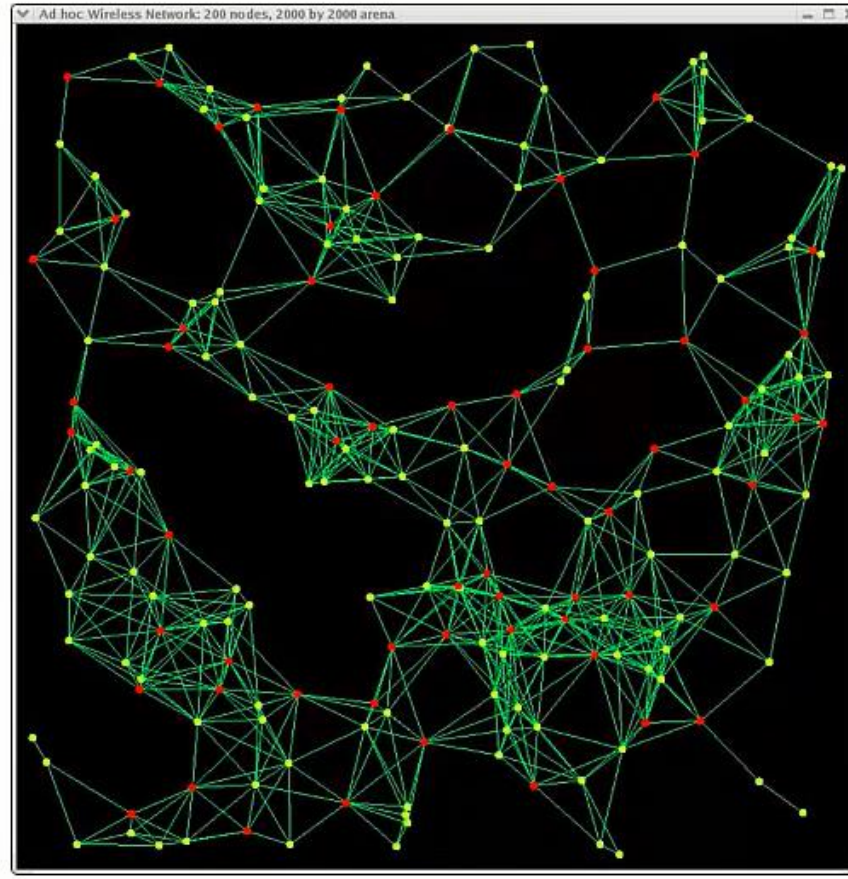


- We must choose a vertex in A with highest covering no.
- 4 is not eligible as it neither adjacent to 5 or 6 in G.
- The eligible vertices are 1, 2 and 3.
- We select 3 as it has the highest covering no.
- Now the output set C = {3,5,6} is a minimal connected dominating set.



Applications

- Mobile ad hoc networks are represented by a connected graph.
- Minimum connected dominating sets are used to do routing in wireless ad hoc networks.
- The dominating sets induce a virtual backbone.



Ad Hoc Network

Conclusions

- Finding a minimum connected dominating set is an NP-complete problem.
- Several approximation algorithms including greedy algorithms have been proposed.
- Connected dominating sets have lots of practical applications in wireless networks.
- Connected dominating sets are often used as a virtual backbone in mobile ad hoc networks for routing.

Thank you