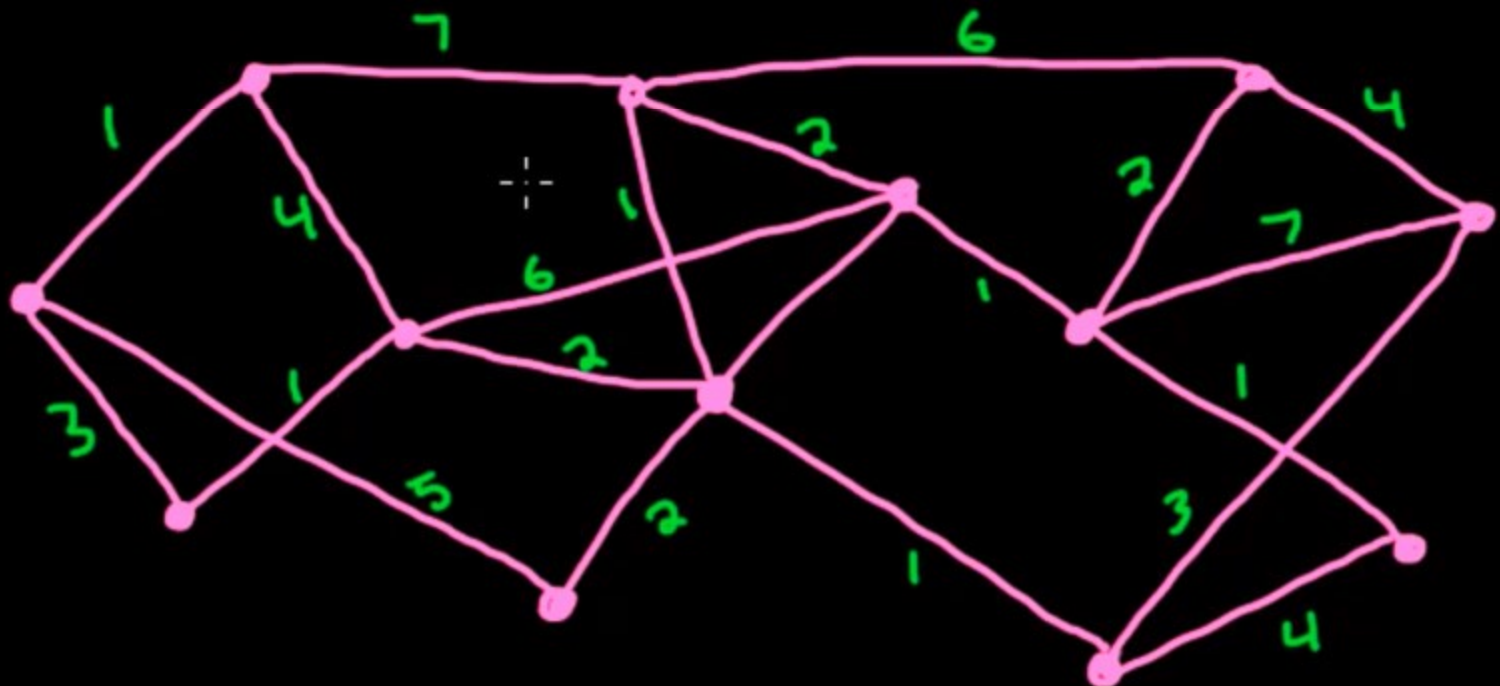


- 1. Dijkstra Algorithm with Examples**
- 2. Bellman Ford Algorithm with Examples**
- 3. All Pairs shortest path (Warshal –Floyd) Algorithm**

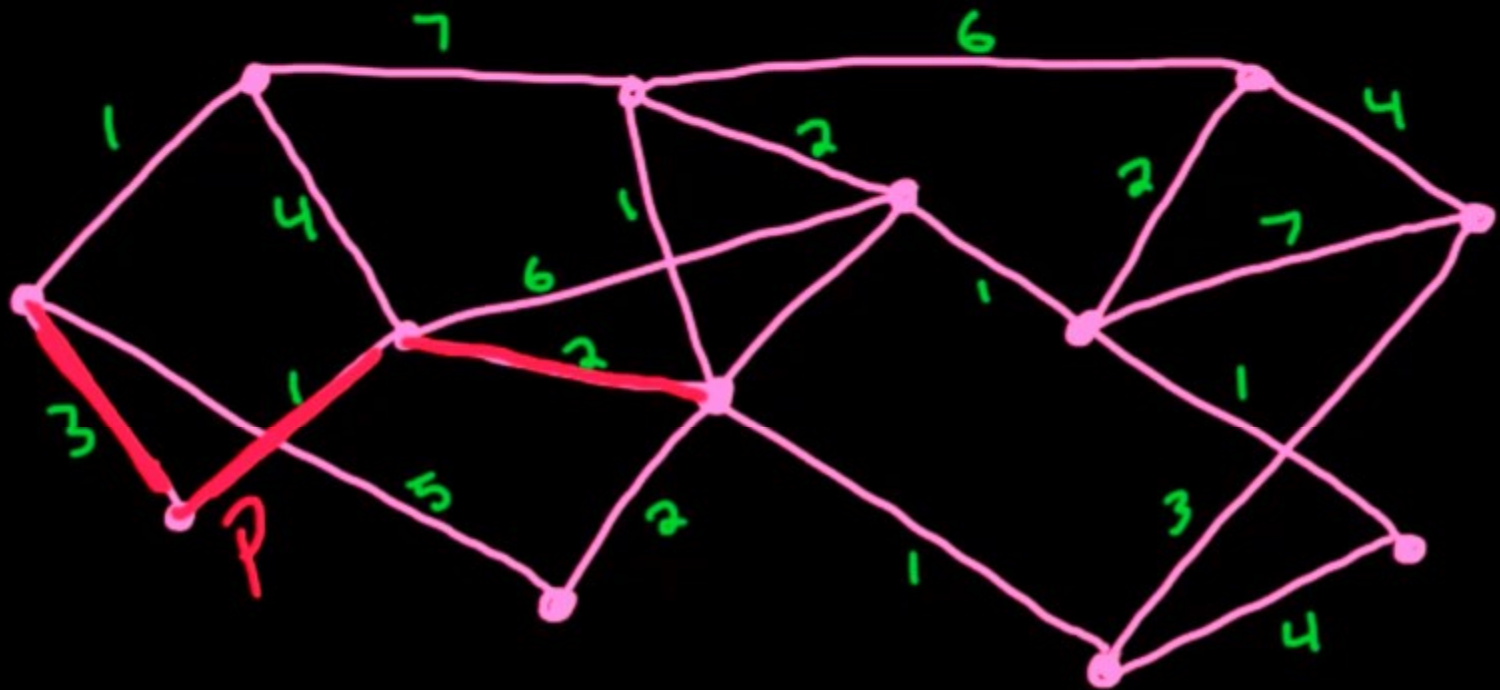
The Shortest Path Problem

Edge weighted Graph: a graph G together with

$$\alpha: E(G) \rightarrow \underbrace{\mathbb{N}}_{\{0,1\}}$$



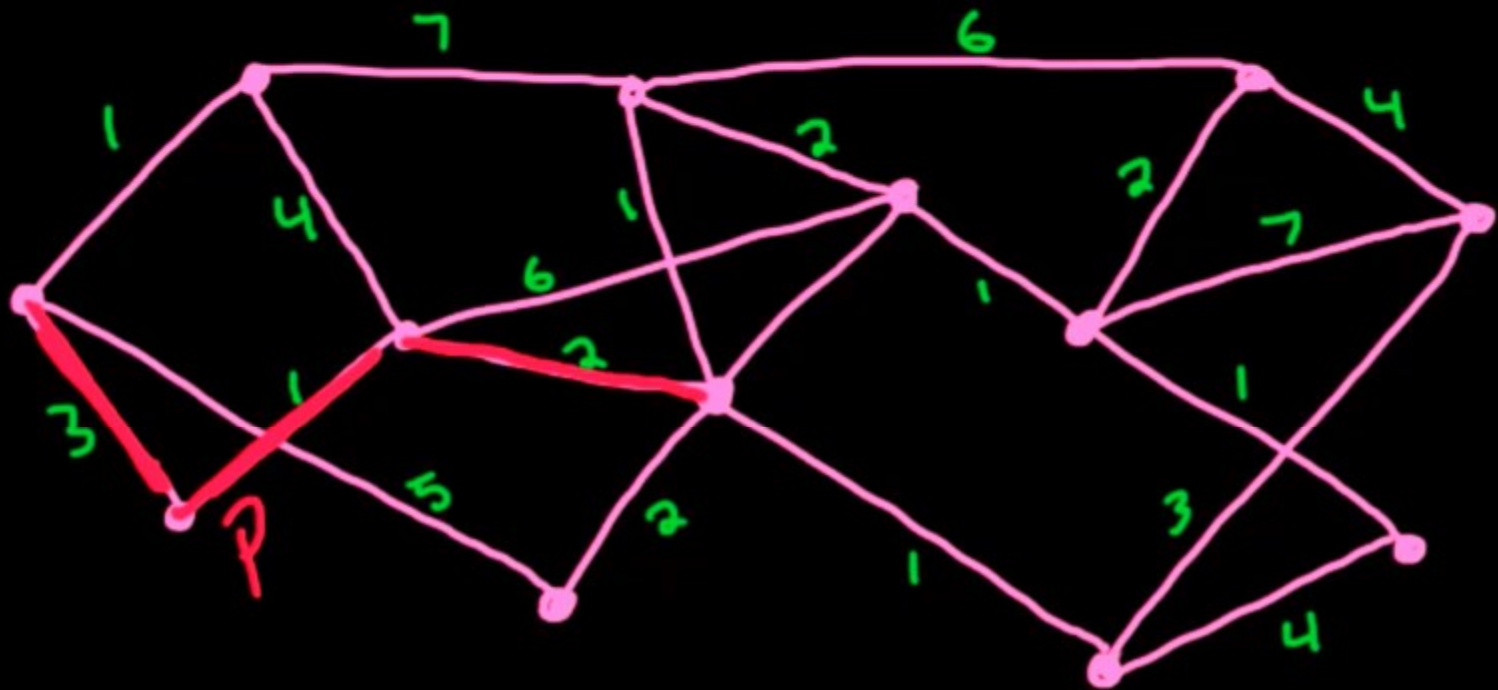
$$\alpha: E(G) \rightarrow \mathbb{N} \cup \{0\}$$



$\alpha(uv)$ called the length of uv

$$\alpha(P) = \sum_{e \in P} \alpha(e) \quad \left(\begin{array}{l} \text{for any} \\ \alpha(H) = \end{array} \right.$$

$$\alpha: E(G) \rightarrow \mathbb{N} \cup \{0\}$$



$\alpha(uv)$ called the length of uv

$$\alpha(P) = \sum_{e \in P} \alpha(e) \quad \left(\begin{array}{l} \text{for any} \\ \alpha(H) = \end{array} \right.$$



$\alpha(uv)$ called the length of uv

$$\alpha(P) = \sum_{e \in P} \alpha(e) \quad \left(\begin{array}{l} \text{for any} \\ \alpha(H) = \end{array} \right.$$

Shortest Path Problem: Given a connected graph G with weight function $\alpha: E(G) \rightarrow \mathbb{N}$, find for given

$$d_G^\alpha(u, v) = \min \{ \alpha(P) \mid P: u \rightarrow v \}$$

For a connected edge weighted graph G and a source vertex $u \in V(G)$, find the minimum weight of a path from u to every other vertex in G . Let $\alpha: E(G) \rightarrow \mathbb{N}$ be the weight function.

Dijkstra's Algorithm:

① Set $S = \emptyset$ which will store the vertices of G for which a shortest path has been found

② Set $t(u) = 0$ and $t(v) = \infty$ for all $v \in V(G)$ $v \neq u$ and add u to S .

③ Let w be the newest member of S .

For each $v \notin S$ $v \in N_G(w)$ set

$$t(v) = \min \{ t(v), t(w) + \alpha(vw) \}$$

④ Pick any $w \notin S$ with minimum $t(w)$ and add w to S
Repeat step ③



function.

Dijkstra's Algorithm:

① Set $S = \emptyset$ which will store the vertices of G for which a shortest path has been found.

② Set $t(u) = 0$ and $t(v) = \infty$ for all $v \in V(G)$ $v \neq u$ and add u to S .

③ Let w be the newest member of S .

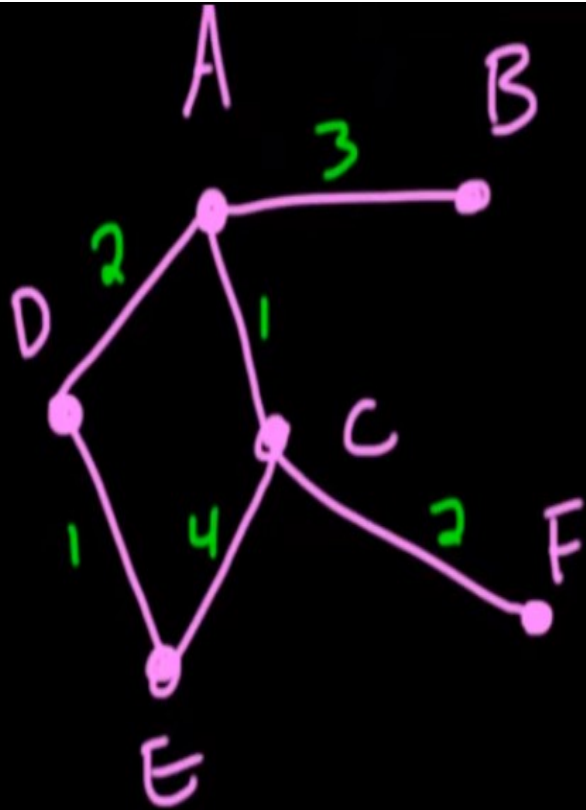
For each $v \notin S$ $v \in N_G(w)$ set

$$t(v) = \min \{ \underline{t(v)}, \underline{t(w) + \alpha(vw)} \}$$

④ Pick any $w \notin S$ with minimum $t(w)$ and add w to S .
Repeat step ③

Solution: For each $v \in S$, $t(v)$ is the minimum weight of a path in G with weight function α .





① $S = \emptyset$

② $t(A) = 0$

$t(B) = t(C) = t(D) = t(E) = t(F) = \infty$

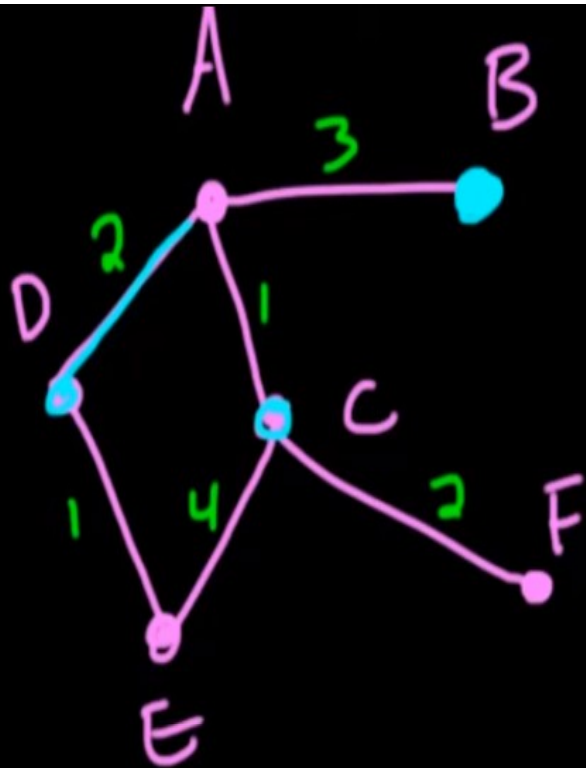
$S = \{A\}$

+

③

$$t(v) = \min \left\{ \underline{t(v)}, \underline{t(w) + \alpha(vw)} \right\}$$





$$\textcircled{1} S = \emptyset$$

$$\textcircled{2} t(A) = 0$$

$$\underline{t(B)} = \underline{t(C)} = \underline{t(D)} = t(E) = t(F) = \infty$$

$$S = \{A\}$$

$$\textcircled{3} t(B) = \min\{\infty, 0 + 3\} = 3$$

$$t(C) = \min\{t(C), t(A) + \alpha(AC)\}$$

$$= \min\{\infty, 0 + 1\} = 1$$

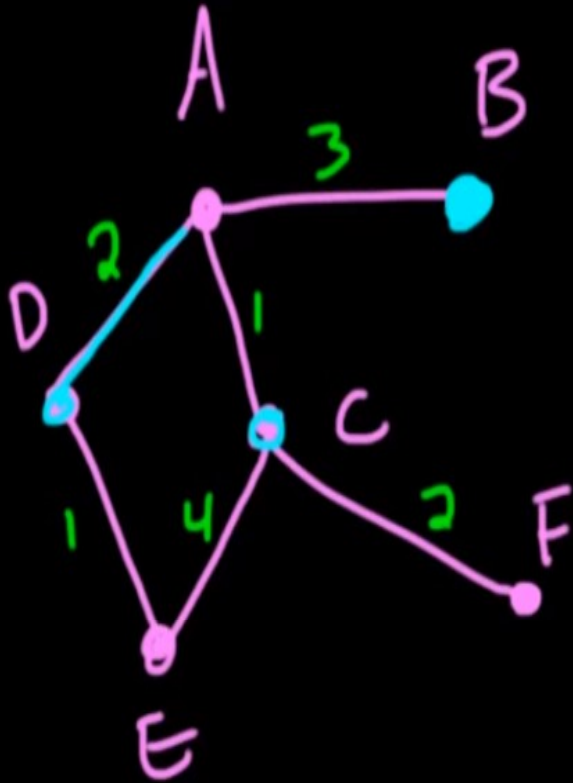
$$t(D) = \min\{\infty, 0 + 2\} = 2$$

$$t(v) = \min\{\underline{t(v)}, \underline{t(w) + \alpha(vw)}\}$$



$\alpha: E(G) \rightarrow \mathbb{N}$

Source vertex: A



① $S = \emptyset$

② $t(A) = 0$

$\underline{t(B)} = \underline{t(C)} = \underline{t(D)} = t(E) = t(F) = \infty$

$S = \{A\}$

③ $t(B) = \min\{\infty, 0 + 3\} = 3$

$t(C) = \min\{t(C), t(A) + \alpha(AC)\}$
 $= \min\{\infty, 0 + 1\} = 1$

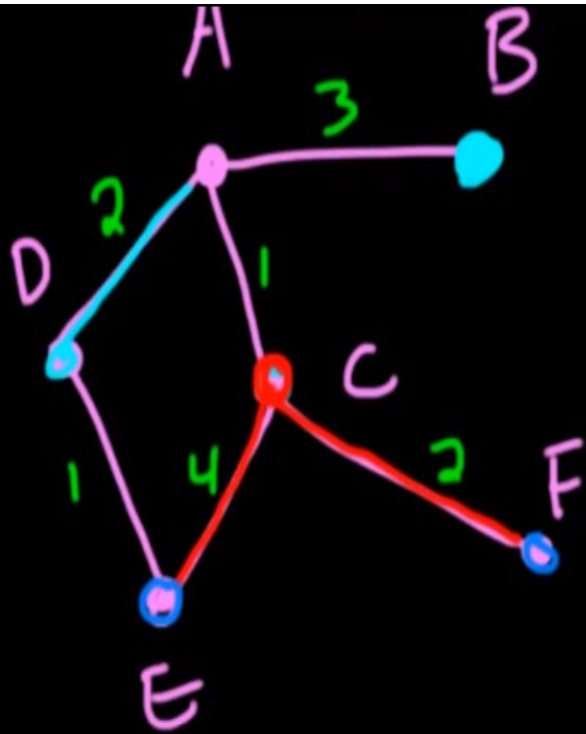
$t(D) = \min\{\infty, 0 + 2\} = 2$

④ Add C to S $S = \{A, C\}$

$t(v) = \min\{\underline{t(v)}, \underline{t(w) + \alpha(vw)}\}$

Dijkstra Algorithm with Examples





$$\textcircled{1} S = \emptyset$$

$$\textcircled{2} t(A) = 0$$

$$\underline{t(B)} = \underline{t(C)} = \underline{t(D)} = t(E) = t(F) = \infty$$

$$S = \{A\}$$

$$\textcircled{3} t(B) = \min\{\infty, 0 + 3\} = 3$$

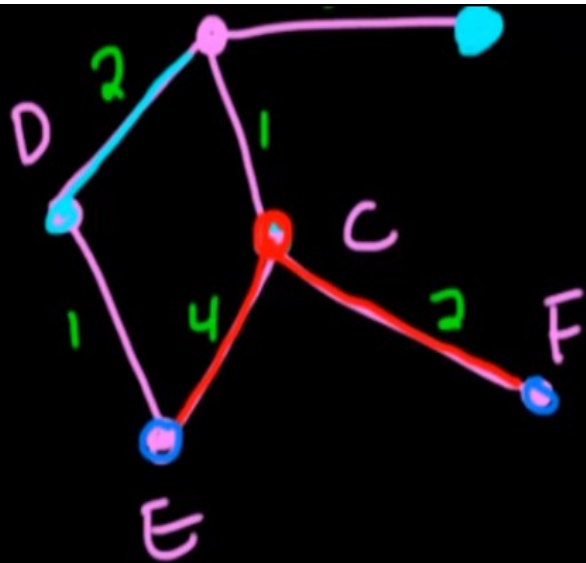
$$t(C) = \min\{t(C), t(A) + w(A, C)\} \\ = \min\{\infty, 0 + 1\} = 1$$

$$t(D) = \min\{\infty, 0 + 2\} = 2$$

$$\textcircled{4} \text{Add } C \text{ to } S \quad S = \{A, C\}$$

$$\textcircled{3} t(E) = \min\{\infty, t(C) + w(C, E)\} = \min\{\infty, 1 + 4\} = 5$$





$$(2) \quad t(A) = 0$$

$$\underline{t(B)} = \underline{t(C)} = \underline{t(D)} = t(E) = t(F) = \infty$$

$$S = \{A\}$$

$$(3) \quad t(B) = \min \{ \infty, 0 + 2 \} = 2$$

$$\begin{aligned} t(C) &= \min \{ t(C), t(A) + \alpha(AC) \} \\ &= \min \{ \infty, 0 + 1 \} = 1 \end{aligned}$$

$$t(D) = \min \{ \infty, 0 + 1 \} = 1$$

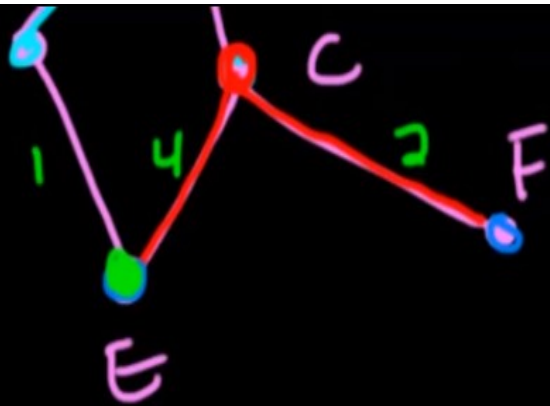
$$(4) \quad \text{Add C to S} \quad S = \{A, C\}$$

$$(3) \quad t(E) = \min \{ \infty, t(C) + \alpha(CE) \} = \min \{ \infty, 1 + 4 \} = 5$$

$$t(F) = 3$$

$$(4) \quad \text{Add D to S} \quad S = \{A, C, D\}$$





$$S = \{A\}$$

$$(3) \quad t(B) = \min \{\infty, 0 + 3\} = 3$$

$$t(C) = \min \{t(C), t(A) + \alpha(AC)\} \\ = \min \{\infty, 0 + 1\} = 1$$

$$t(D) = \min \{\infty, 0 + 2\} = 2$$

$$(4) \quad \text{Add } C \text{ to } S \quad S = \{A, C\}$$

$$(3) \quad t(E) = \min \{\infty, t(C) + \alpha(CE)\} = \min \{\infty, 1 + 4\} = 5 \\ t(F) = 3$$

$$(4) \quad \text{Add } D \text{ to } S \quad S = \{A, C, D\}$$

$$(3) \quad t(E) = \min \{t(E), t(D) + \alpha(DE)\} = \min \{5, 2 + 1\} = 3$$

E

$$t(C) = \min \{t(C), t(A) + \alpha(AC)\} \\ = \min \{\infty, 0 + 1\} = 1$$

$$t(D) = \min \{\infty, 0 + 2\} = 2$$

④ Add C to S $S = \{A, C\}$

$$\textcircled{3} \quad t(E) = \min \{\infty, t(C) + \alpha(CE)\} = \min \{\infty, 1 + 4\} = 5 \\ t(F) = 3$$

④ Add D to S $S = \{A, C, D\}$

$$\textcircled{3} \quad t(E) = \min \{t(E), t(D) + \alpha(DE)\} = \min \{5, 2 + 1\} = 3 \\ \text{Add E to S} \quad S = \{A, C, D, E\} \quad \text{no change}$$



$$= \min\{\infty, 0 + 1\} = 1$$

$$t(D) = \min\{\infty, 0 + 2\} = 2$$

④ Add C to S $S = \{A, C\}$

$$\textcircled{3} \quad t(E) = \min\{\infty, t(C) + \alpha(CE)\} = \min\{\infty, 1 + 4\} = 5$$

$$t(F) = 3$$

④ Add D to S $S = \{A, C, D\}$

$$\textcircled{3} \quad t(E) = \min\{t(E), t(D) + \alpha(DE)\} = \min\{5, 2 + 1\} = 3$$

Add B to S $S = \{A, C, D, B\}$ no change

Add E to S $S = \{A, C, D, B, E\}$ no change

Add F to S $S = \{A, C, D, B, E, F\}$ no change



④ Add C to S

$$\textcircled{3} \quad t(E) = \min \{ \infty, t(C) + \alpha(CE) \} = \min \{ \infty, 1 + 2 \} = 3$$
$$t(F) = 3$$

④ Add D to S $S = \{A, C, D\}$

$$\textcircled{3} \quad t(E) = \min \{ t(E), t(D) + \alpha(DE) \} = \min \{ 3, 3 + 0 \} = 3$$

Add B to S $S = \{A, C, D, B\}$ no

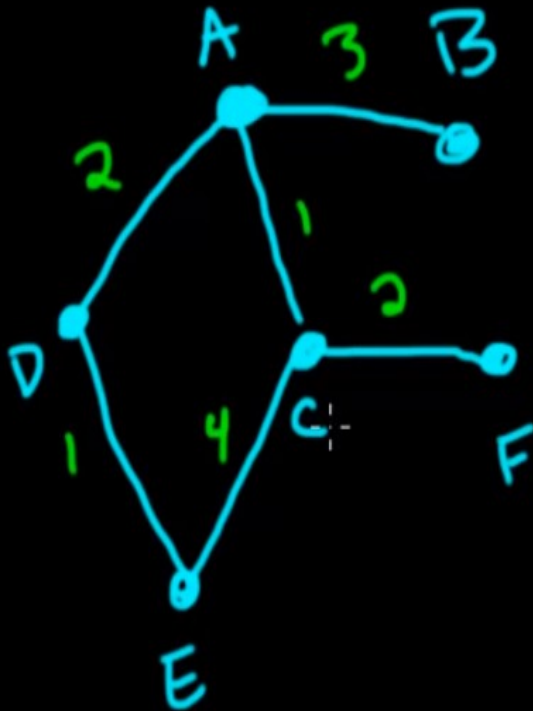
Add E to S $S = \{A, C, D, B, E\}$ no

Add F to S $S = \{A, C, D, B, E, F\}$

0 / 1 1 1 1 1
1 2 3 3 3

Use Dijkstra's Algorithm to find the shortest path weights for the following edge weighted graphs.
Use A as the source vertex.

S



A	0
B	∞
C	∞
D	∞
E	∞
F	∞



Use Dijkstra's Algorithm to find the shortest path weights for the following edge weighted graphs.
Use A as the source vertex.

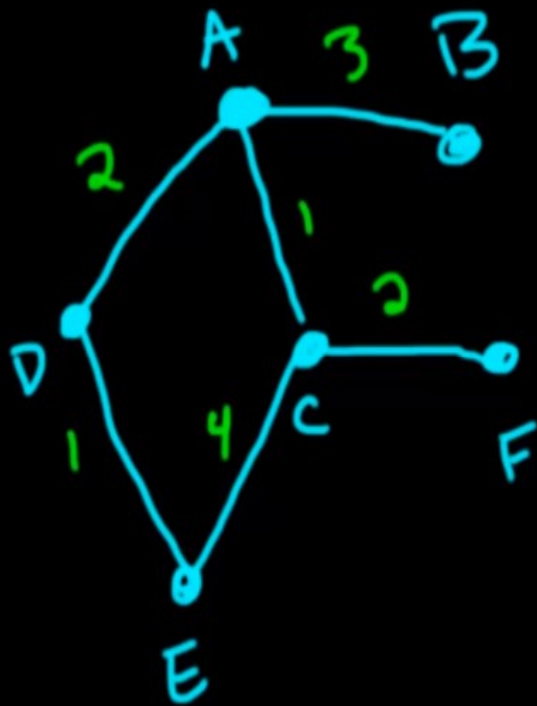
5



A	0	
B	∞	3
C	∞	1
D	∞	2
E	∞	∞
F	∞	∞



Use Dijkstra's Algorithm to find the shortest path weights for the following edge weighted graphs.
Use A as the source vertex.



A	0		
B	∞	3	
C	∞	1	
D	∞	2	
E	∞	∞	5
F	∞	∞	3



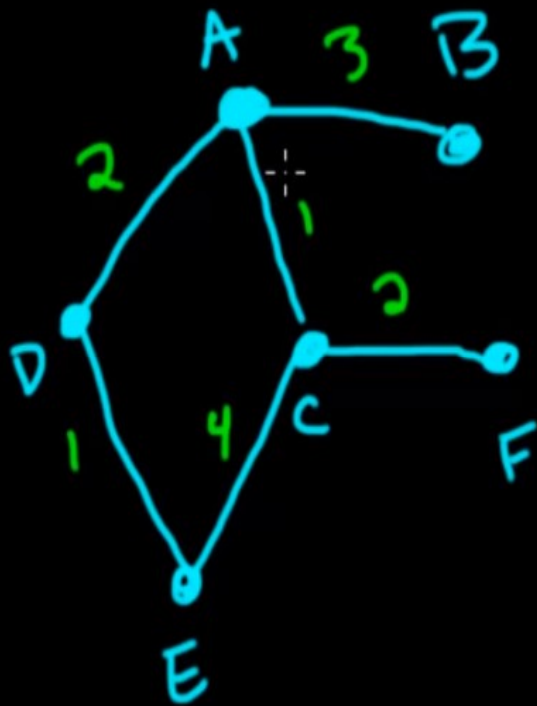
Use Dijkstra's Algorithm to find the shortest path weights for the following edge weighted graphs. Use A as the source vertex.



5

A	0				
B	∞	3	3	3	
C	∞	1			
D	∞	2	2		
E	∞	∞	5	3	
F	∞	∞	3	3	

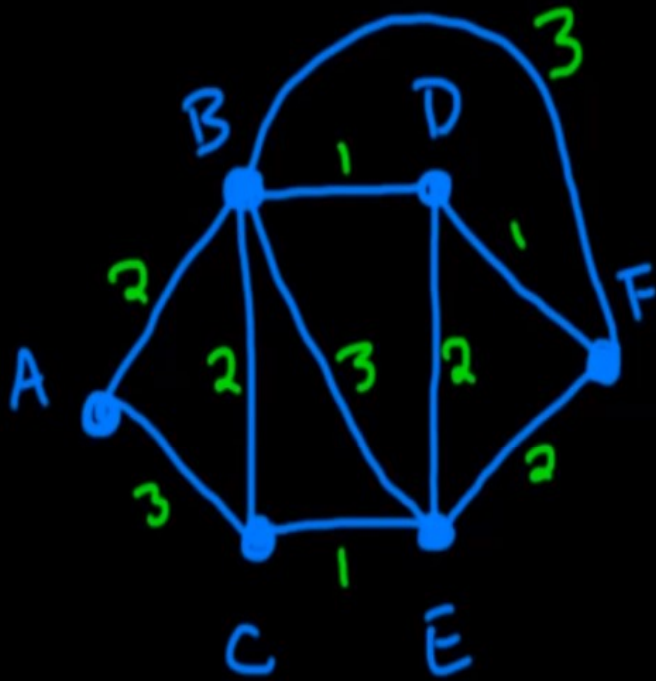
Use Dijkstra's Algorithm to find the shortest path weights for the following edge weighted graphs.
Use A as the source vertex.



5

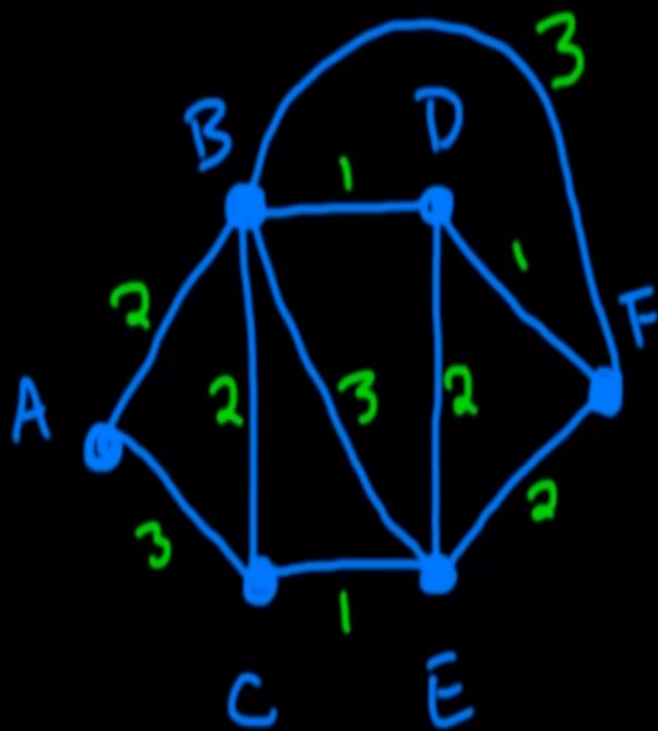
A	0					
B	∞	3	3	3		
C	∞	1				
D	∞	2	2			
E	∞	∞	5	3	3	
F	∞	∞	3	3	3	3





A	0
B	∞
C	∞
D	∞
E	∞
F	∞

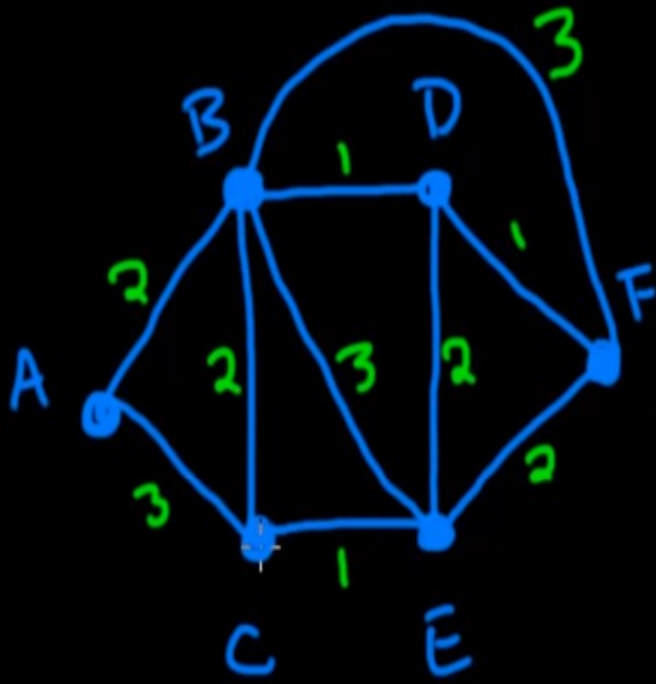




+

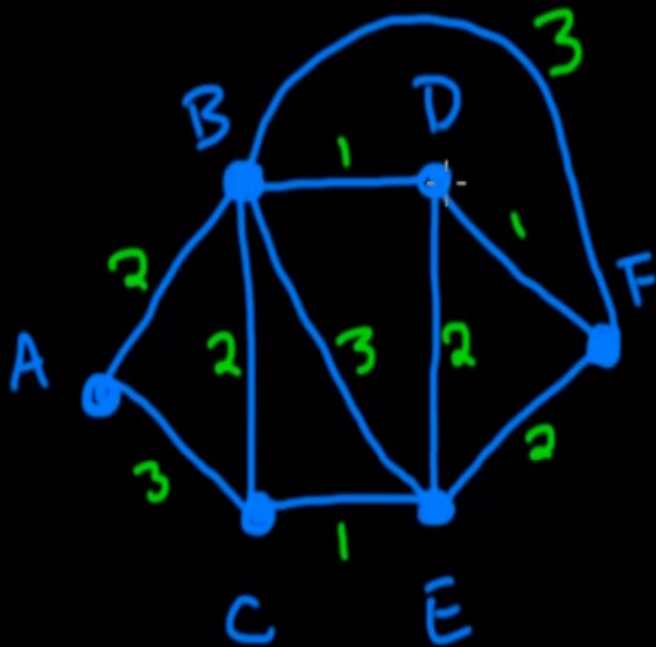
A	0	
B	∞	2
C	∞	3
D	∞	∞
E	∞	∞
F	∞	∞





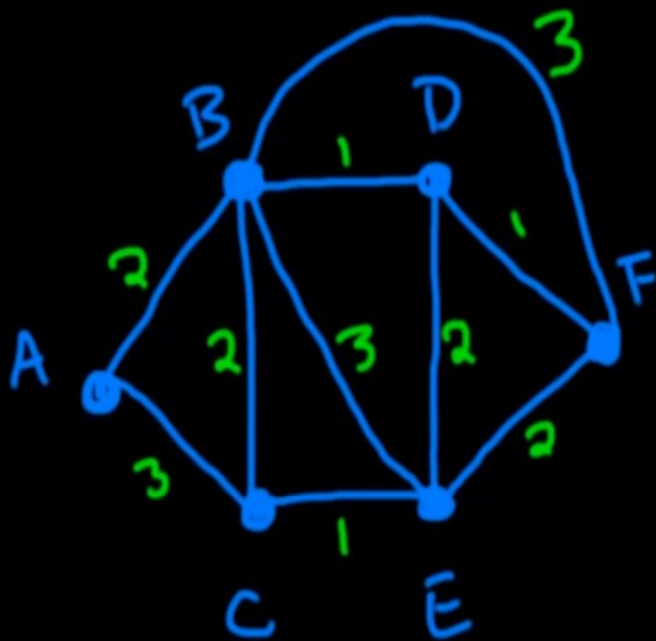
A	0			
B	∞	2		
C	∞	3	3	
D	∞	∞	3	
E	∞	∞	5	
F	∞	∞	5	





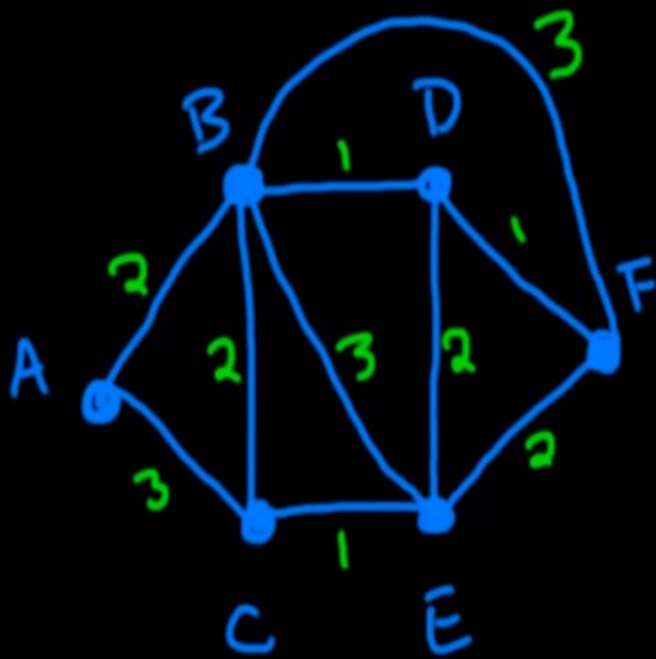
A	0				
B	∞	2			
C	∞	3	3		
D	∞	∞	3	3	
E	∞	∞	5	4	
F	∞	∞	5	5	





A	0					
B	∞	2				
C	∞	3	3			
D	∞	∞	3	3		
E	∞	∞	5	4	4	
F	∞	∞	5	5	4	

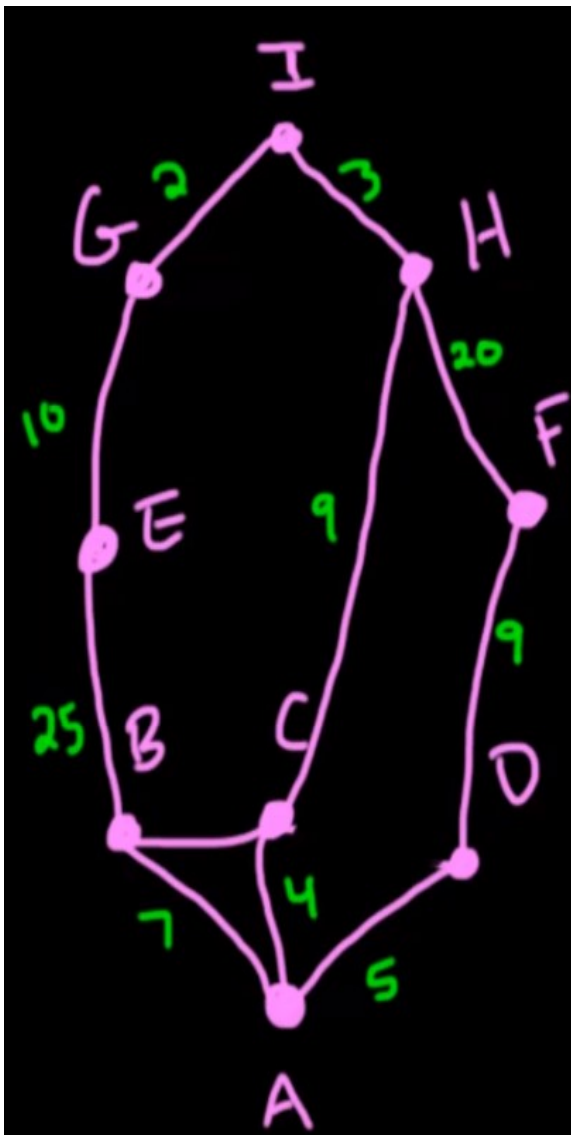




A	0					
B	∞	2				
C	∞	3	3			
D	∞	∞	3	3		
E	∞	∞	5	4	4	
F	∞	∞	5	5	4	4

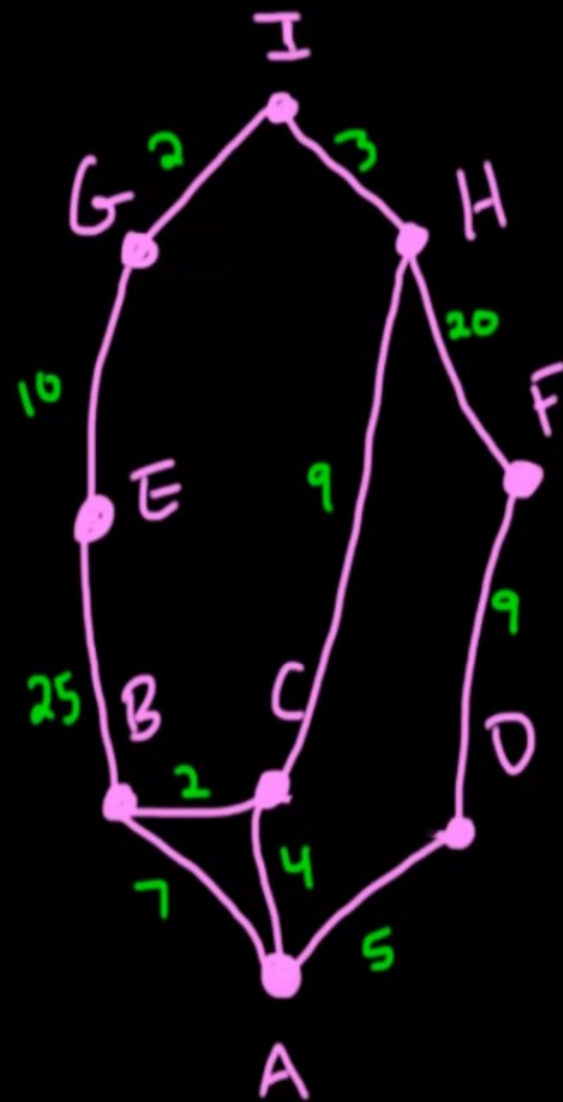
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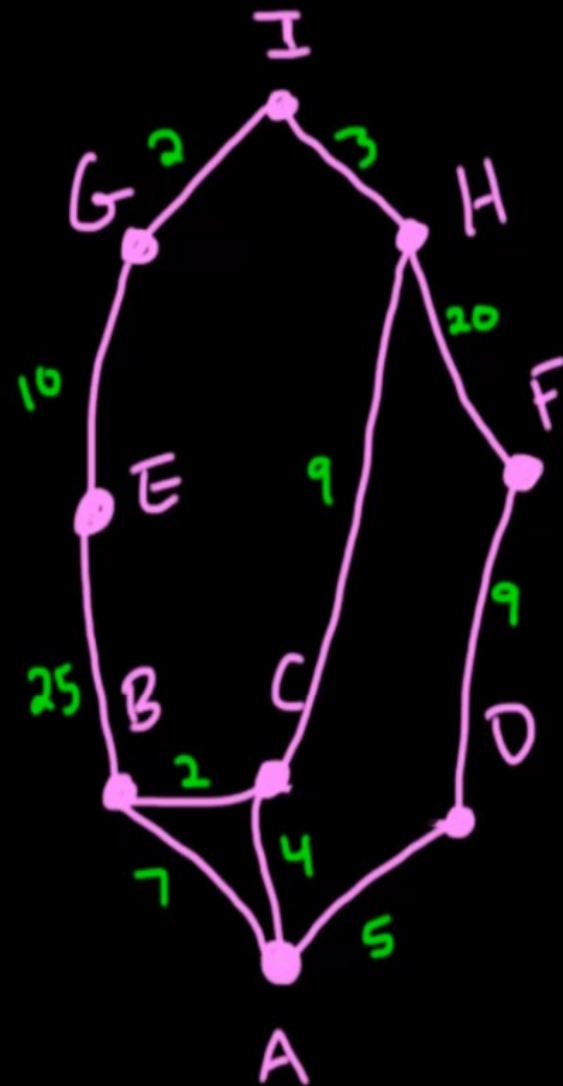
A	0
B	∞
C	∞
D	∞
E	∞
F	∞
G	∞
H	∞
I	∞





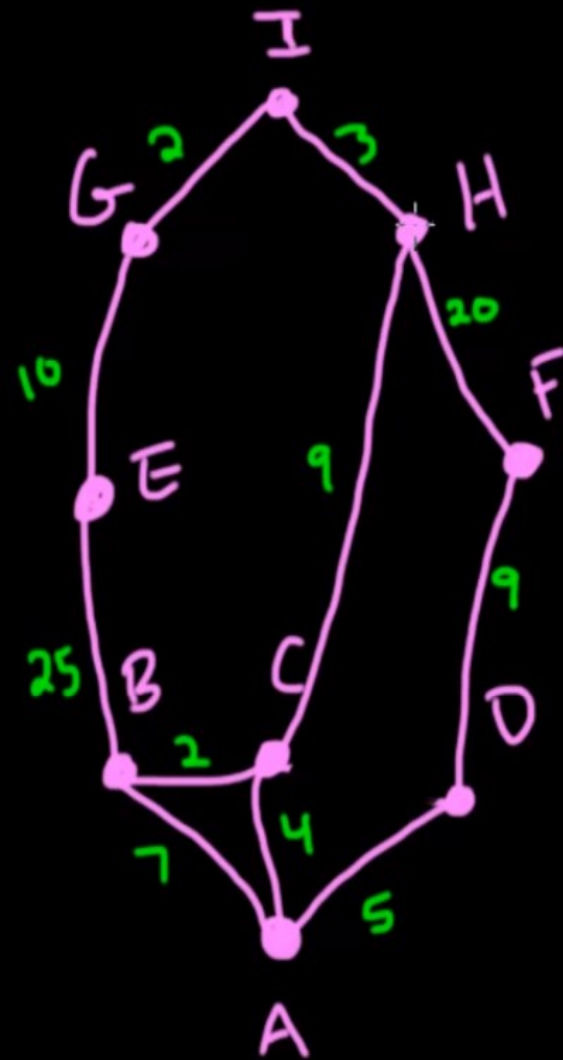
A	0	
B	∞	7
C	∞	4
D	∞	5
E	∞	∞
F	∞	∞
G	∞	∞
H	∞	∞
I	∞	∞





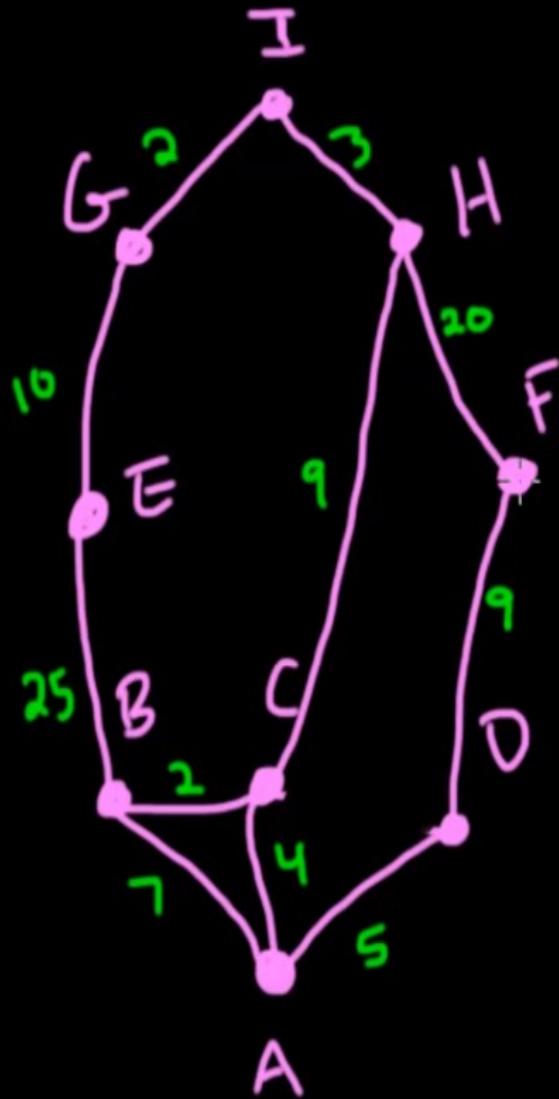
A	0			
B	∞	7		6
C	∞	4		
D	∞	5	5	
E	∞	∞	∞	
F	∞	∞	∞	
G	∞	∞	∞	
H	∞	∞	∞	13
I	∞	∞	∞	∞





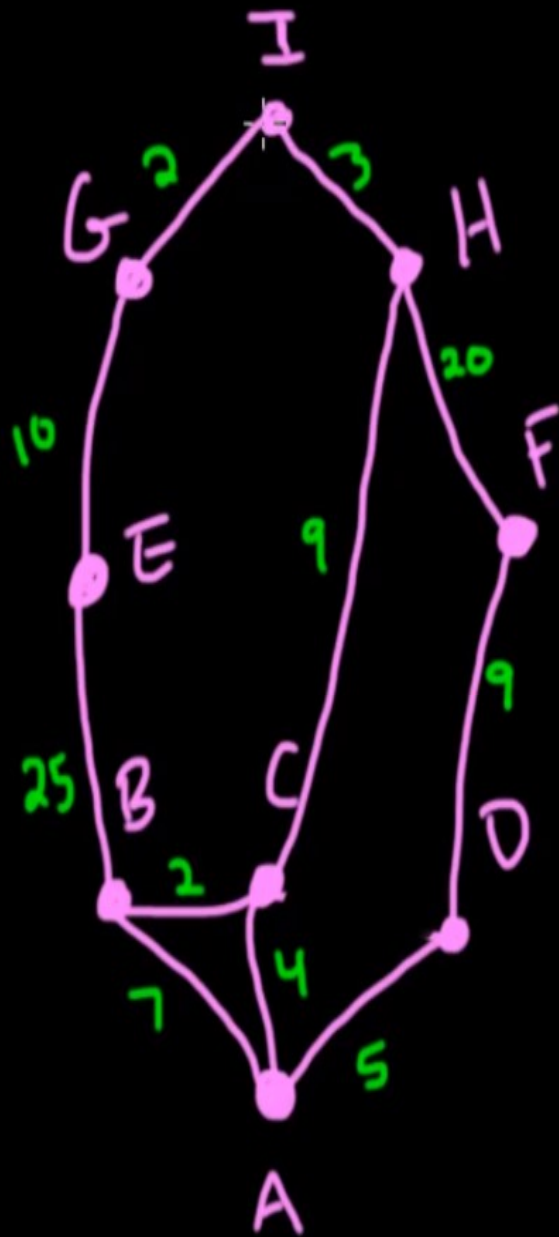
A	0							
B	∞	7	6					
C	∞	4						
D	∞	5						
E	∞	∞	∞	∞	31			
F	∞	∞	∞	14	14			
G	∞	∞	∞	∞	∞			
H	∞	∞	13	13	13			
I	∞	∞	∞	∞	∞	∞		





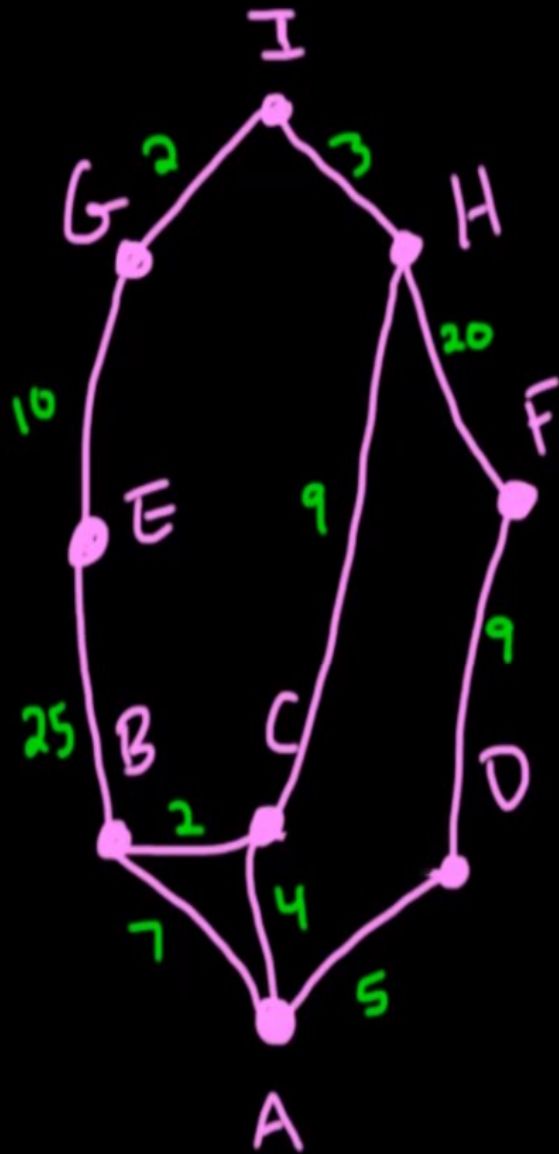
A	0							
B	∞	7	6	6				
C	∞	4						
D	∞	5	5					
E	∞	∞	∞	∞	31	31		
F	∞	∞	∞	14	14	14		
G	∞	∞	∞	∞	∞	∞		
H	∞	∞	13	13	13	13		
I	∞	∞	∞	∞	∞	∞	16	





A	0							
B	∞	7	6					
C	∞	4						
D	∞	5						
E	∞	∞	∞	∞	31	31	31	
F	∞	∞	∞	14	14	14		
G	∞	∞	∞	∞	∞	∞	∞	
H	∞	∞	13	13	13			
I	∞	∞	∞	∞	∞	16	16	

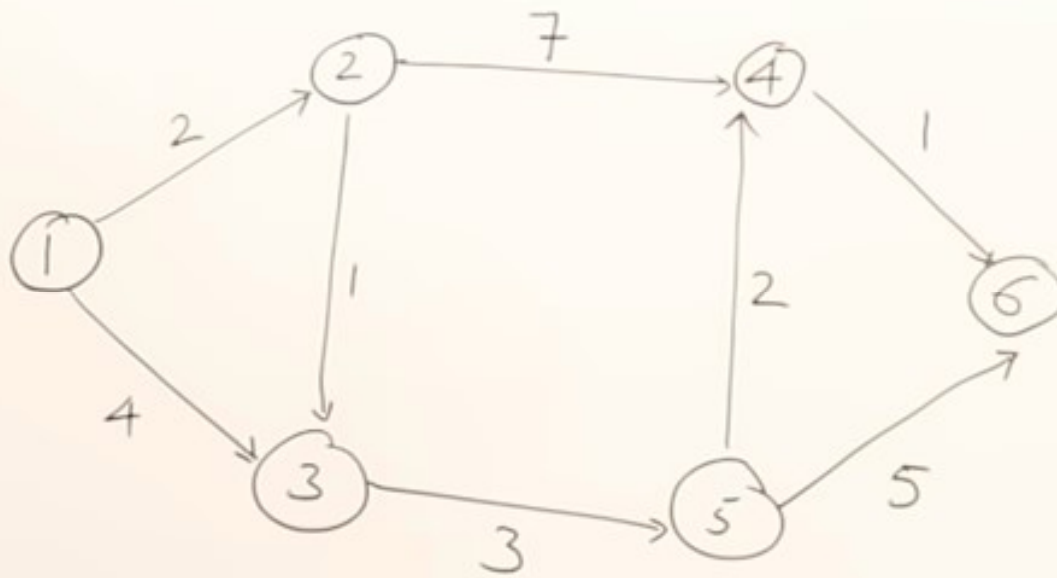




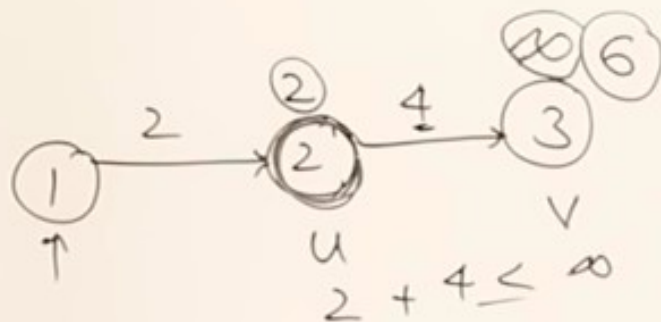
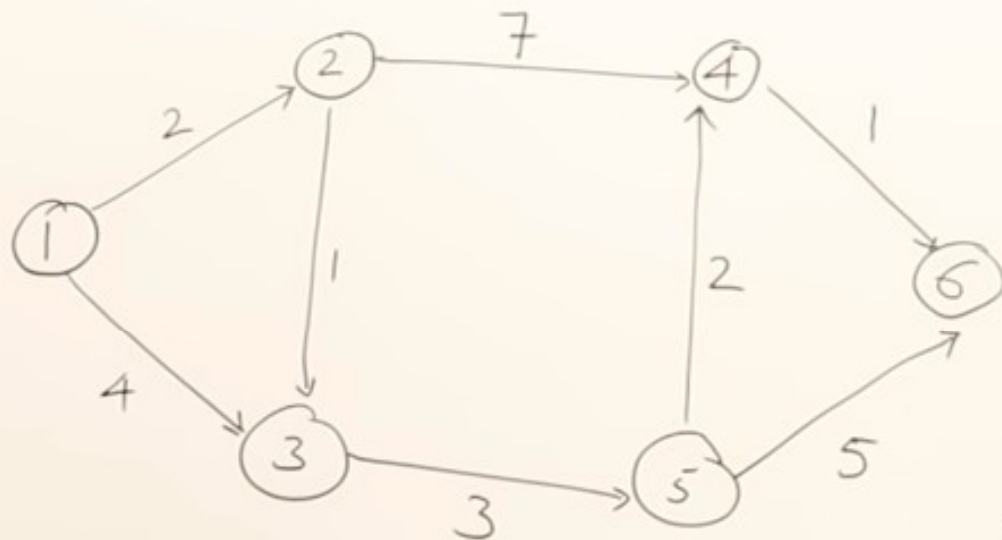
A	0								
B	∞	7	6	6					
C	∞	4							
D	∞	5	5						
E	∞	∞	∞	∞	31	31	31	31	28
F	∞	∞	∞	14	14	14			
G	∞	∞	∞	∞	∞	∞	∞	18	
H	∞	∞	13	13	13				
I	∞	∞	∞	∞	∞	16	16		



Dijkstra Algorithm

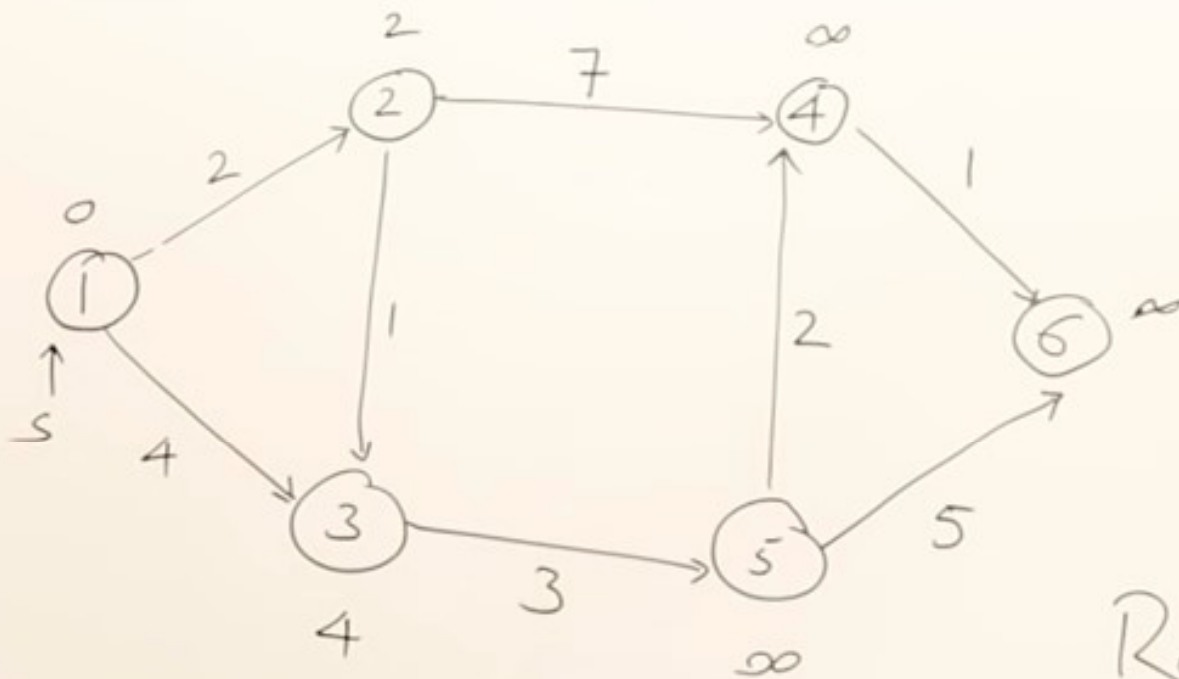


Dijkstra Algorithm



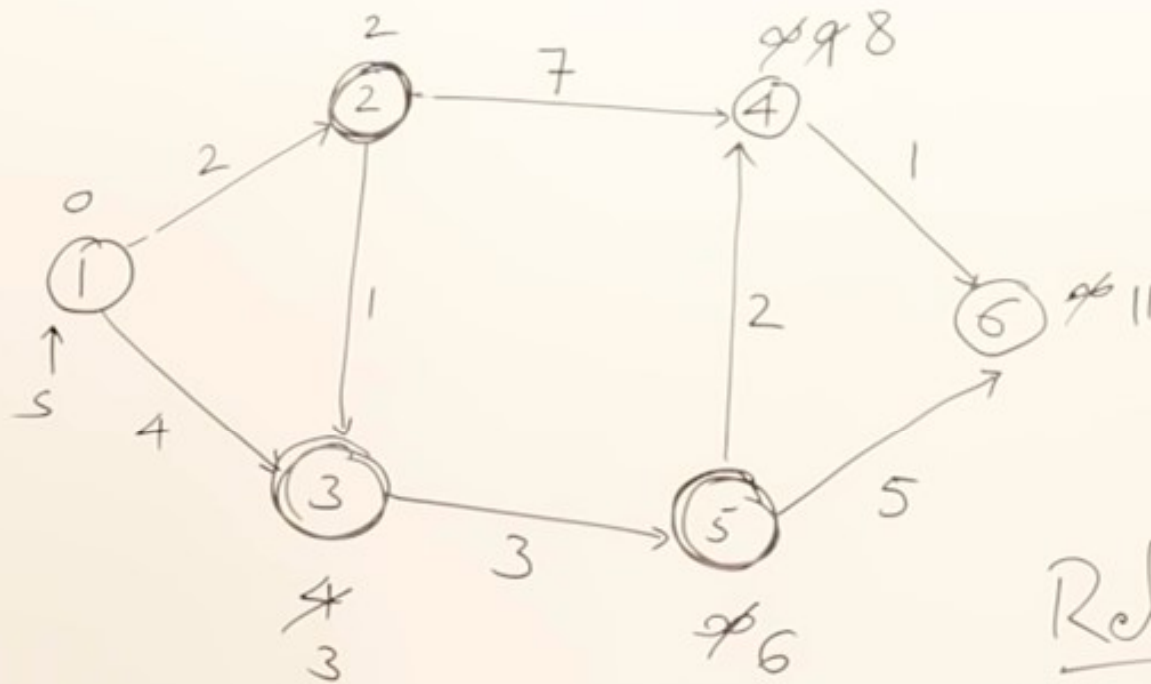
Relaxation
if $(d[u] + c(u, v) < d[v])$
 $d[v] = d[u] + c(u, v)$

Dijkstra Algorithm



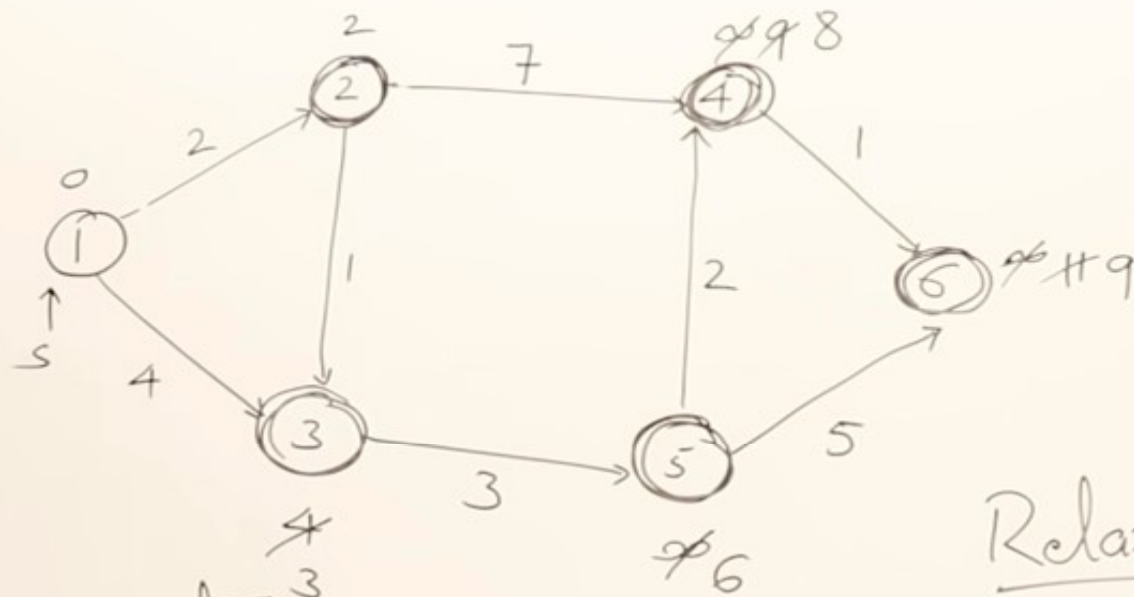
Relaxation
if $(d[u] + c(u, v) < d[v])$
 $d[v] = d[u] + c(u, v)$

Dijkstra Algorithm



Relaxation
if $(d[u] + c(u, v) < d[v])$
 $d[v] = d[u] + c(u, v)$

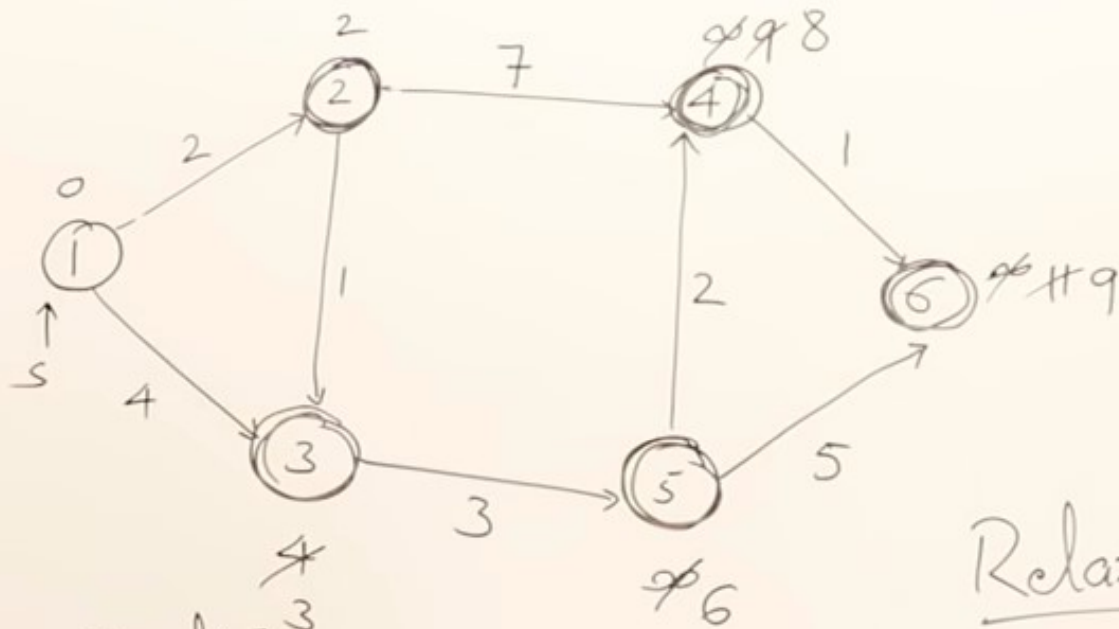
Dijkstra Algorithm



v	$d[v]$
2	2
3	3
4	8
5	6
6	9

Relaxation
 if $(d[u] + c(u, v) < d[v])$
 $d[v] = d[u] + c(u, v)$

Dijkstra Algorithm



v	d[v]
2	2
3	3
4	8
5	6
6	9

$$n = |V| \quad |V|$$

$$n \times n = n^2 \quad O(|V|^2)$$

$$O(n^2)$$

Relaxation

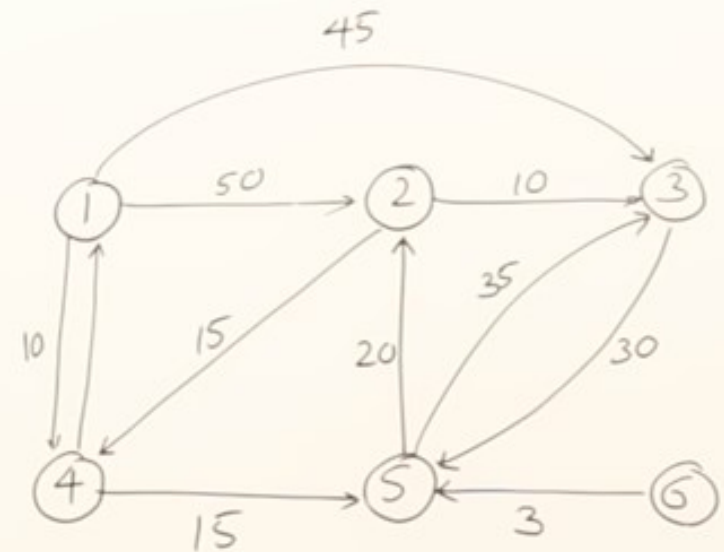
$$\text{if } (d[u] + c(u, v) < d[v])$$

$$d[v] = d[u] + c(u, v)$$

Dijkstra Algorithm

starting vertex 1

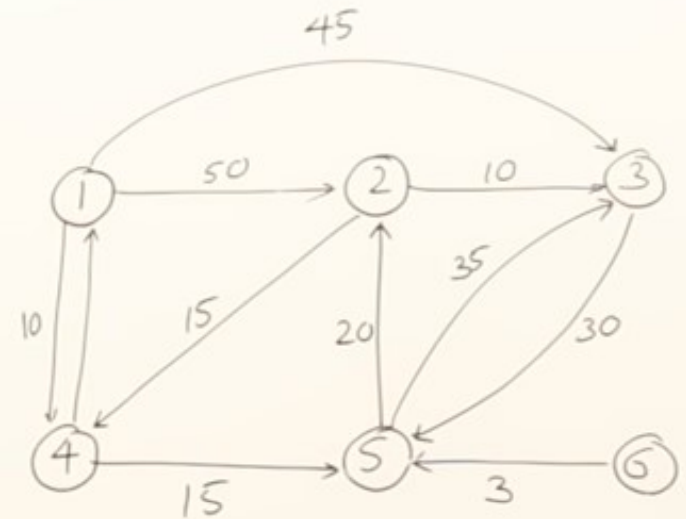
selected vertex	2	3	4	5	6
4	50	45	(10)	∞	∞
5	50	45	(10)	(25)	∞
	45	45	(10)	(25)	∞



Dijkstra Algorithm

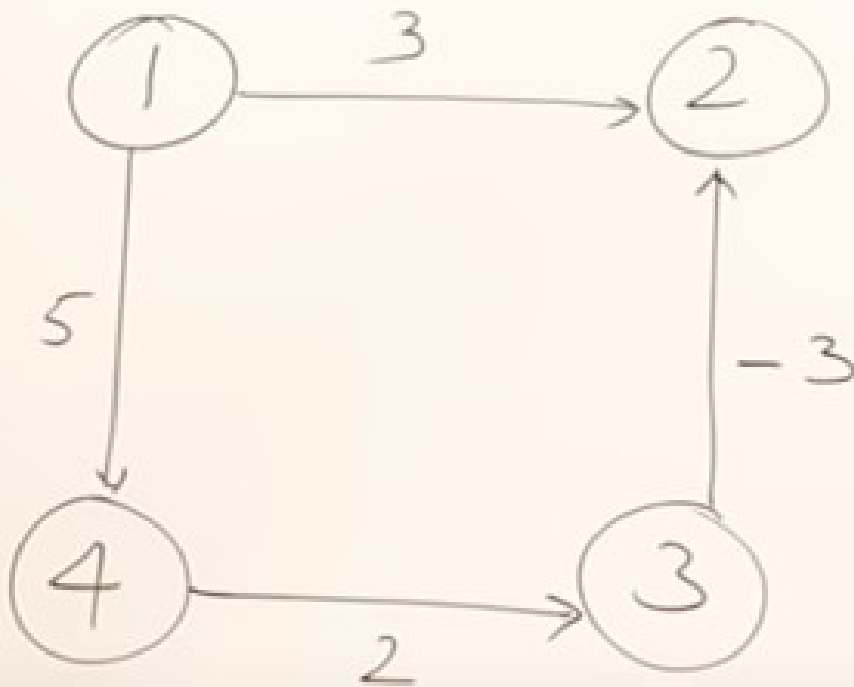
starting vertex 1

selected vertex	2	3	4	5	6
4	50	45	(10)	∞	∞
5	50	45	(10)	(25)	∞
2	(45)	45	(10)	(25)	∞
3	(45)	(45)	(10)	(25)	∞
6	(45)	(45)	(10)	(25)	(20)

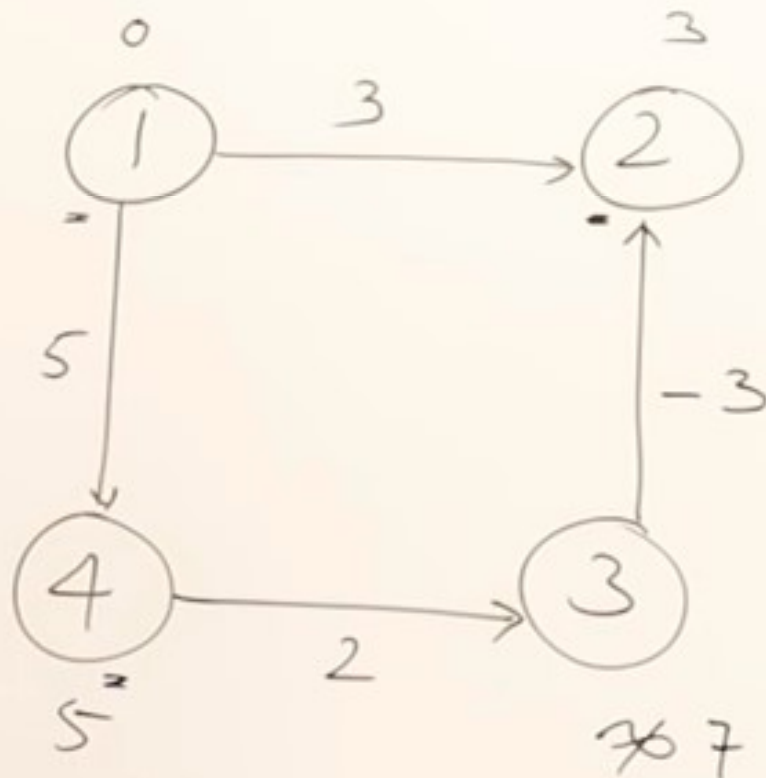


Dijkstra Algorithm

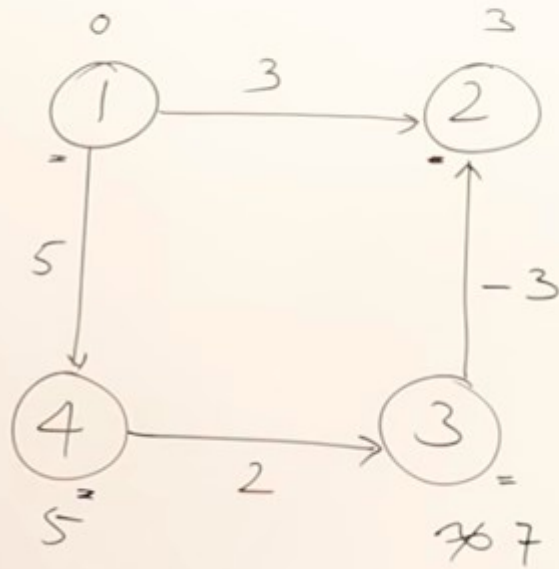
Negative
weight
(demerit)



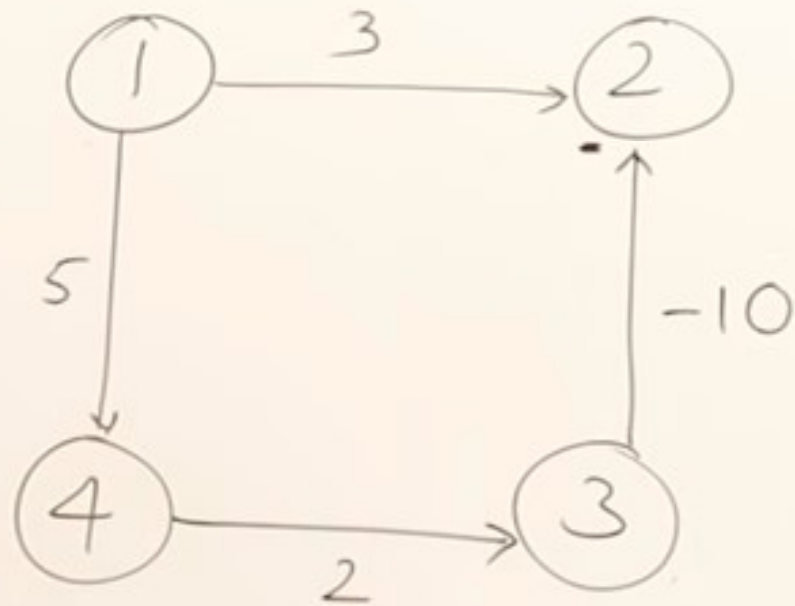
Dijkstra Algorithm



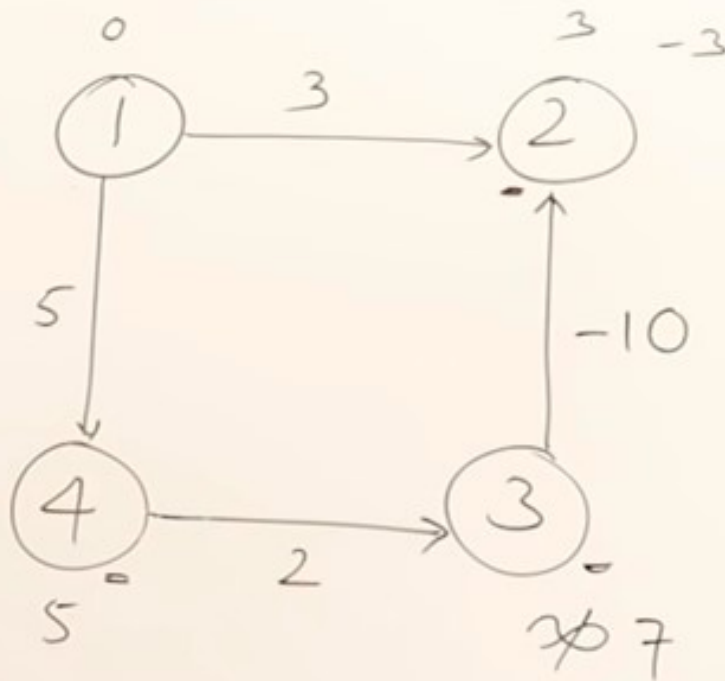
Dijkstra Algorithm



Dijkstra Algorithm

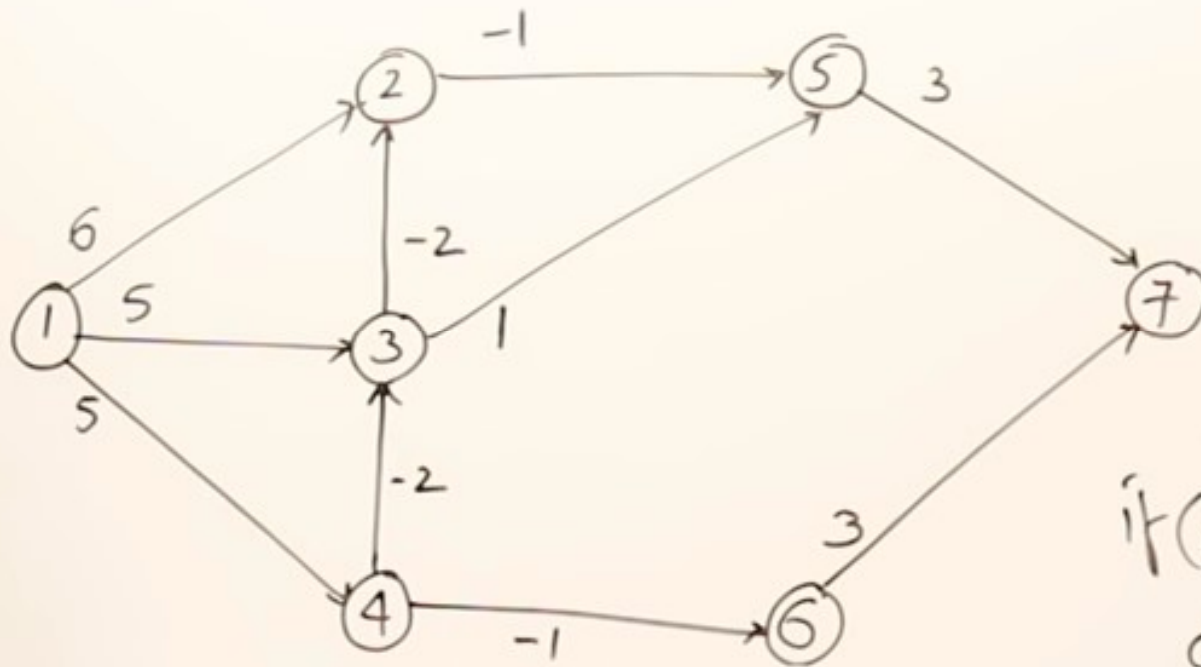


Dijkstra Algorithm



Single Source Shortest Path

Bellman-Ford

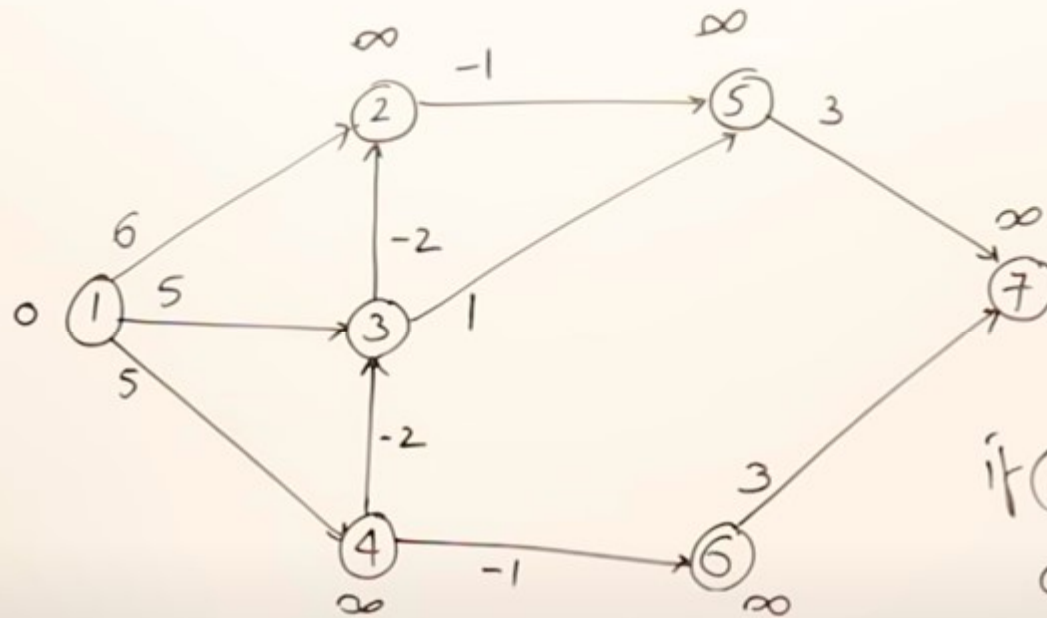


(u, v)

Relaxation

if $(d[u] + c(u, v) < d[v])$
 $d[v] = d[u] + c(u, v)$

Single Source Shortest Path
Bellman-Ford



(u, v)

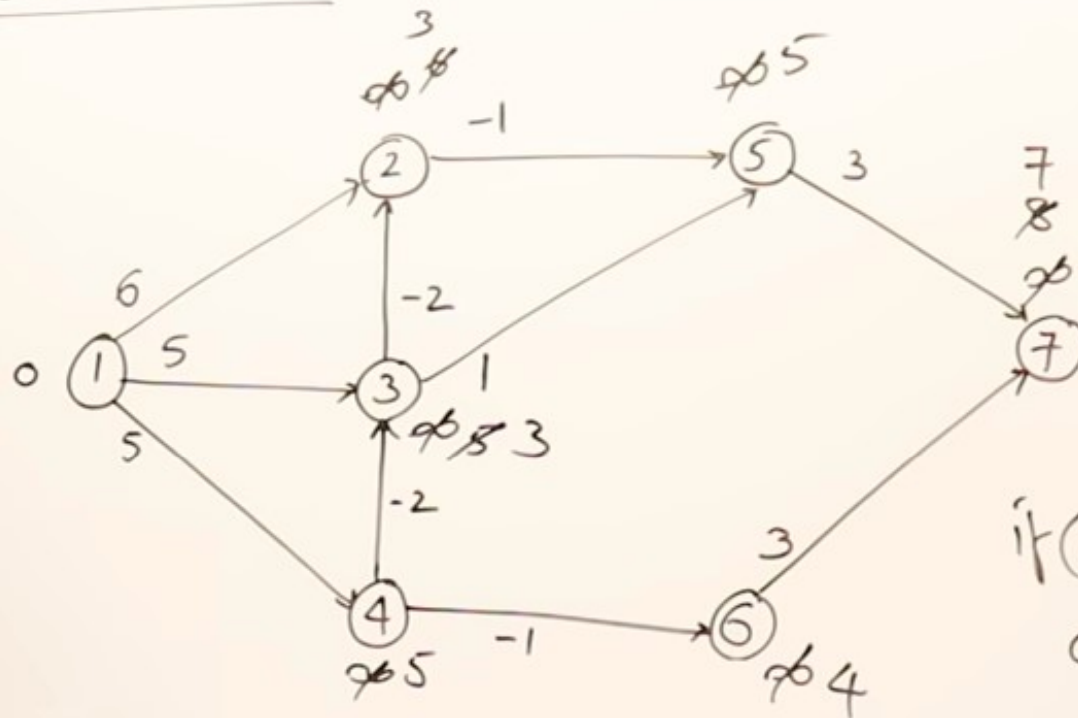
Relaxation

if $(d[u] + c(u, v) < d[v])$
 $d[v] = d[u] + c(u, v)$

edgeList $\rightarrow (1, 2), (1, 3), (1, 4), (2, 5), (3, 2), (3, 5), (4, 3), (4, 6), (5, 7), (6, 7)$

Single Source Shortest Path

Bellman-Ford



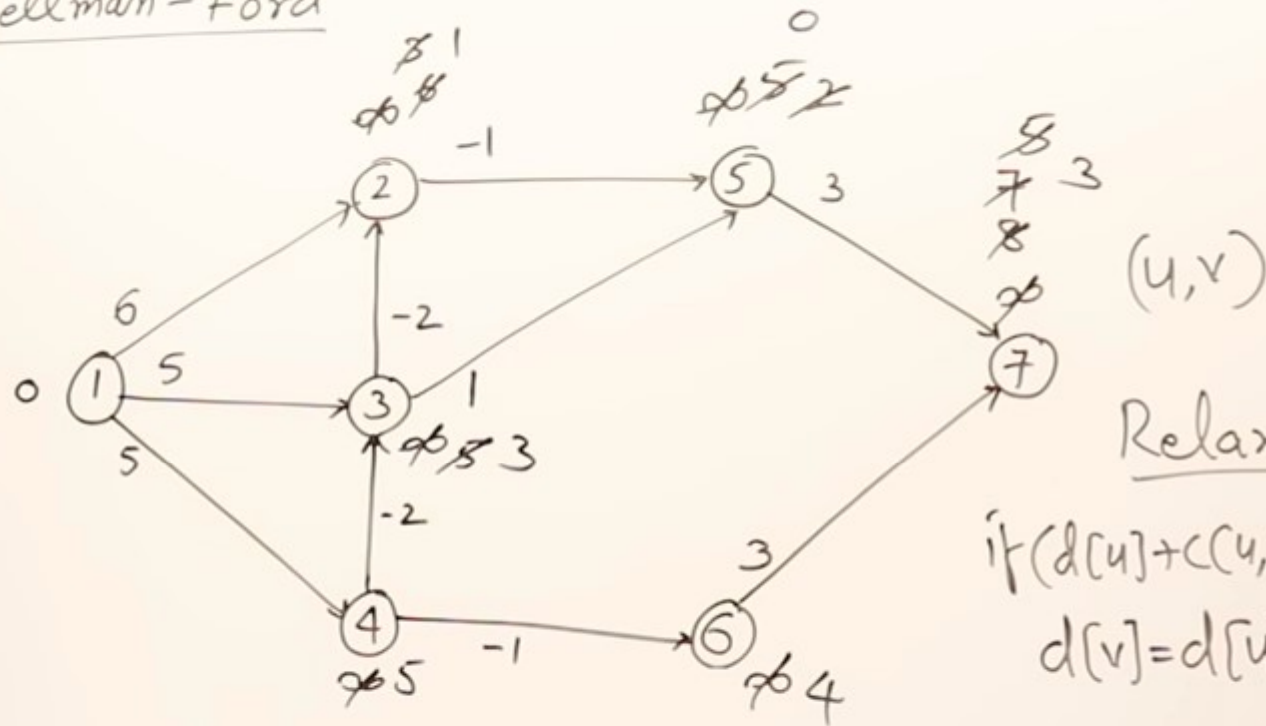
(u,v)

Relaxation

if ($d[u] + c(u,v) < d[v]$)
 $d[v] = d[u] + c(u,v)$

edgeList $\rightarrow (1,2), (1,3), (1,4), (2,5), (3,2), (3,5), (4,3), (4,6), (5,7), (6,7)$

Single Source Shortest Path Bellman-Ford



(u,v)

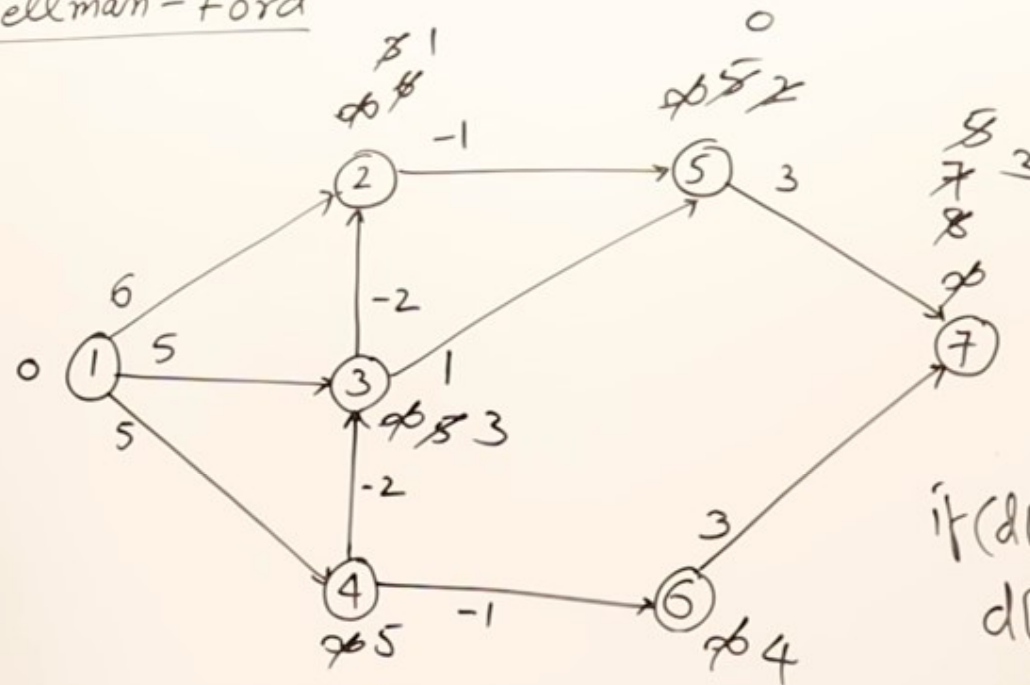
Relaxation

if ($d[u] + c(u,v) < d[v]$)
 $d[v] = d[u] + c(u,v)$

edgeList → (1,2), (1,3), (1,4), (2,5), (3,2), (3,5), (4,3), (4,6), (5,7), (6,7)

Single Source Shortest Path Bellman-Ford

1-0
 2-1
 3-3
 4-5
 5-0
 6-4
 7-3



(u,v)

Relaxation

if $(d[u] + c(u,v) < d[v])$
 $d[v] = d[u] + c(u,v)$

edgeList $\rightarrow (1,2), (1,3), (1,4), (2,5), (3,2), (3,5), (4,3), (4,6), (5,7), (6,7)$

Single Source Shortest Path

Bellman-Ford

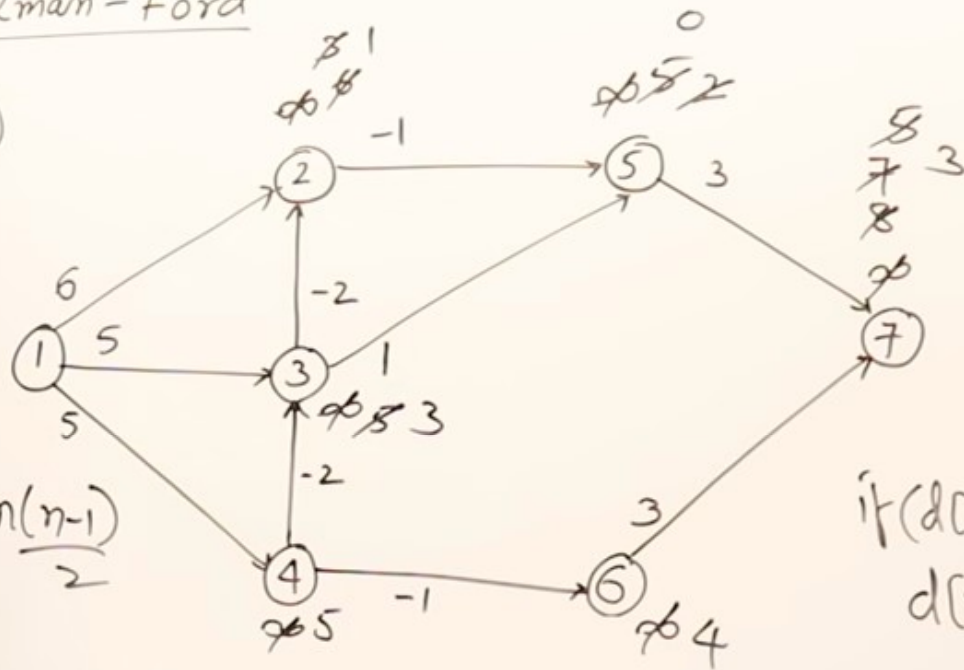
- 1-0
- 2-1
- 3-3
- 4-5
- 5-0
- 6-4
- 7-3

$$O(|E||V|-1)$$

$$O(|V||E|)$$

$$O(n^2)$$

$$|E| \frac{n(n-1)}{2}$$



(u,v)

Relaxation

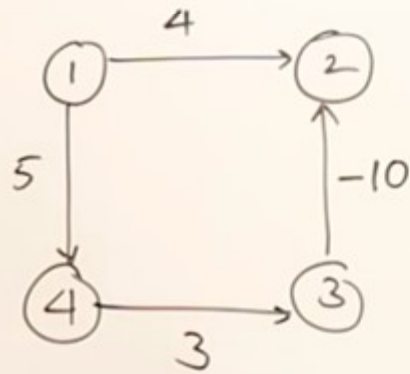
$$\text{if } (d[u] + c(u,v) < d[v])$$

$$d[v] = d[u] + c(u,v)$$

edgeList $\rightarrow (1,2), (1,3), (1,4), (2,5), (3,2), (3,5), (4,3), (4,6), (5,7), (6,7)$

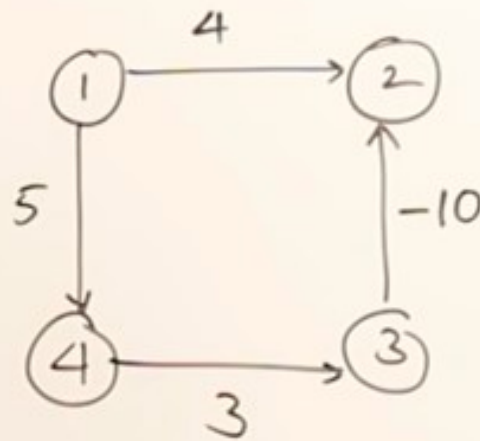
$$\frac{n(n-1)}{2} \times (n-1) = O(n^3)$$

Single Source Shortest Path Bellman-Ford



Single Source Shortest Path

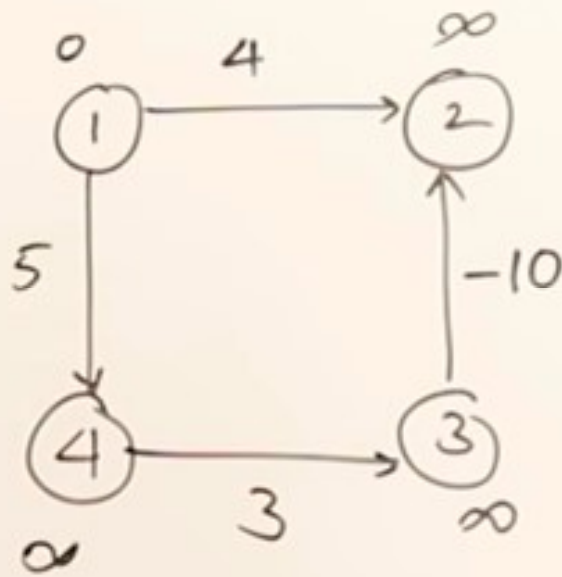
Bellman-Ford



edges $\rightarrow (3,2) (4,3) (1,4) (1,2)$

Single Source Shortest Path

Bellman - Ford

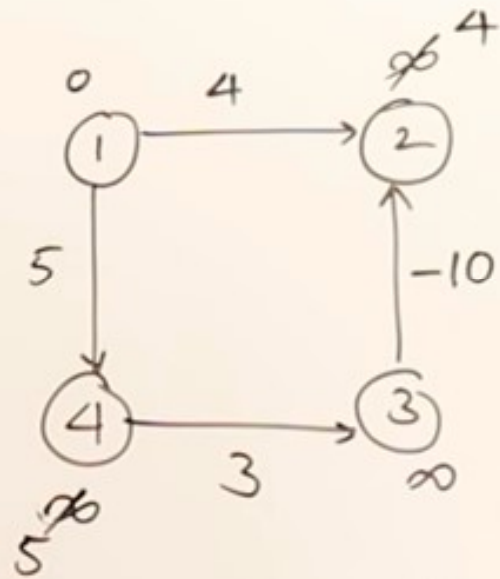


edges $\rightarrow (3,2) (4,3) (1,4) (1,2)$

$$n = 4 - 1 = 3$$

Single Source Shortest Path

Bellman-Ford

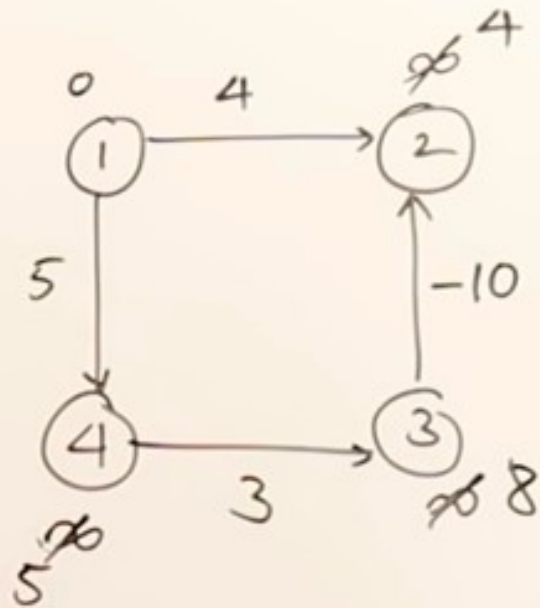


edges $\rightarrow (3,2) (4,3) (1,4) (1,2)$

$$n = 4 - 1 = 3$$

Single Source Shortest Path

Bellman-Ford

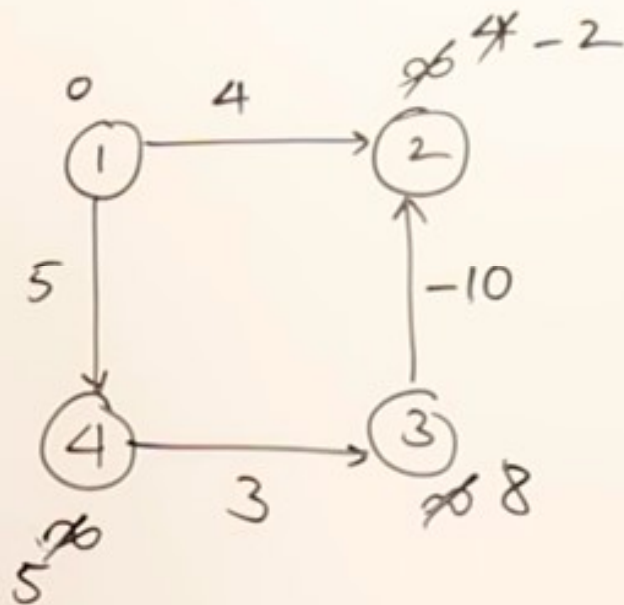


edges $\rightarrow (3,2) (4,3) (1,4) (1,2)$

$$n = 4 - 1 = 3$$

Single Source Shortest Path

Bellman-Ford



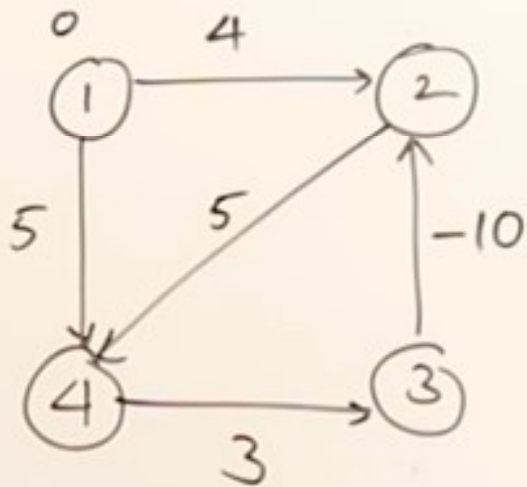
1 - 0
2 - (-2)
3 - 8
4 - 5

edges → (3,2) (4,3) (1,4) (1,2)

$$n = 4 - 1 = 3$$

Single Source Shortest Path

Bellman-Ford

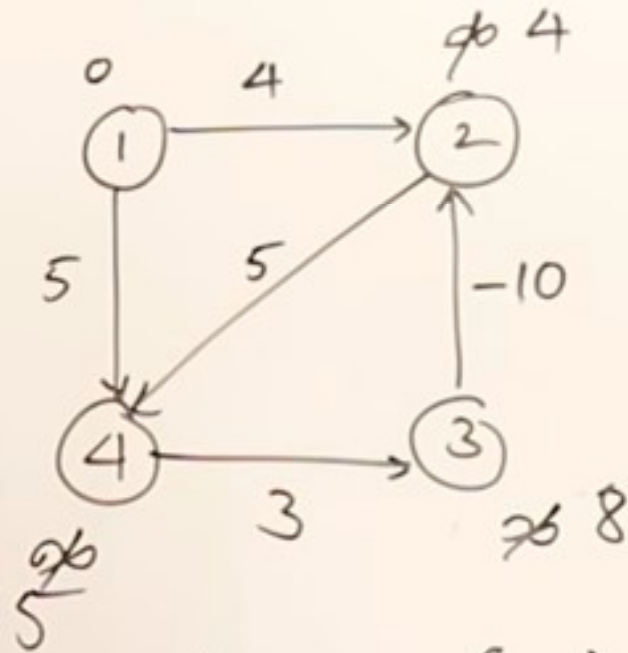


Demerit

edges $\rightarrow (3,2) (4,3) (1,4) (1,2) (2,4)$

Single Source Shortest Path

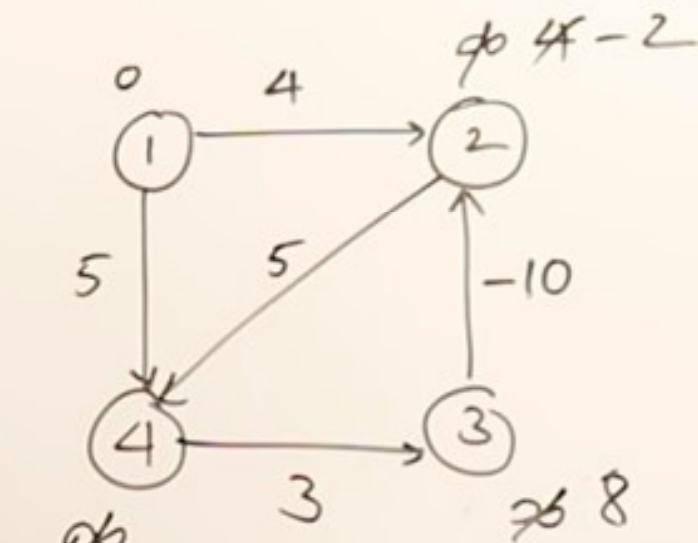
Bellman-Ford



edges $\rightarrow (3,2) (4,3) (1,4) (1,2) (2,4)$

Single Source Shortest Path

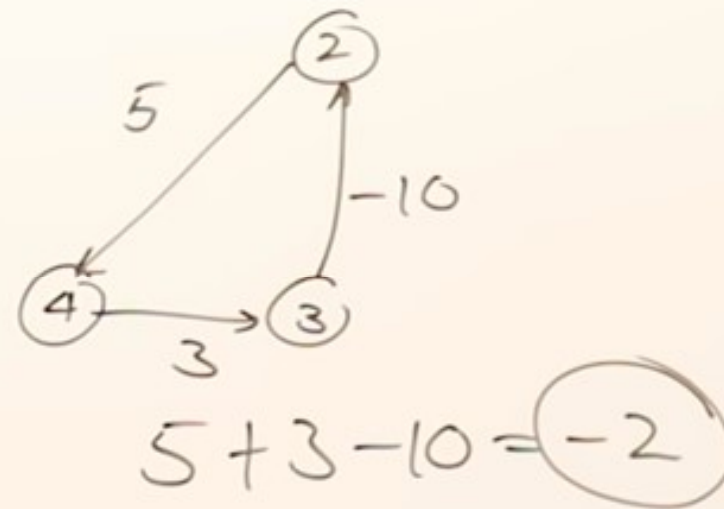
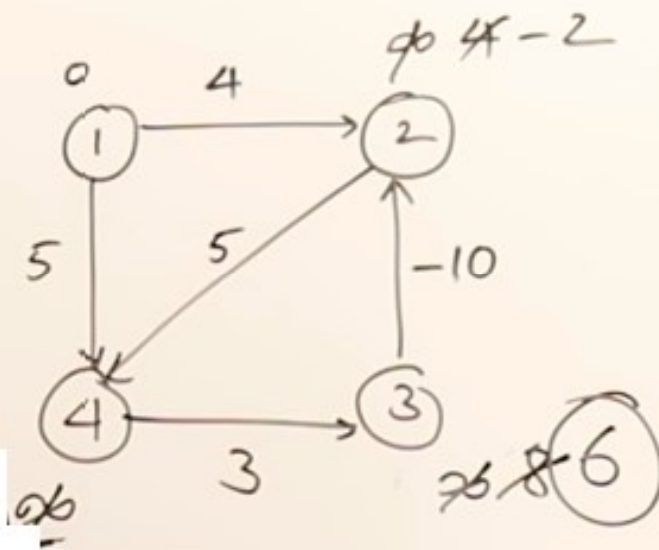
Bellman-Ford



edges $\rightarrow (3, 2) (4, 3) (1, 4) (1, 2) (2, 4)$

Single Source Shortest Path

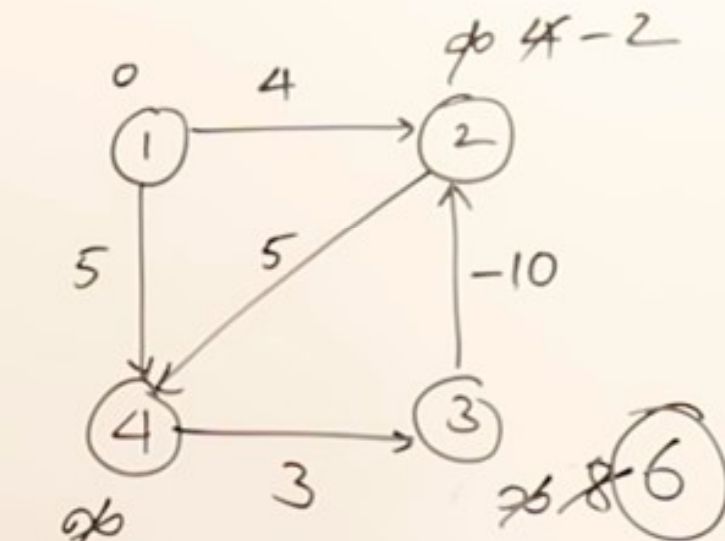
Bellman-Ford



edges $\rightarrow (3,2) (4,3) (1,4) (1,2) (2,4)$

Single Source Shortest Path

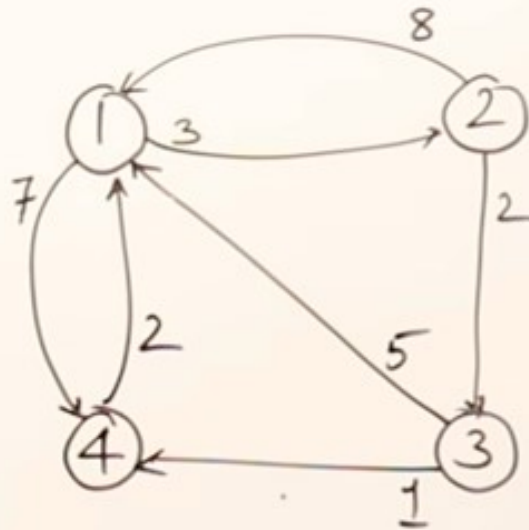
Bellman-Ford



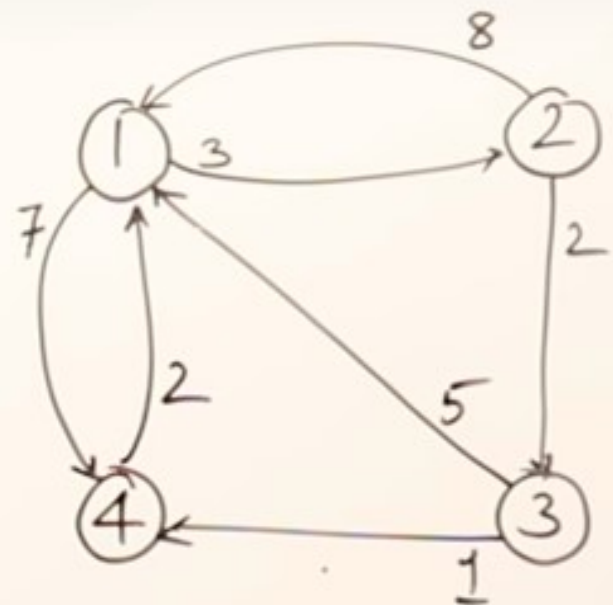
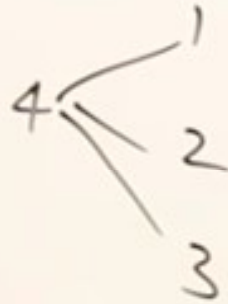
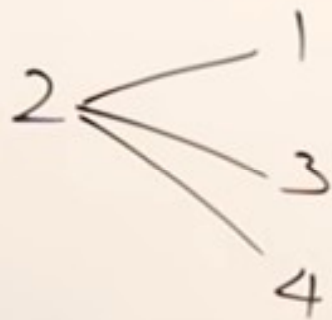
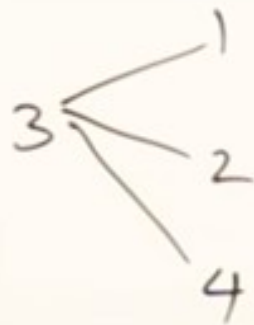
edges $\rightarrow (3,2) (4,3) (1,4) (1,2) (2,4)$

Thank you

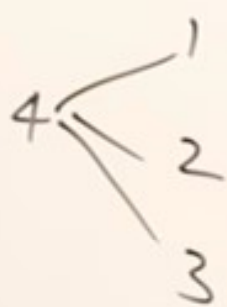
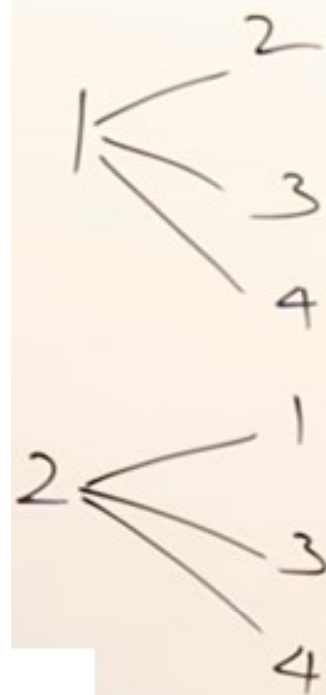
All Pairs Shortest Path (Floyd-Warshall Algorithm)



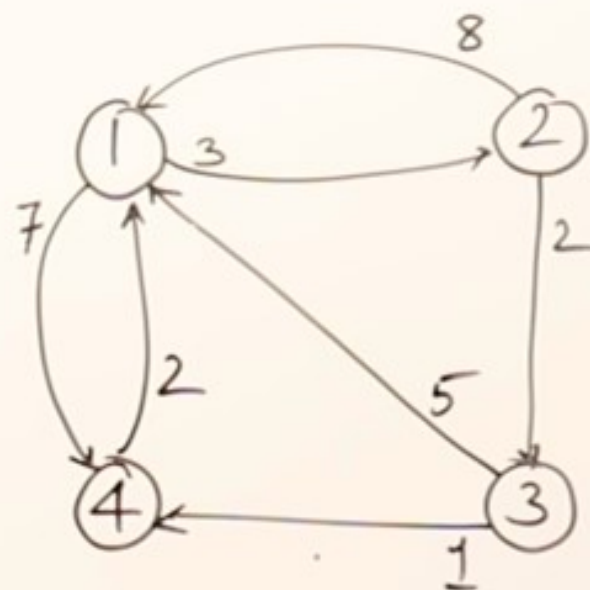
$$A^0 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{matrix}$$



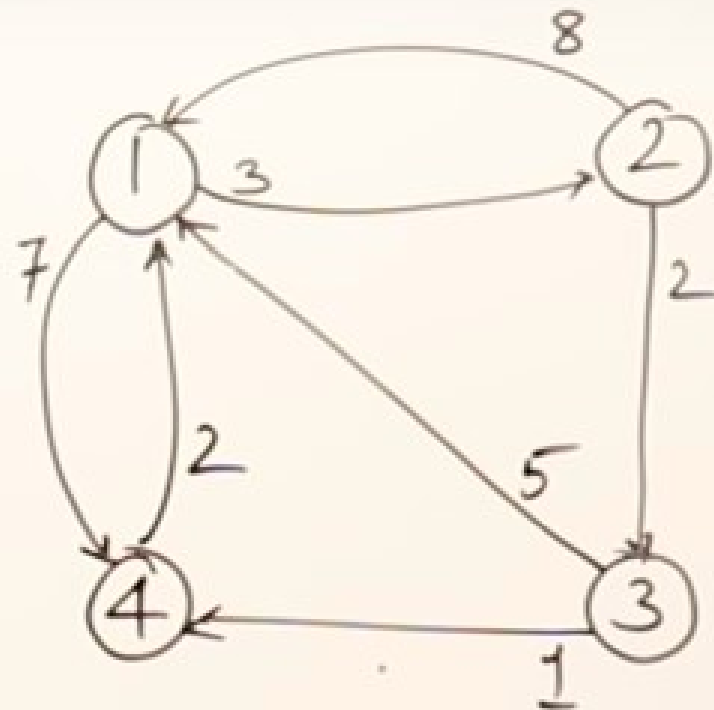
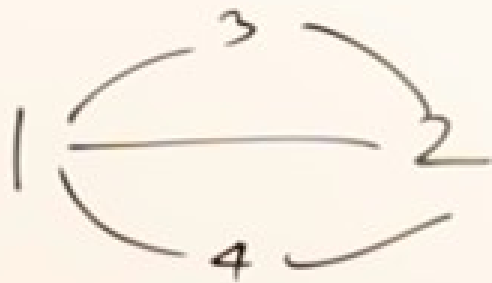
$$A^0 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{matrix}$$



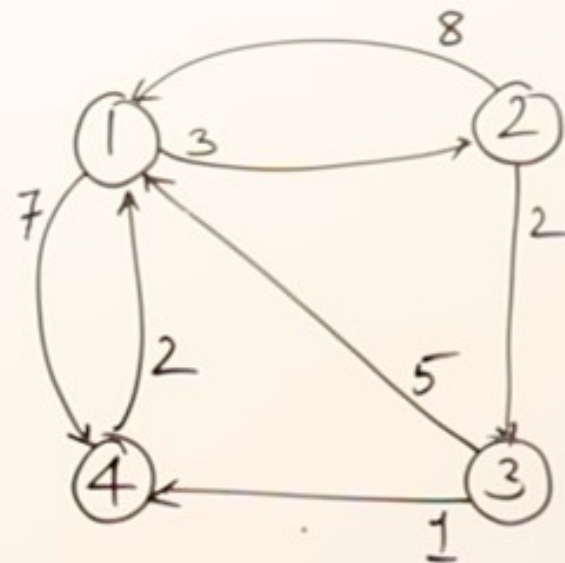
$$n^2 \times n = O(n^3) \quad A^0 =$$



	1	2	3	4
1	0	3	∞	7
2	8	0	2	∞
3	5	∞	0	1
4	2	∞	∞	0



$$A^1 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & & \\ 5 & & 0 & \\ 2 & & & 0 \end{bmatrix} \end{matrix}$$

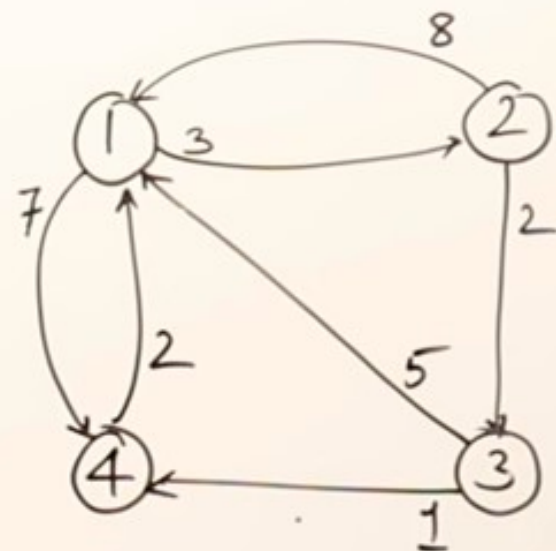


$$A^0 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{matrix}$$

$$A' = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \\ 5 & & 0 & \\ 2 & & & 0 \end{bmatrix} \end{matrix}$$

$$A^0[2,3] \quad A^0[2,1] + A^0[1,3]$$

$$2 < 8 + \infty$$



$$A^0 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{matrix}$$

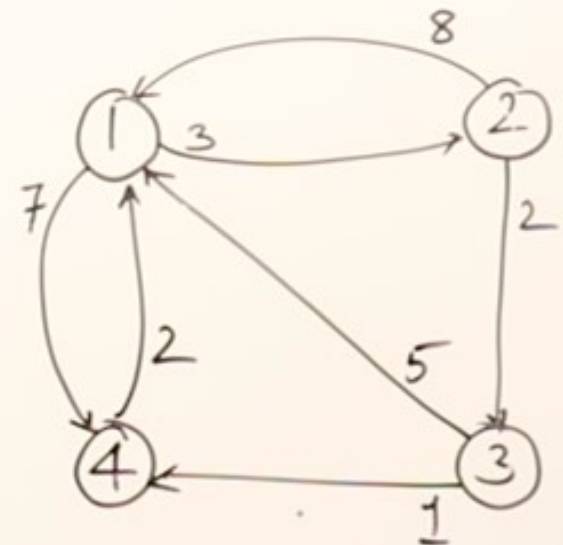
$$A' = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & & 0 & \\ 2 & & & 0 \end{bmatrix} \end{matrix}$$

$$A^0[2,3] \quad A^0[2,1] + A^0[1,3]$$

$$2 < 8 + \infty$$

$$A^0[2,4] \quad A^0[2,1] + A^0[1,4]$$

$$\infty > 8 + 7$$



$$A^0 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{matrix}$$

$$A' = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & - & 0 & \\ 2 & & & 0 \end{bmatrix} \end{matrix}$$

$$A^0[2,3] \quad A^0[2,1] + A^0[1,3]$$

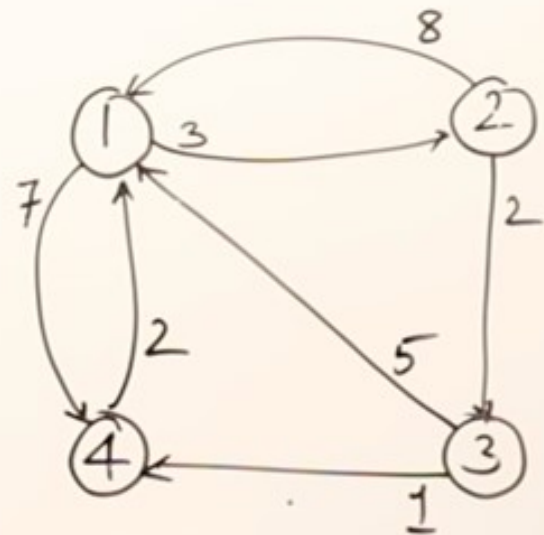
$$2 < 8 + \infty$$

$$A^0[2,4] \quad A^0[2,1] + A^0[1,4]$$

$$\infty > 8 + 7$$

$$A^0[3,2] \quad A^0[3,1] + A^0[1,2]$$

$$\infty \quad 5 + 3$$



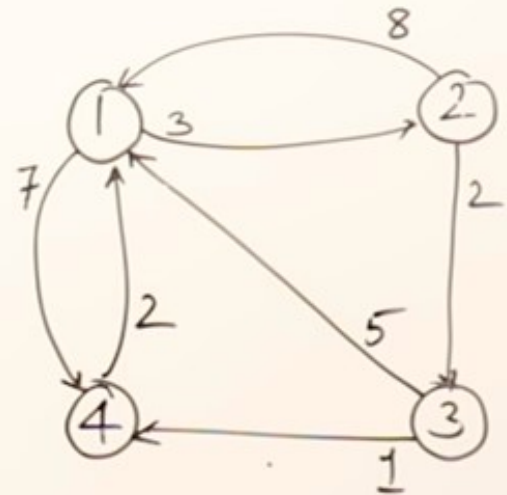
$$A^0 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{matrix}$$

$$A^1 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 8 & \infty & 0 \end{bmatrix} \end{matrix}$$

$$A^2 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & & \\ 8 & 0 & 2 & 15 \\ & 8 & 0 & \\ & 8 & & 0 \end{bmatrix} \end{matrix}$$

$$A^1[1,3] \quad A^1[1,2] + A^1[2,3]$$

∞



$$A^0 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{matrix}$$

$$A = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 8 & \infty & 0 \end{bmatrix} \end{matrix}$$

$$A^2 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 7 \\ 8 & 0 & 2 & 15 \\ & 8 & 0 & \\ & 8 & & 0 \end{bmatrix} \end{matrix}$$

$$A'[1,3] \quad A'[1,2] + A'[2,3]$$

$$\infty > 3 + 2$$

$$A'[1,4] \quad A'[1,2] + A'[2,4]$$

$$7 < 3 + 15$$

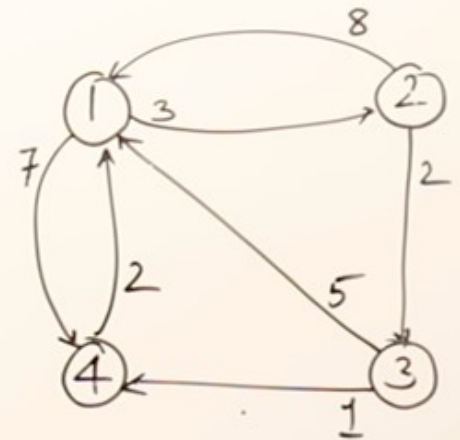
$$A^1 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & \infty & 0 \end{bmatrix} \end{matrix}$$

$$A^2 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{matrix}$$

$$A^3 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & \\ & 0 & 2 & \\ 5 & 8 & 0 & 1 \\ & & 7 & 0 \end{bmatrix} \end{matrix}$$

$$A^2[1,2] \quad A^2[1,3] + A^2[3,2]$$

$$3 < 5 + 8$$



$$A^0 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{matrix}$$

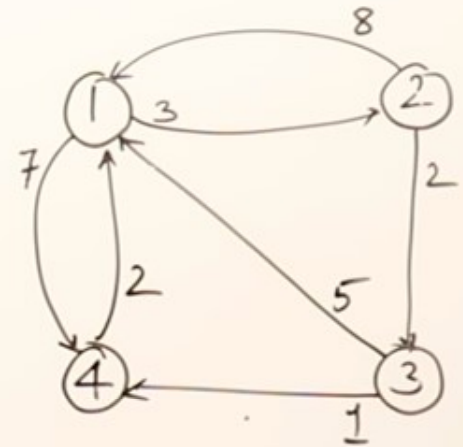
All Pairs Shortest Path

$$A^1 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & \infty & 0 \end{bmatrix} \end{matrix}$$

$$A^2 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{matrix}$$

$$A^3 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 6 \\ 7 & 0 & 2 & 3 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{matrix}$$

$$A^4 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 6 \\ 5 & 0 & 2 & 3 \\ 3 & 6 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{matrix}$$



$$A^0 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{matrix}$$

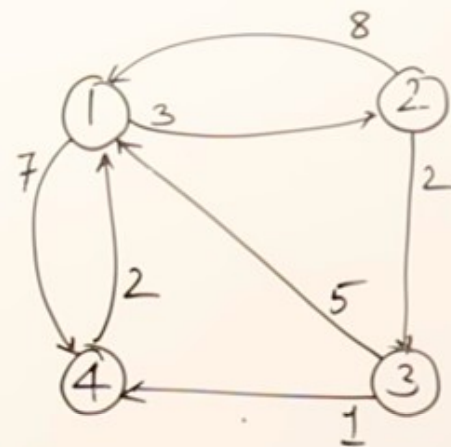
All Pairs Shortest Path

$$A^1 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & \infty & 0 \end{bmatrix} \end{matrix}$$

$$A^2 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{matrix}$$

$$A^3 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 6 \\ 7 & 0 & 2 & 3 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{matrix}$$

$$A^4 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 6 \\ 5 & 0 & 2 & 3 \\ 3 & 6 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{matrix}$$



$$A^0 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{matrix}$$

$$A^k[i,j] = \min \left\{ \underbrace{A^{k-1}[i,j]}_{\text{direct path}}, \underbrace{A^{k-1}[i,k] + A^{k-1}[k,j]}_{\text{path via node k}} \right\}$$

2
for($k=1$; $k \leq n$; $k++$)
{

for($i=1$; $i \leq n$; $i++$)
{

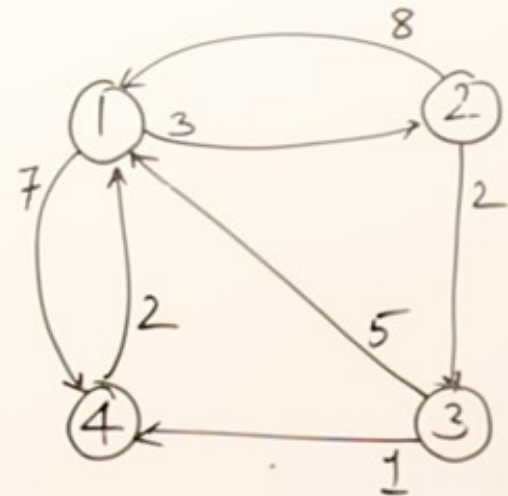
for($j=1$; $j \leq n$; $j++$)
{

$A[i,j] = \min(A[i,j], A[i,k] + A[k,j]);$
}

$A^0 =$

	1	2	3	4
1	0	3	∞	7
2	8	0	2	∞
3	5	∞	0	1
4	2	∞	∞	0

$n \times n$



Thank you