Network Flow

Network Flow

Maximal Flow Problem (Ford Fulkerson Rule)

1. The Ford Fulkerson method is used for solving maximum flow problem.

2. <u>Basic terms</u>:
   - Source
   - Sink
   - Capacity or bottle neck capacity
   - Flow
   - Augumenting path
   - Residual capacity

Source : The source vertex has all outward edge, no inward edge.

Sink : Sink will have all inward edge, no outward edge.

Bottleneck Capacity : Bottle neck capacity of the path is the minimum capacity of any edge on the path.

Residual Capacity : Every edge of a residual graph has a value called residual capacity, which is equal to original capacity minus current flow.

Network flows

- A Network is represented by a weighted graph in which
  vertices — stations
  edges — lines
  through which the given weight assigned to each edge shows the capacity.

edges — lines

through which the given weight assigned to each edge shows the capacity.

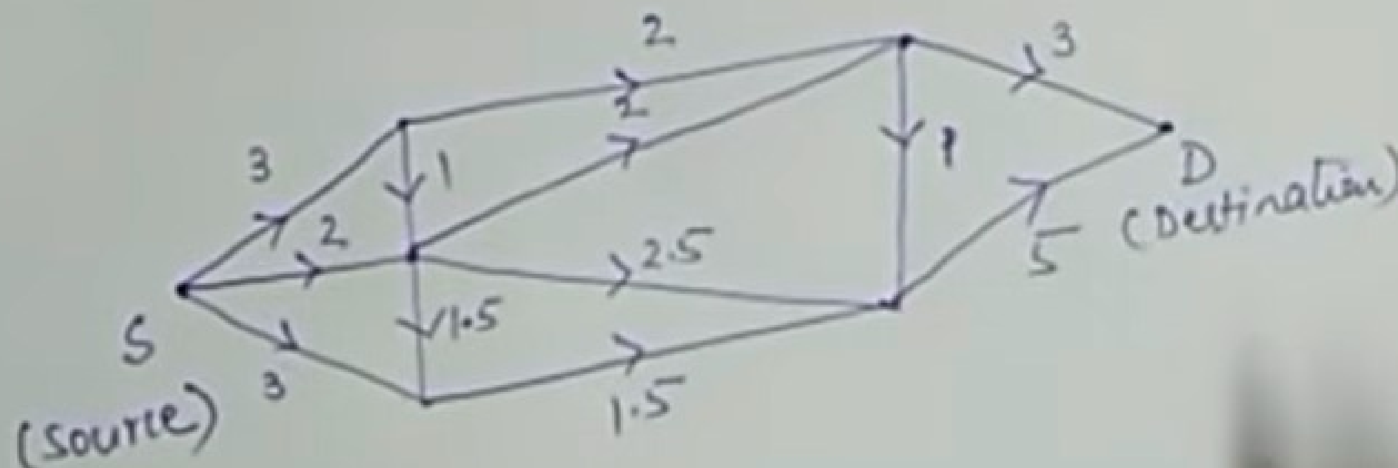
- Capacity is the max amount of flow possible per unit of time through that line.

• Capacity is the ... through
possible per unit of time
that line.

• A Network has satisfies following properties:-

a) Contain one & only one vertex – no incoming edge
Source.

b) Contain one & only one vertex – no outcoming edge.



Flow

a) $0 \leq f(v_i, v_j) \leq c(v_i, v_j)$

b) $\sum_{\forall i} f(v_i, v_j) = \sum_{\forall k} (v_j, v_k)$

c) $f(v_i, v_j) = 0$

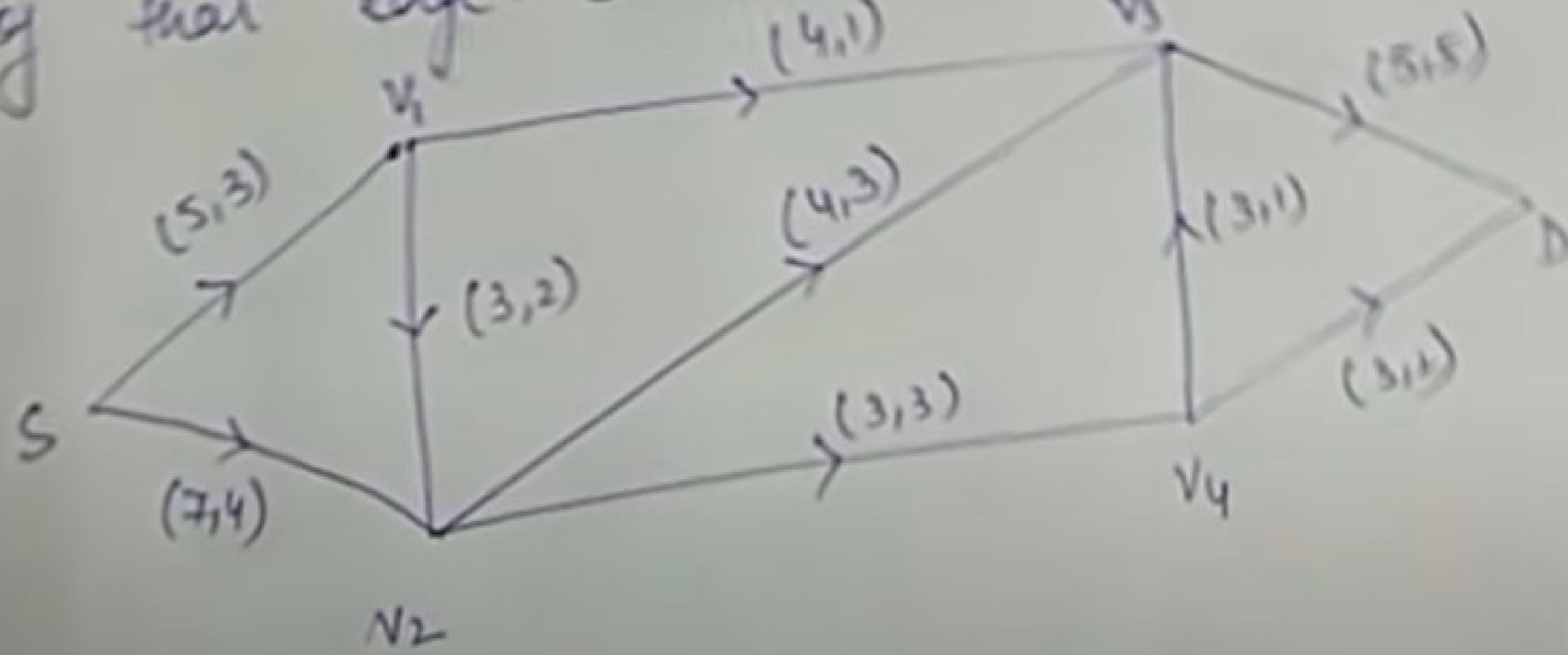
Value of the flow :-

$$f_v = \left[\sum_{\forall i} f(s, v_i) - \sum_{\forall i} f(v_i, s)\right]$$

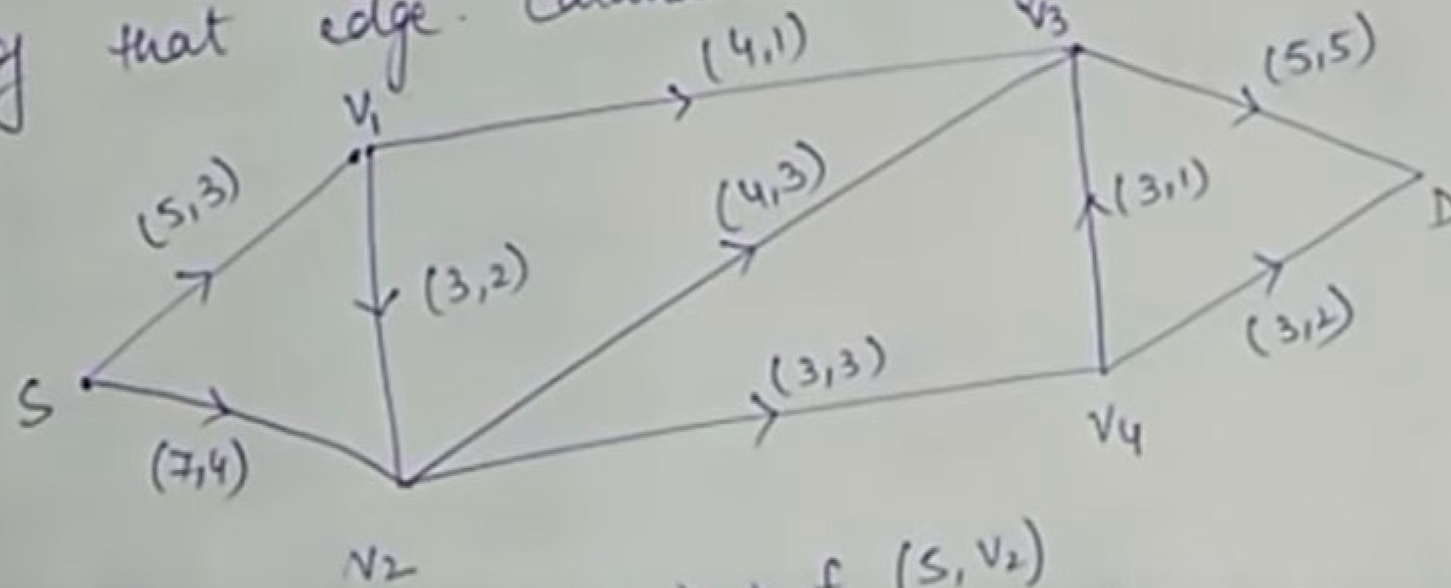
$$= \left[\sum_{\forall k} f(v_k, D) - \sum_{\forall k} f(D, v_k)\right]$$

Network flows

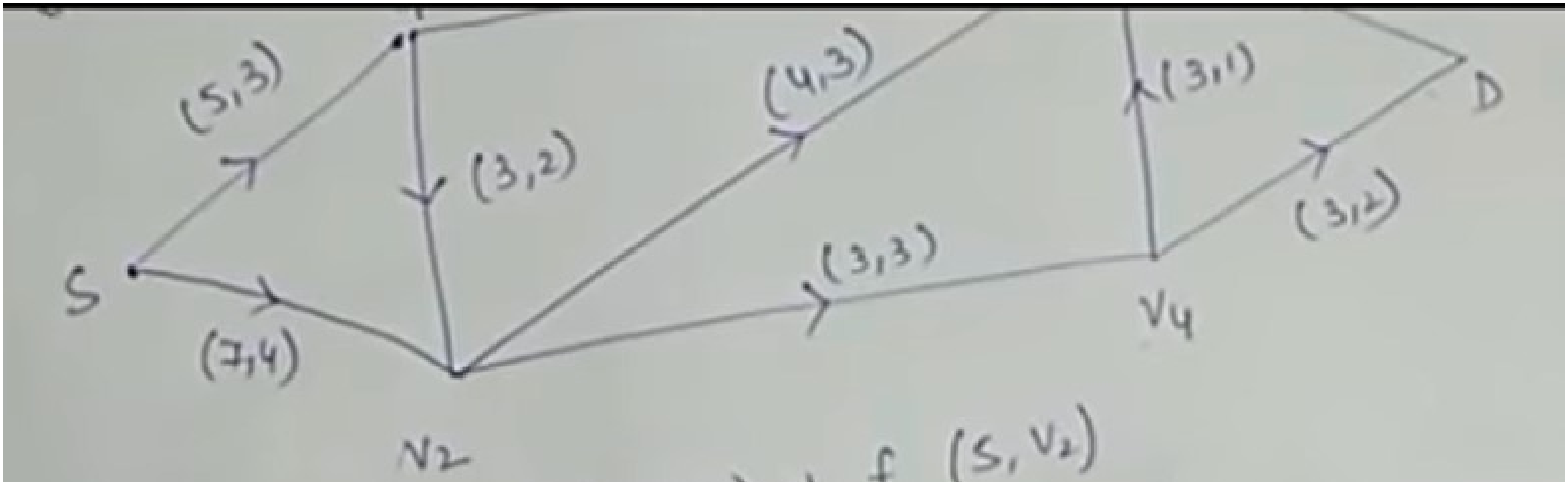
Consider a transport n/w. The pair (C, f) assigned to each edge indicate the capacity and flow of that edge. Calculate value of flow.



Q. Consider a transport and
to each edge indicate the capacity and flow.
of that edge. Calculate value of flow.



$$f_v = f(s, v_1) + f(s, v_2)$$
$$= 3 + 4$$
$$= 7$$

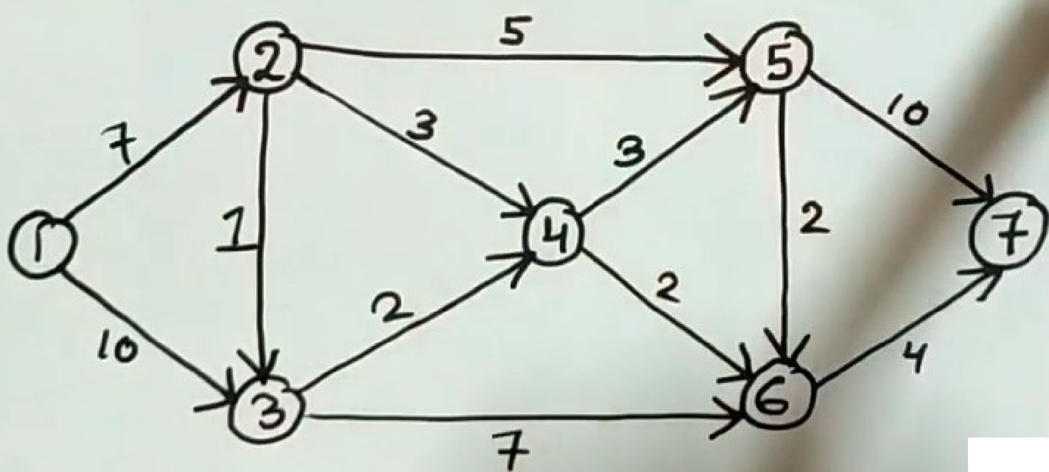


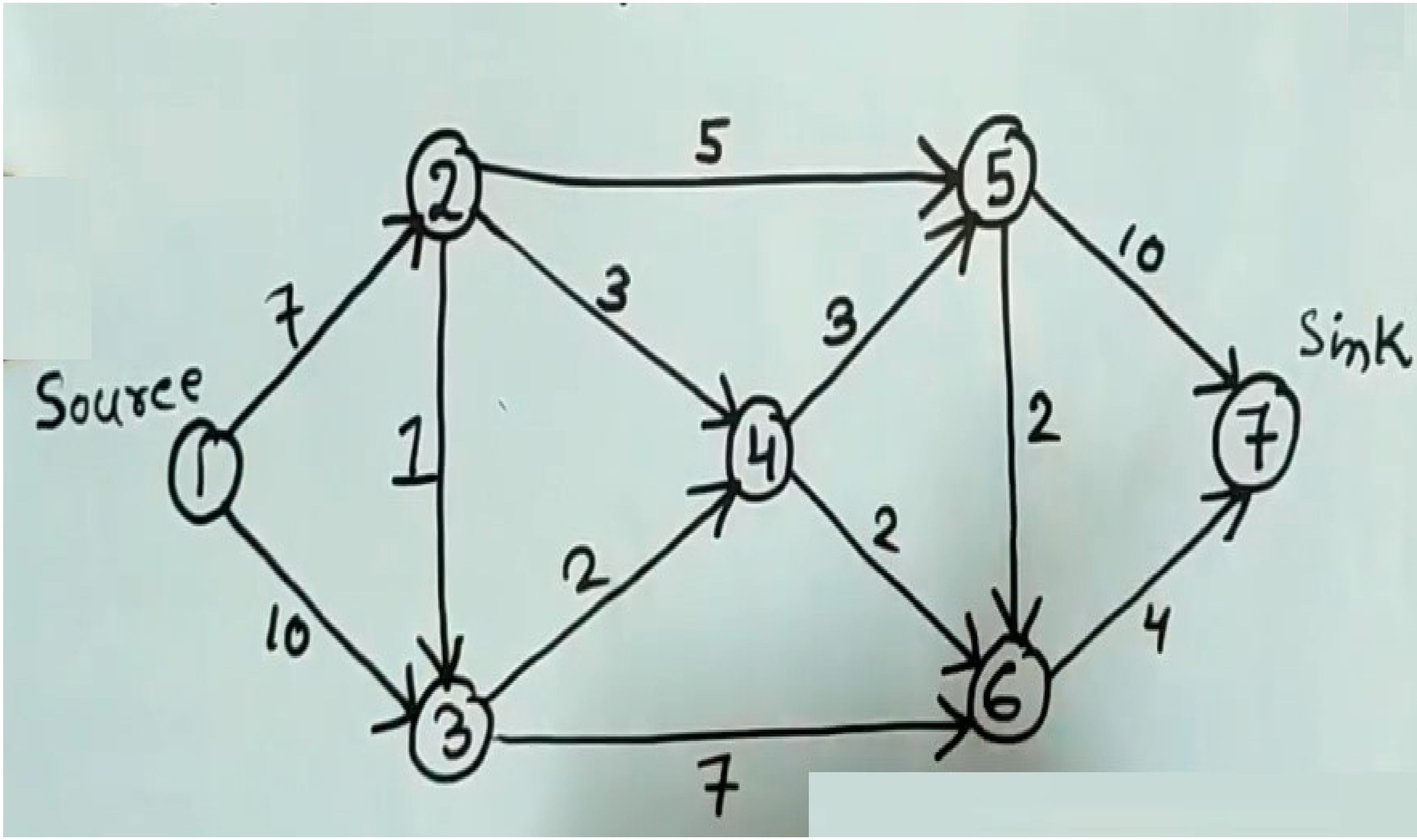
$$f_v = f(S, V_1) + f(S, V_2)$$
$$= 3 + 4$$
$$= 7$$

$$f_v = f(V_3, D) + f(V_4, D)$$
$$= 5 + 2$$
$$= 7$$

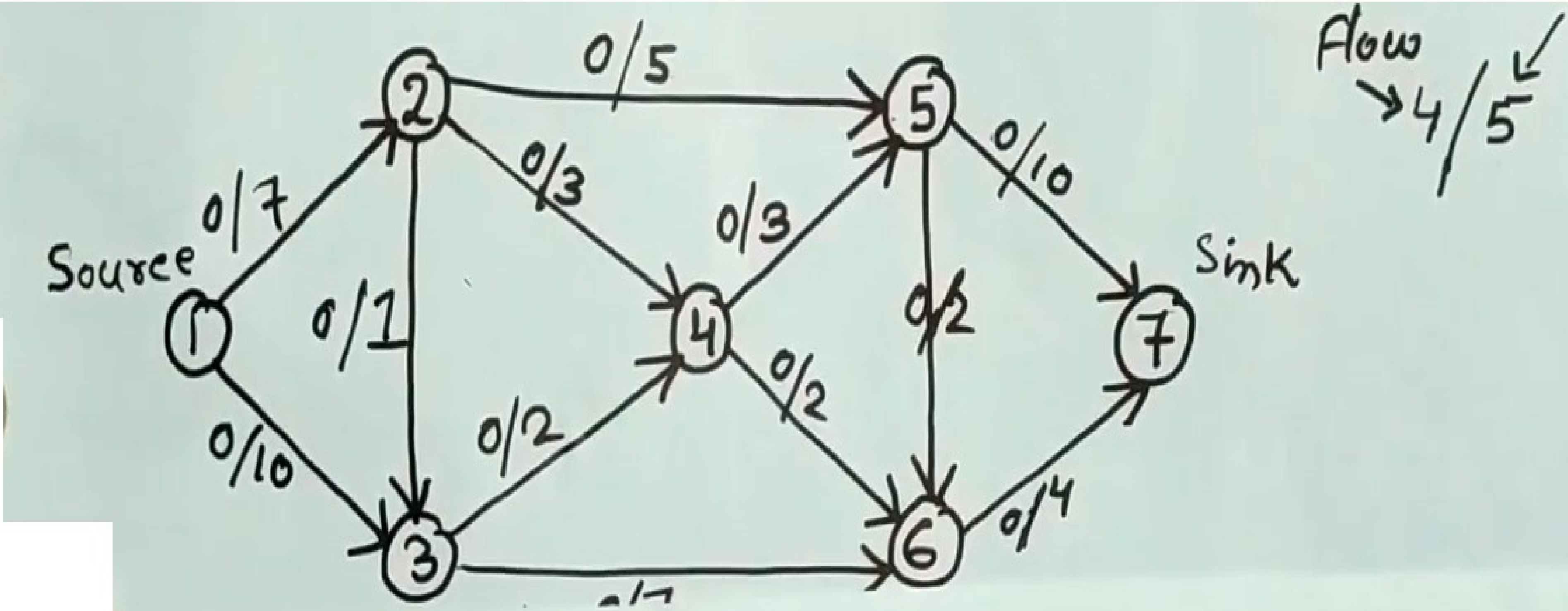
Maximal Flow Problem (Ford Fulkerson Rule)

Q: Find the maximum flow through the given network using Ford Fulkerson algorithm.

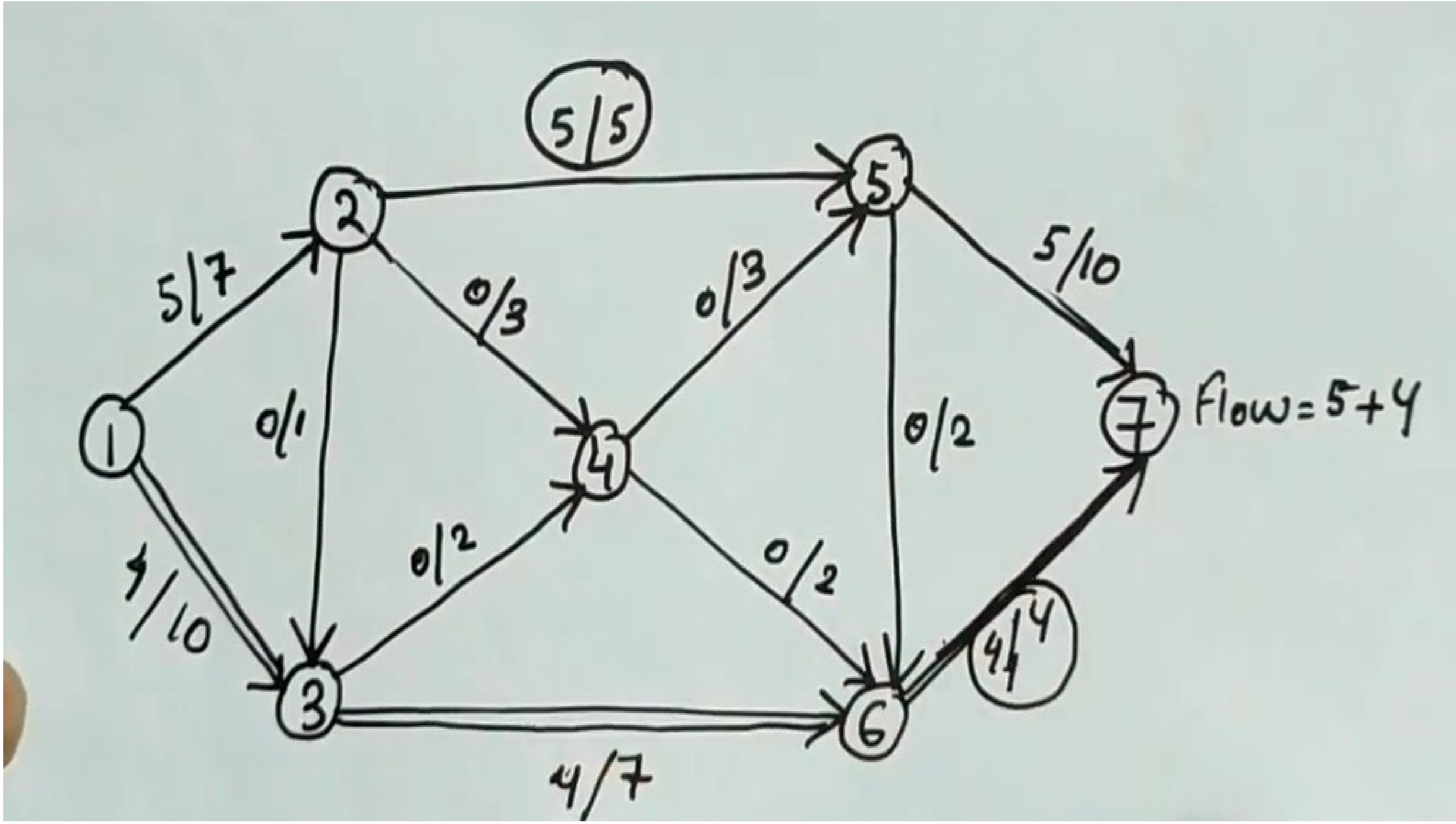




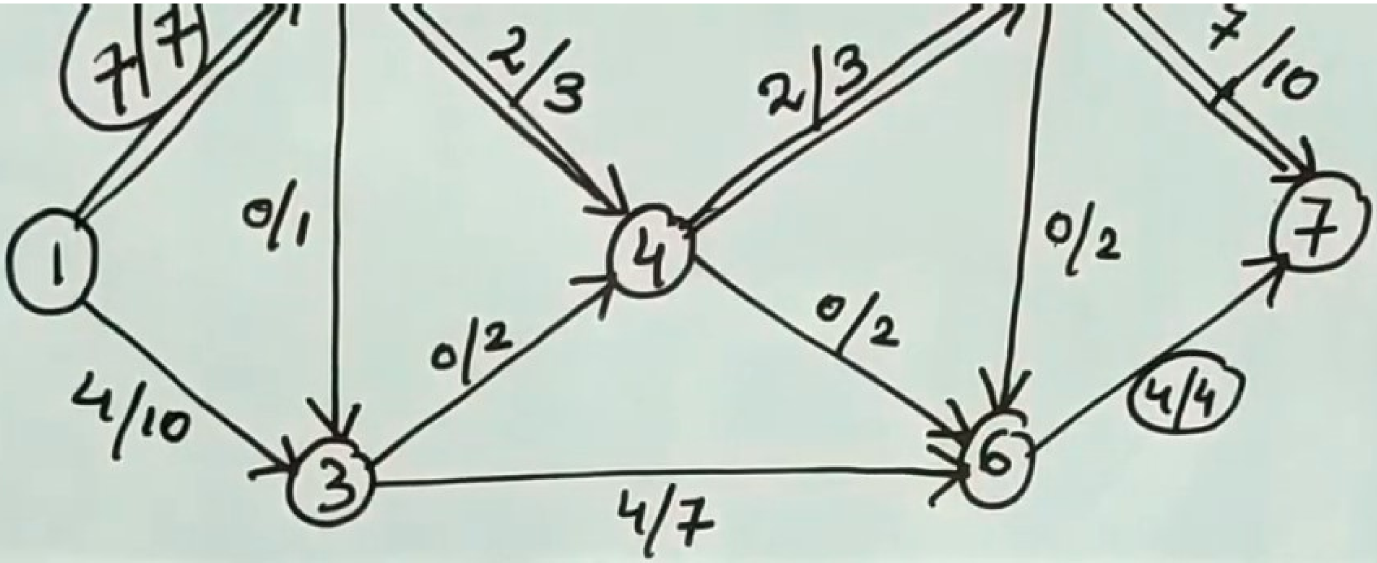
Source
Sink
1
2
3
4
5
6
7
7
10
1
5
3
2
7
3
2
2
10
4



Augumenting path	Bottleneck capacity
1- 2- 5 - 7	5



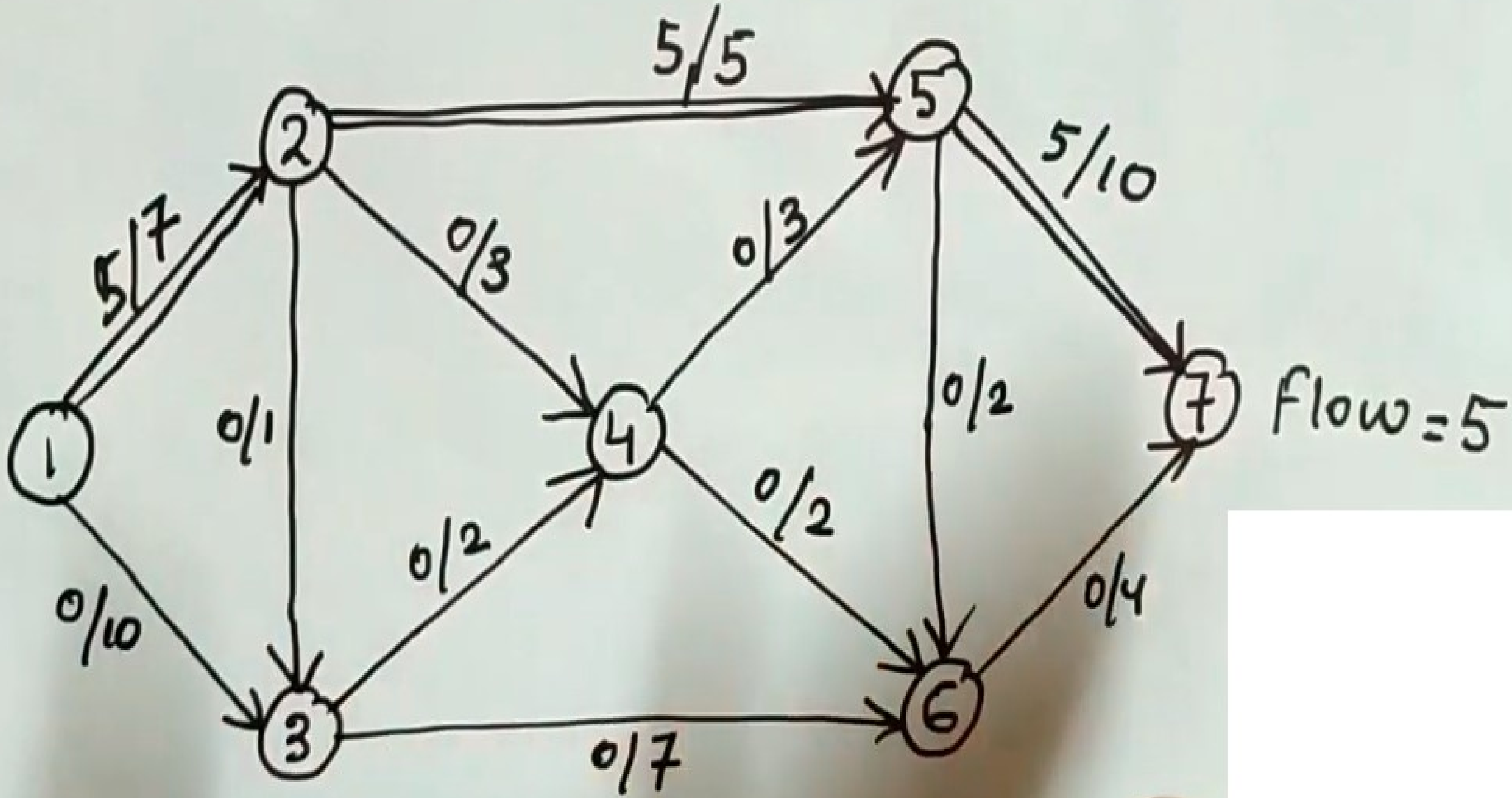
5/5
1
2
3
4
5
6
7
5/7
4/10
0/1
0/3
0/2
0/3
0/2
0/2
5/10
4/4
4/7
Flow = 5+4



Flow = 5+4+2

Augumenting path	Bottleneck capacity
1- 2-5-7	5
1-3-6-7	4
1-2-4-5-7	2

Solution :



Ford-Fulkerson Algorithm for Maximum Flow Problem

Problem

Given a graph which represents a flow network where every edge has a capacity. Also given two vertices Source S and sink t in the graph. Find out the maximum possible flow from S to t with following constraints.

a) Flow on an edge doesn't exceed the given capacity of the edge.

b) In-flow is eaual to Out-flow for every vertex except s and t.

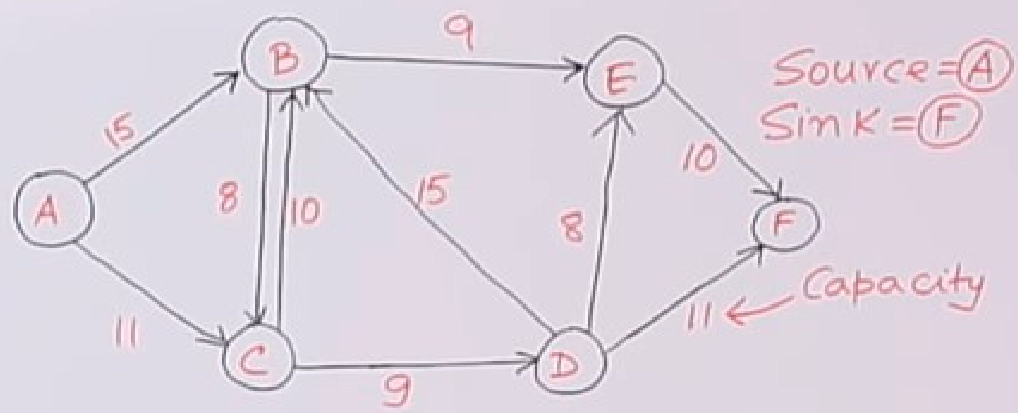
Algorithm

Ford-Fulkerson Algorithm

The following is a simple idea of the algorithm

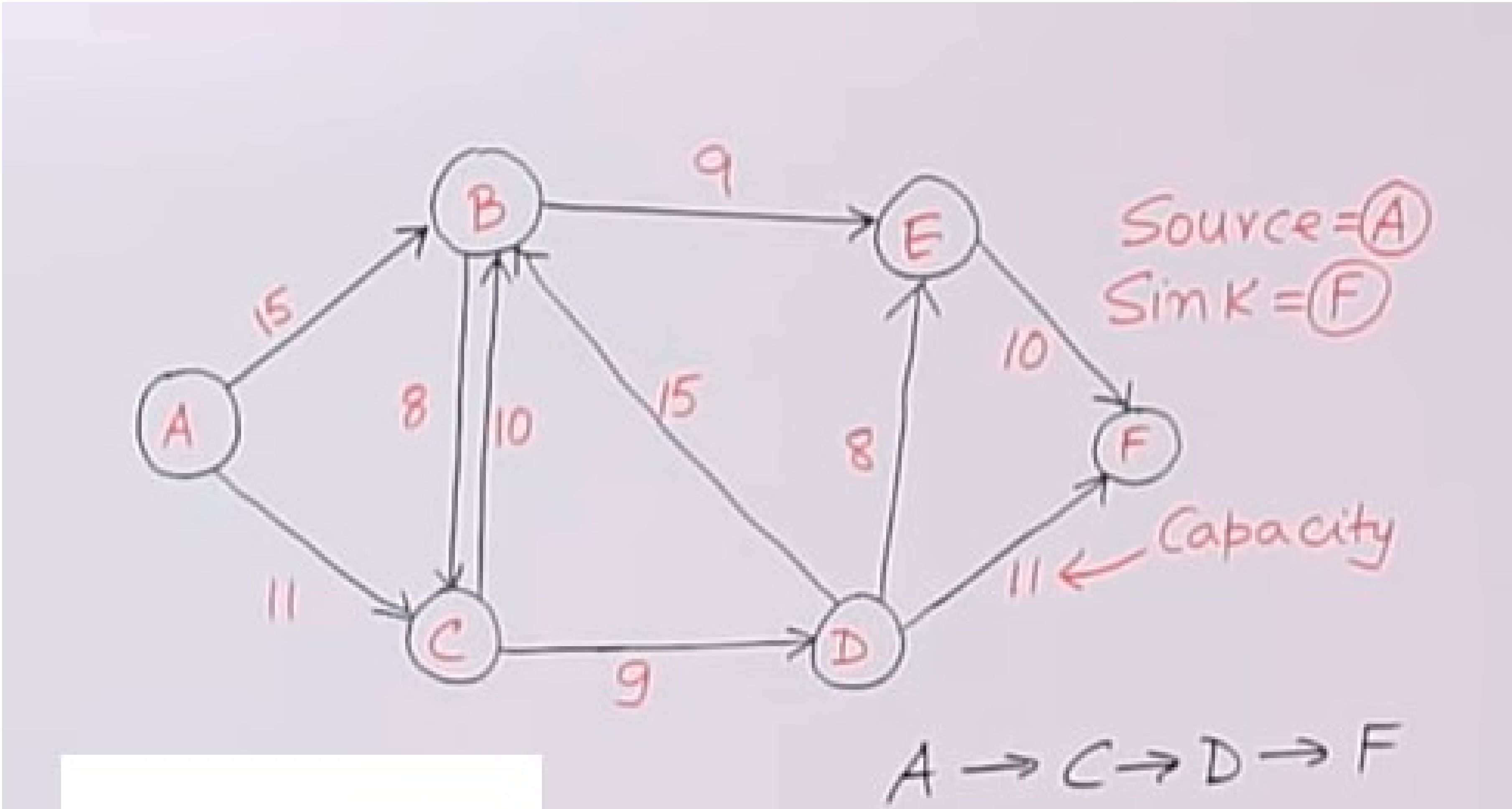
1) Start with a initial flow as 0.
2) While there is an augmeting path from source to sink
   Add this path flow to flow

3) Return flow

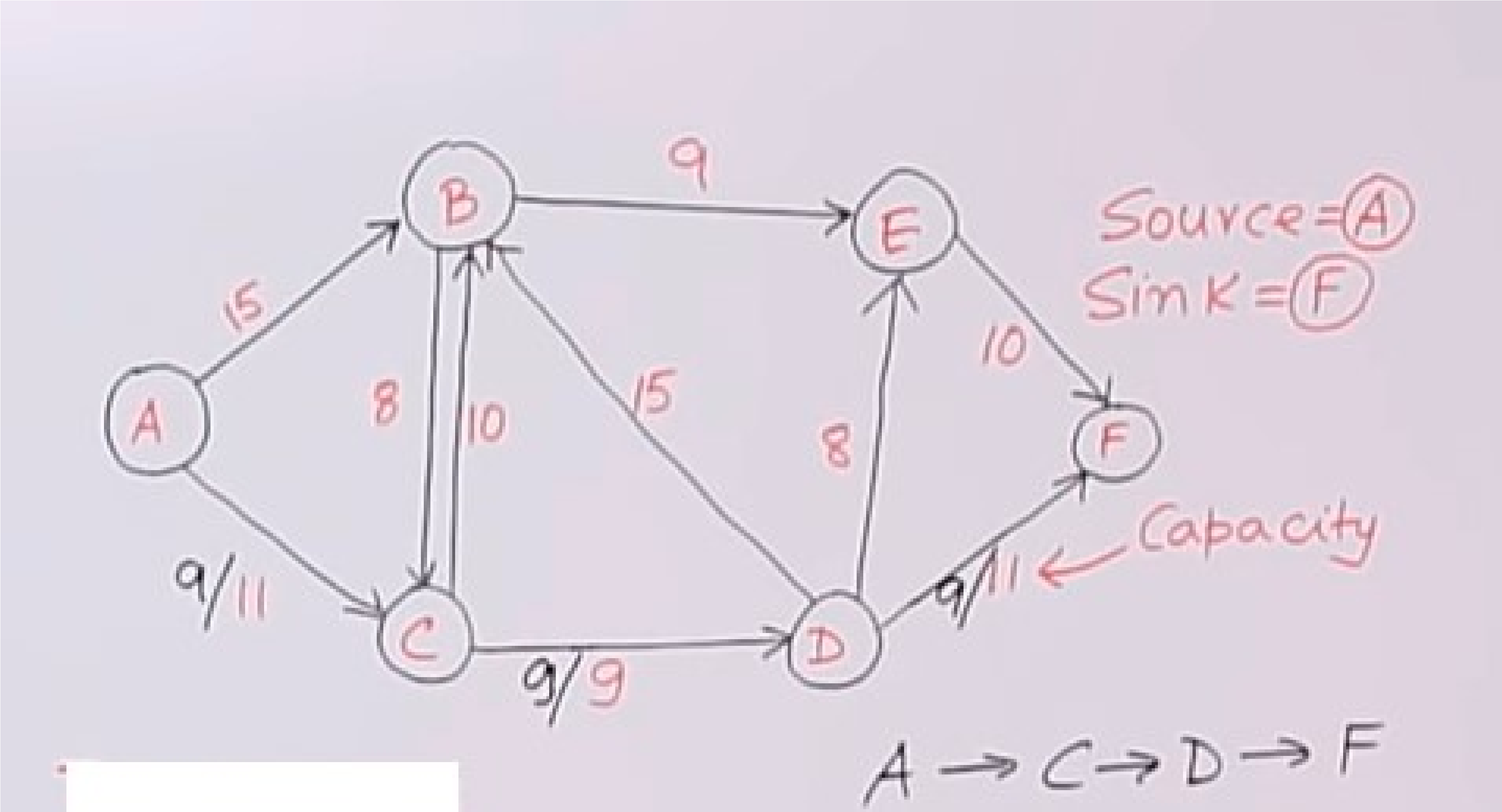


Terminologies

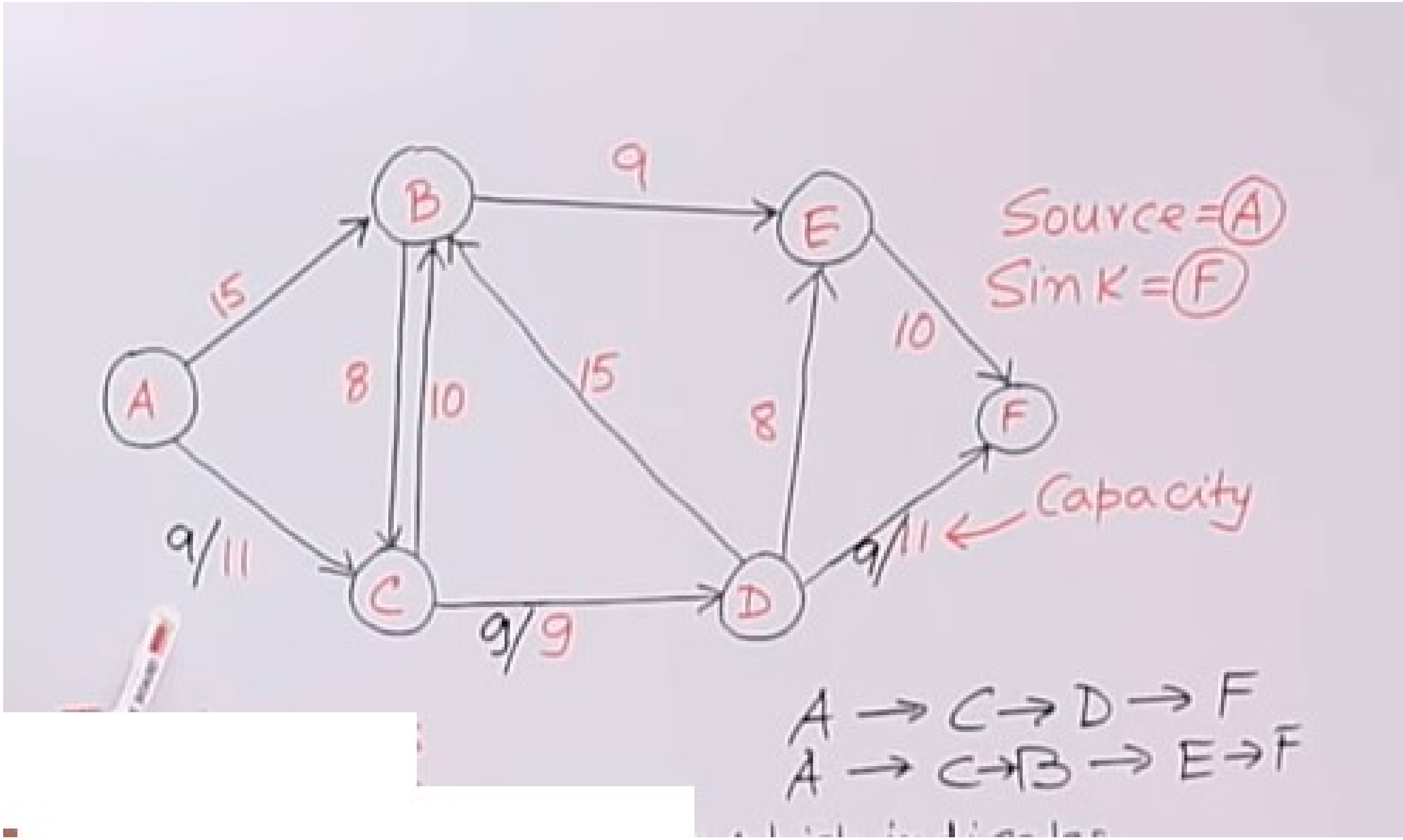
* **Residual Graph:** It's a graph which indicates additional possible flow. If there is such path from source to sink then there is a possibility to add flow
* **Residual Capacity:** It's original capacity of the edge minus flow.
* **Minimal cut:** Also known as bottle neck capacity, which decides maximum possible flow from source to sink through an augmented path
* **Augmenting path:** Augmenting path can be done in two ways -
  1) Non-full forward edges
  2) Non-empty backward edges.



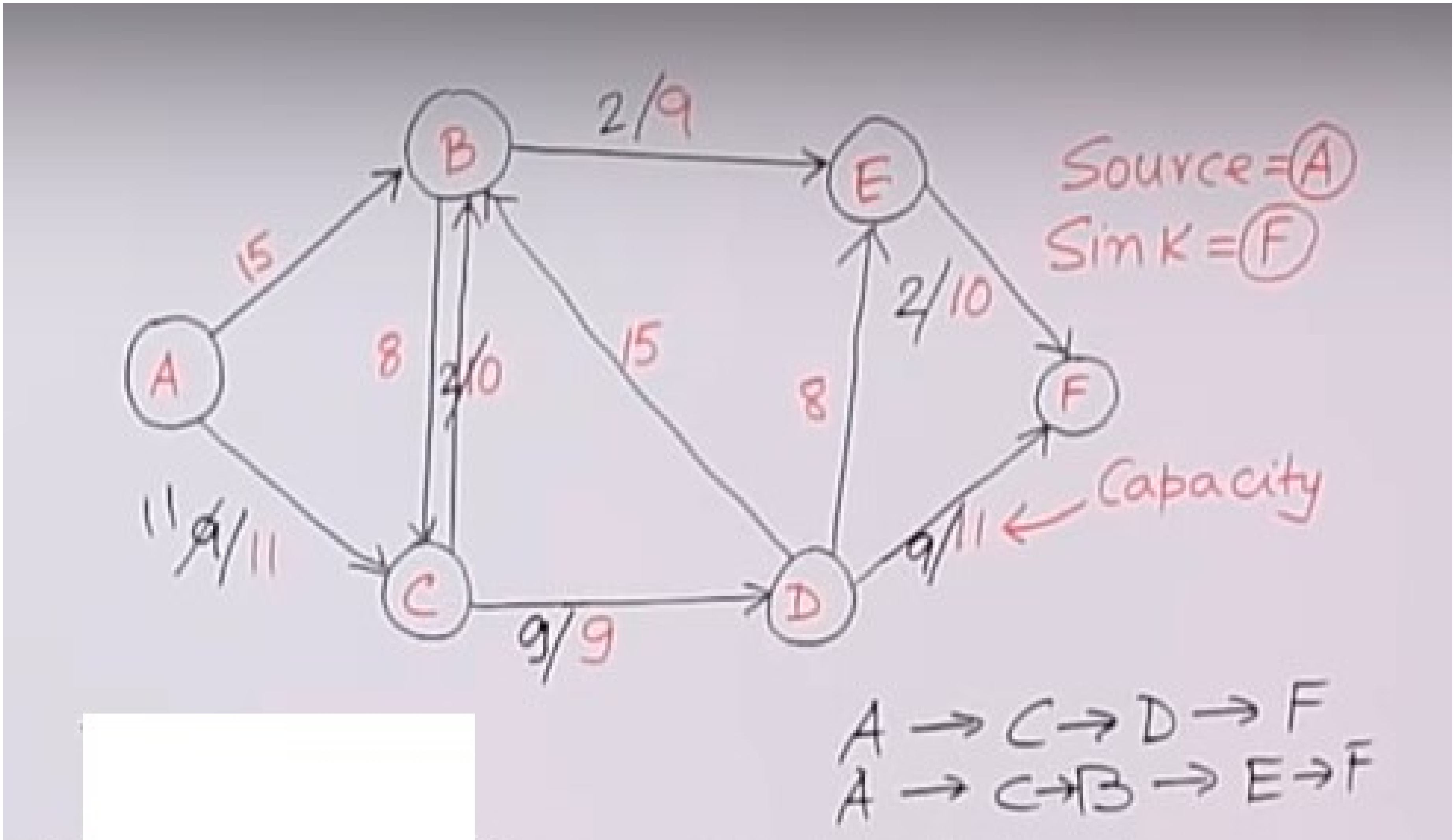
A
B
C
D
E
F
15
11
8
10
9
9
15
8
10
11
Source = A
Sink = F
Capacity
A → C → D → F

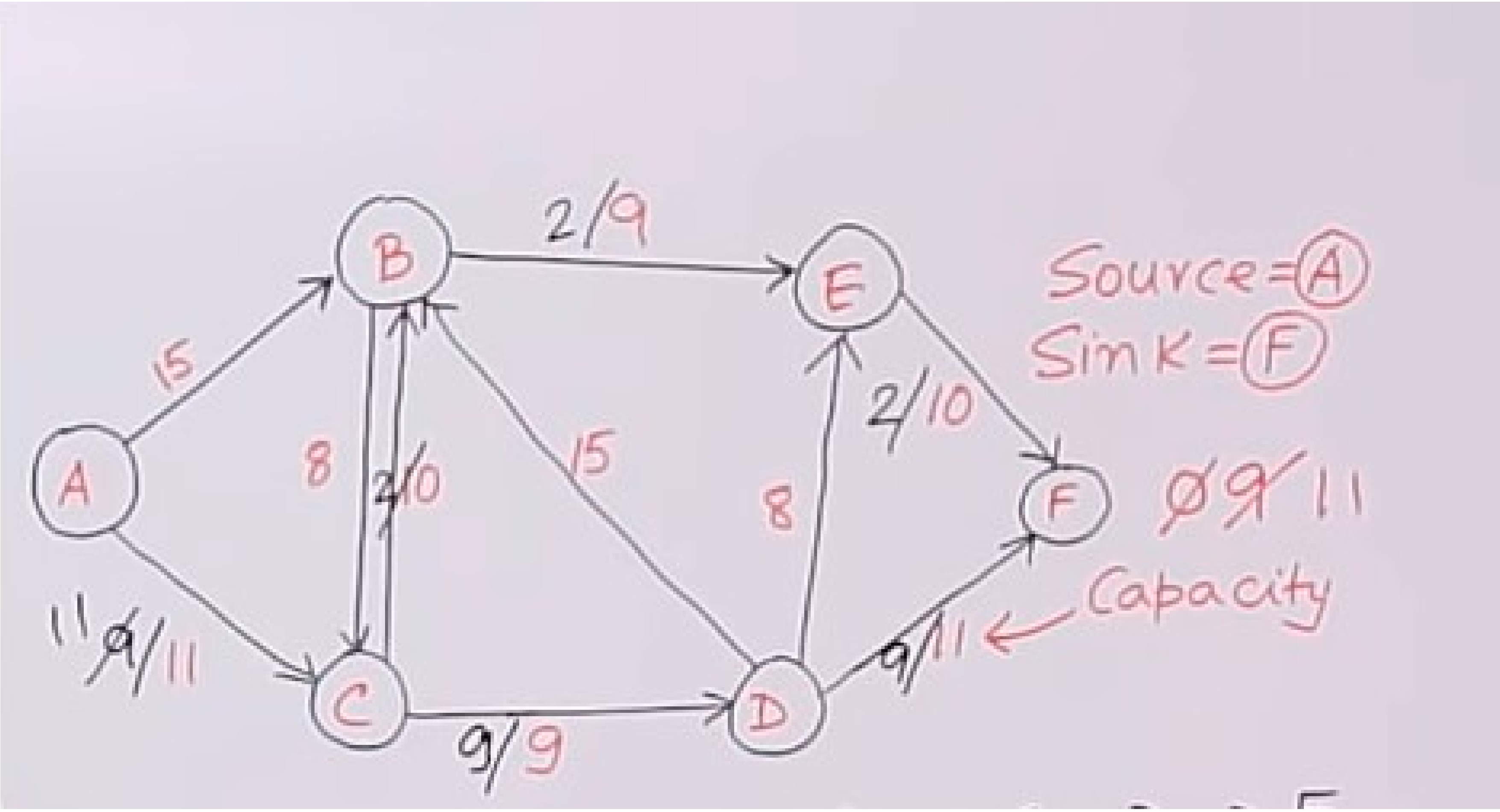


A
B
C
D
E
F
15
9/11
8
10
15
9
8
10
9/9
9/11
Source = A
Sink = F
Capacity
A → C → D → F

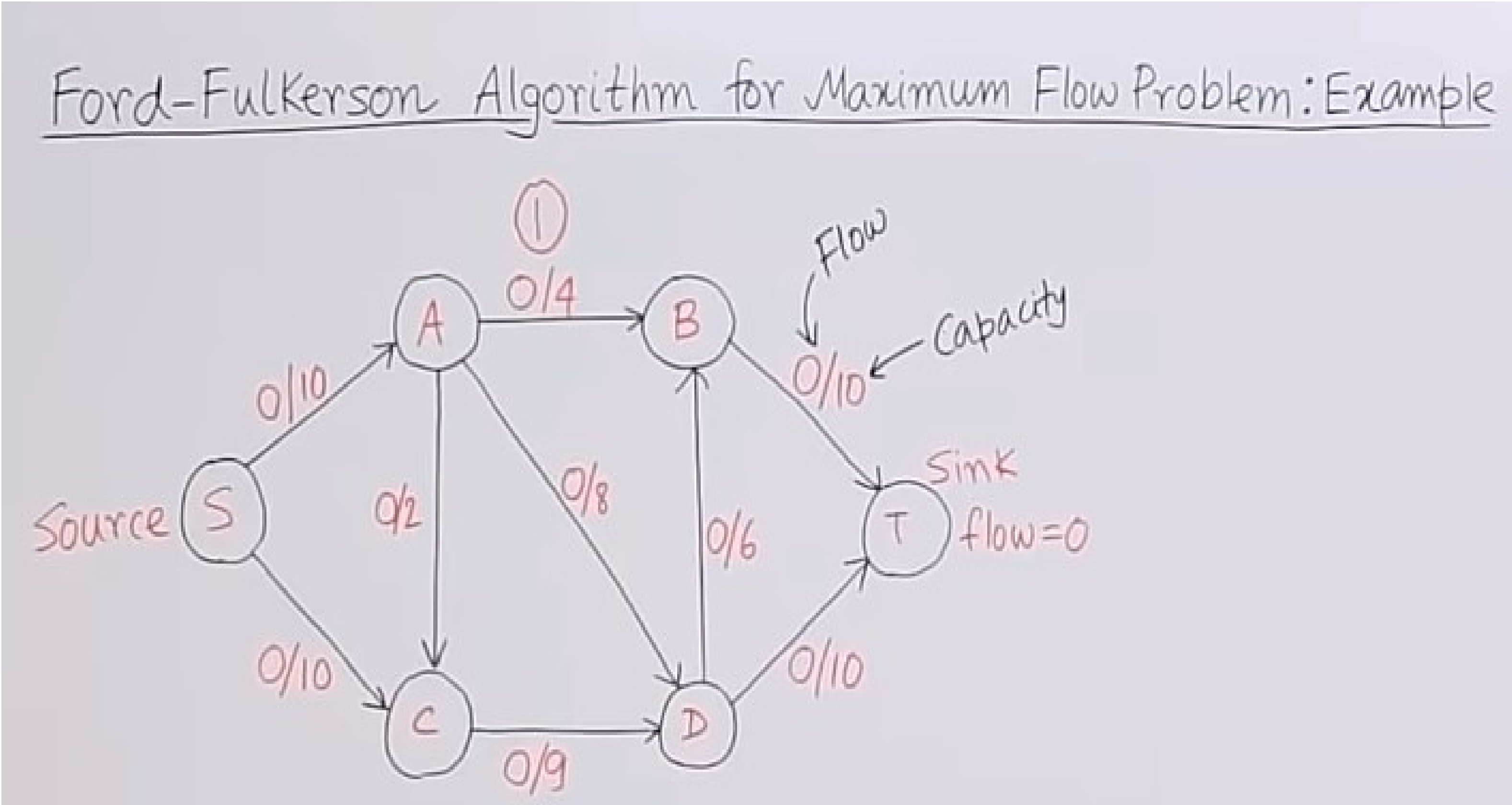


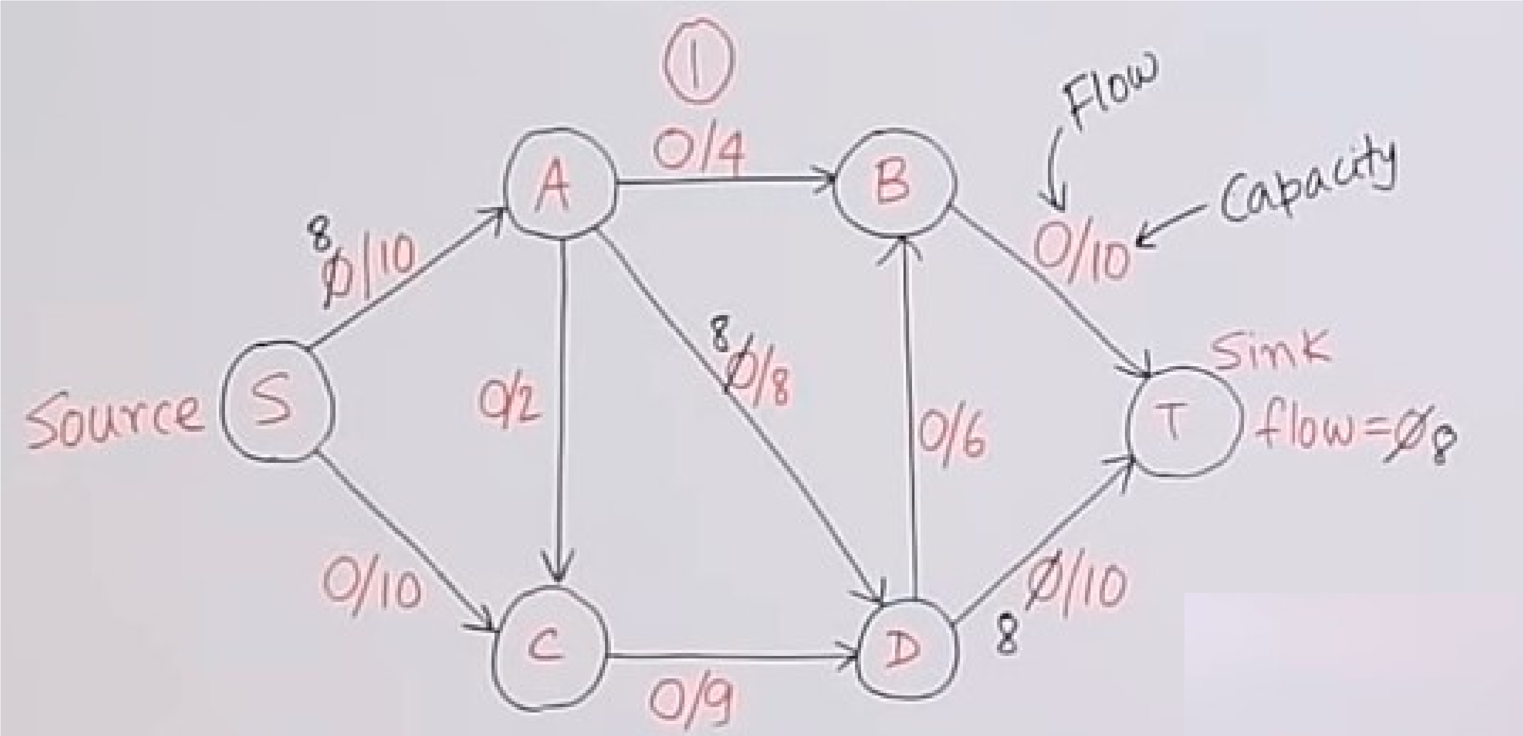
A
B
C
D
E
F
15
9/11
8
10
9
15
9/9
8
10
9/11
Source = A
Sink = F
Capacity
A → C → D → F
A → C → B → E → F


Source = A
Sink = F
Capacity
A
B
C
D
E
F
15
8
2/10
15
2/9
8
2/10
11 9/11
9/9
9/11
A → C → D → F
A → C → B → E → F



A
B
C
D
E
F
15
8
2/10
15
8
2/9
2/10
9/9
11 9/11
9/11
Source = A
Sink = F
11
Capacity

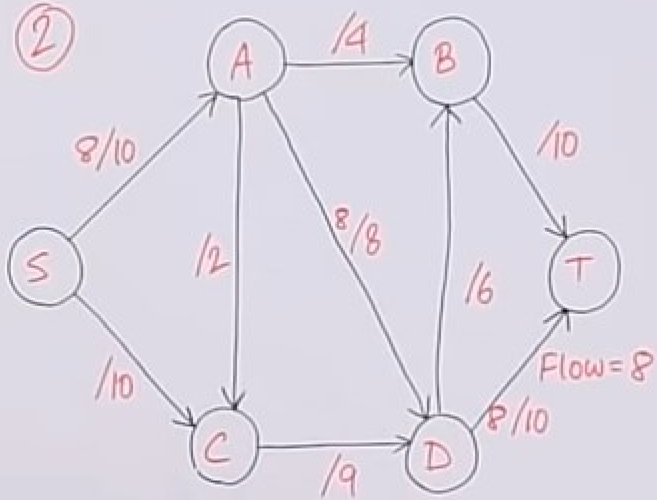

Ford-Fulkerson Algorithm for Maximum Flow Problem: Example
1
Flow
Capacity
Source
S
A
B
C
D
T
Sink
flow=0
0/10
0/4
0/10
0/2
0/8
0/6
0/10
0/10
0/9



Augmenting paths	Bottle neck capacity
① S → A → D → T	8

roblem : Example

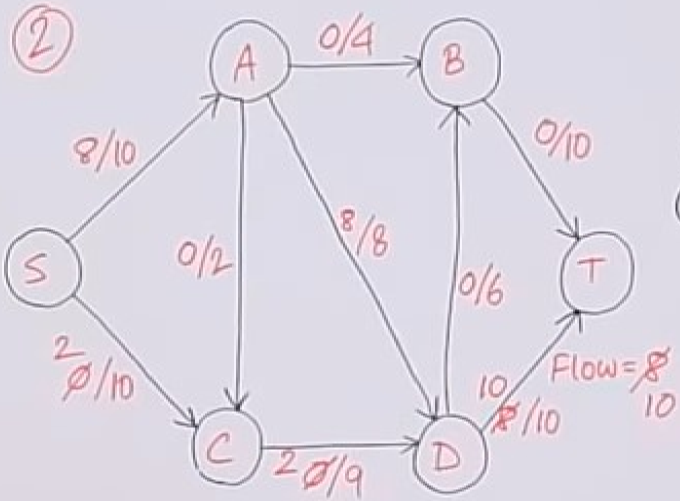
2



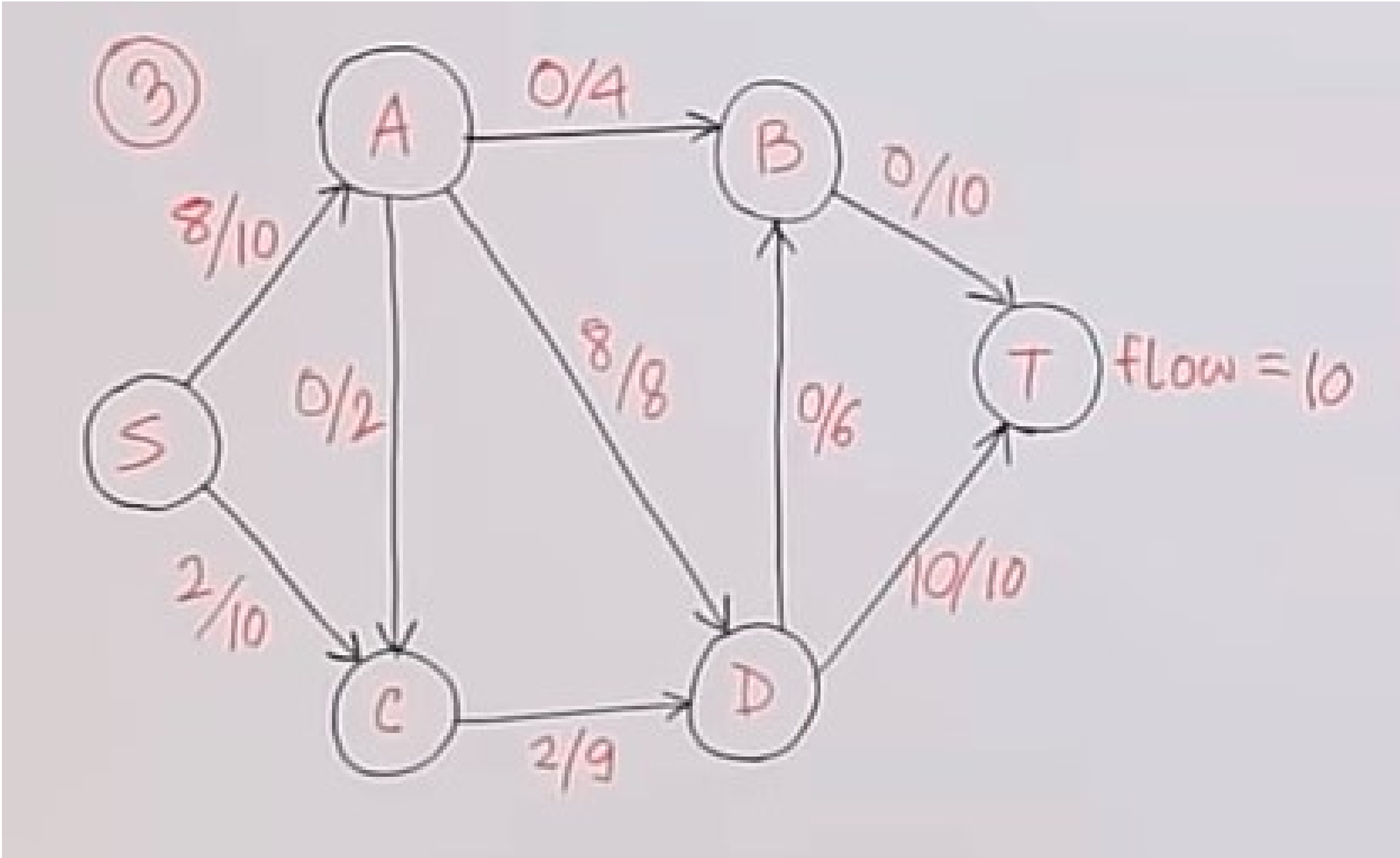
Augmenting paths	Bottle neck capacity
① S → A → D → T	8

Problem : Example

2



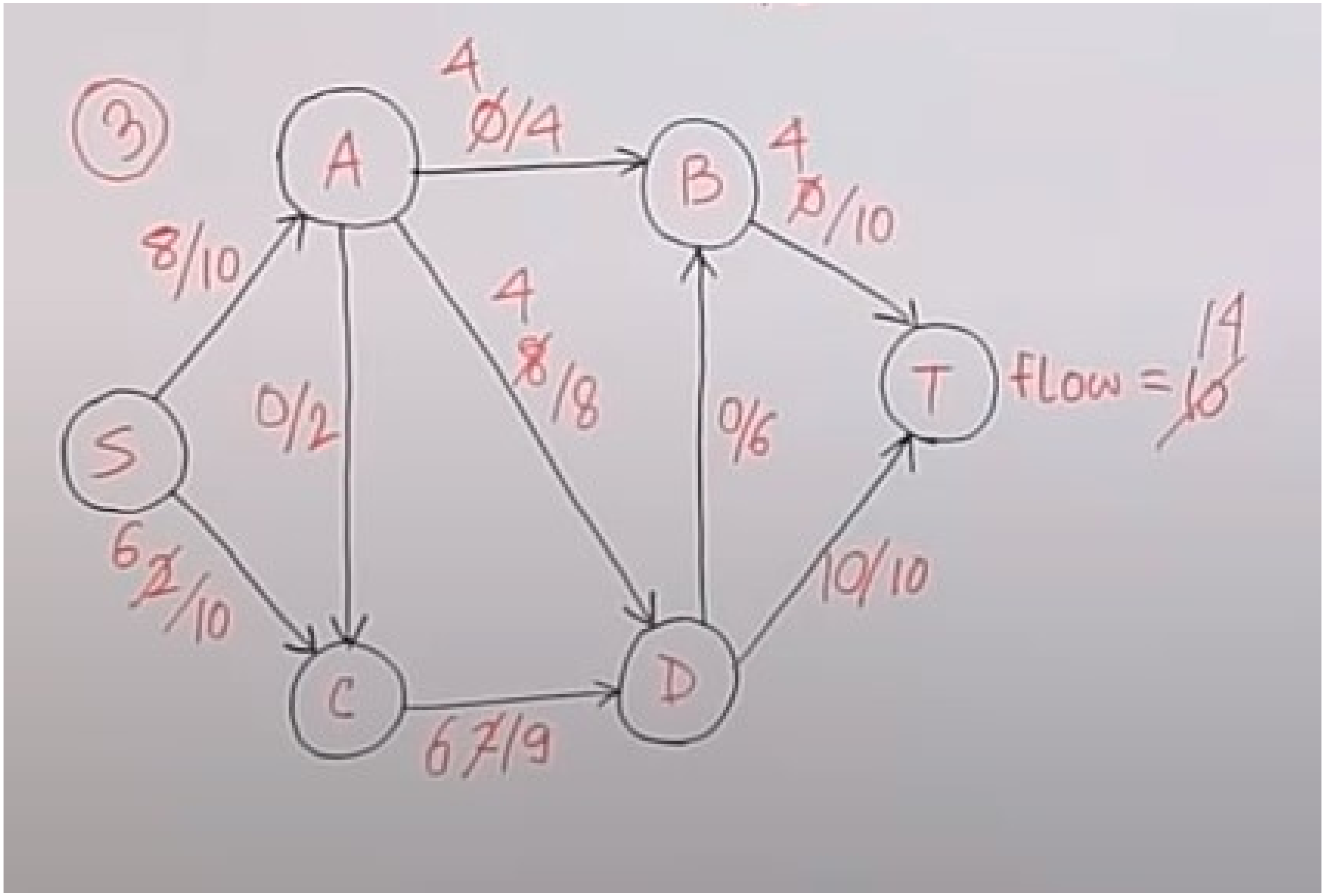
Augmenting paths	Bottle neck capacity
① S → A → D → T	8
② S → C → D → T	2



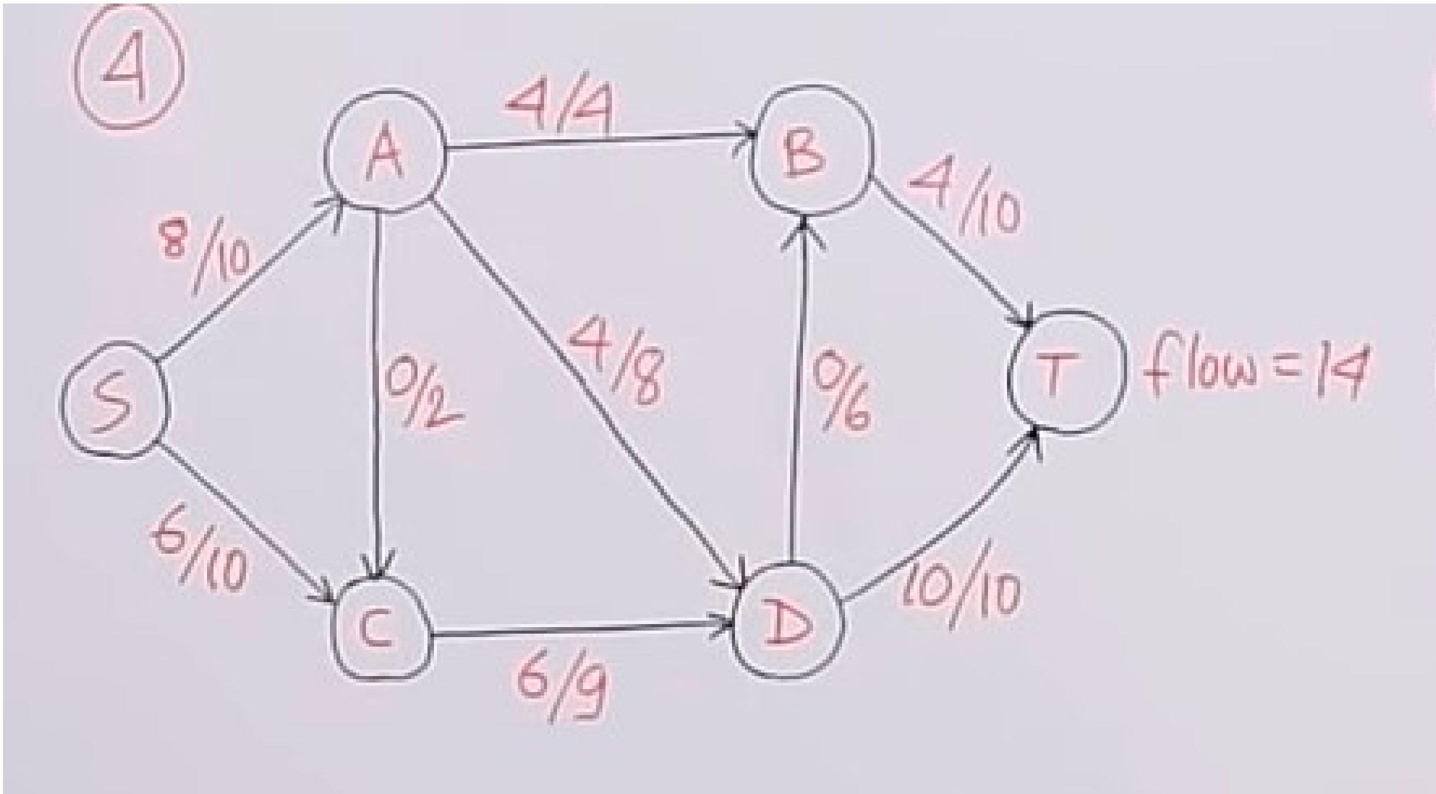
3
A
B
S
T
C
D
0/4
0/10
8/10
8/8
0/2
0/6
2/10
10/10
2/9
flow = 10

Augmenting paths	Bottle neck capacity
① $S \rightarrow A \rightarrow D \rightarrow T$	8
② $S \rightarrow C \rightarrow D \rightarrow T$	2
③ $S \rightarrow C \rightarrow D \rightarrow A \rightarrow B \rightarrow T$	

Augmenting paths	Bottle neck capacity
① $S \rightarrow A \rightarrow D \rightarrow T$	8
② $S \rightarrow C \rightarrow D \rightarrow T$	2
③ $S \rightarrow C \rightarrow D \rightarrow A \rightarrow B \rightarrow T$	4

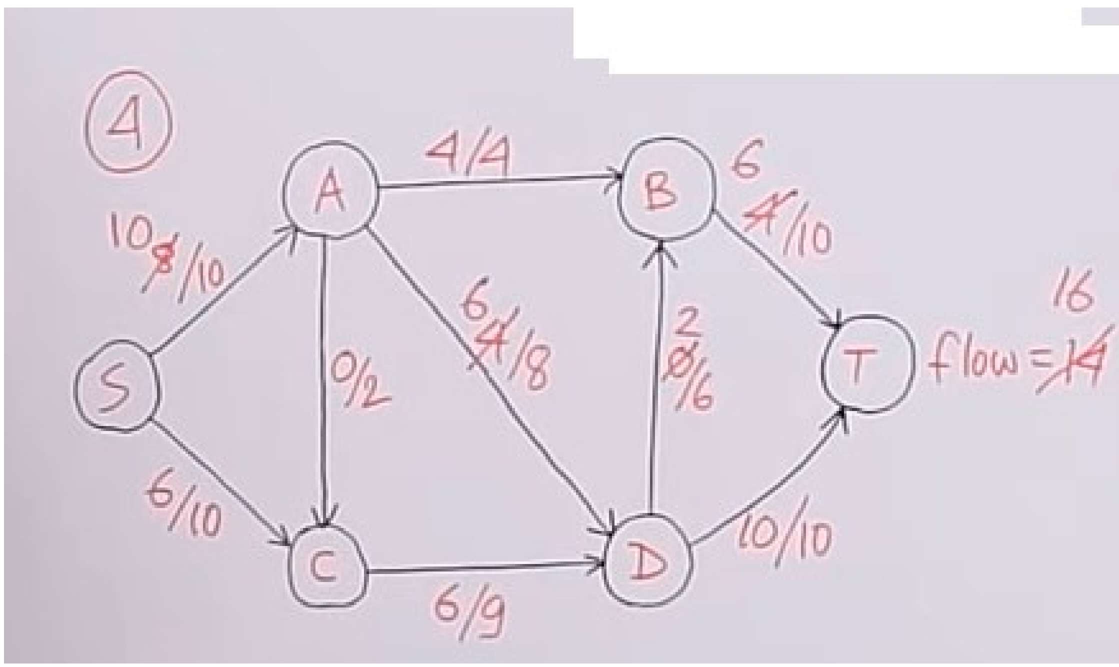


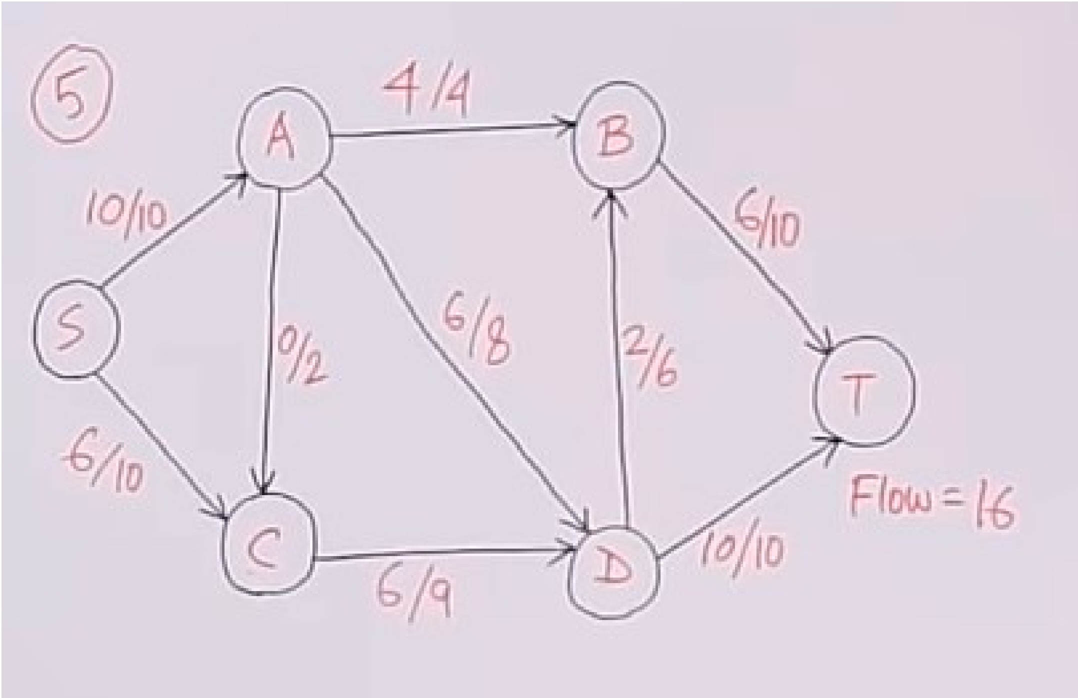
3
A
B
C
D
S
T
8/10
4 0/4
4 0/10
0/2
4 8/8
0/6
6 2/10
10/10
6 7/9
flow = 14 10


4
S
A
B
C
D
T
8/10
6/10
4/4
0/2
4/8
6/9
0/6
4/10
10/10
flow = 14

Augmenting paths	Bottle neck capacity
① S → A → D → T	8
② S → C → D → T	2
③ S → C → D → A → B → T	4
④ S → A → D → B → T	2

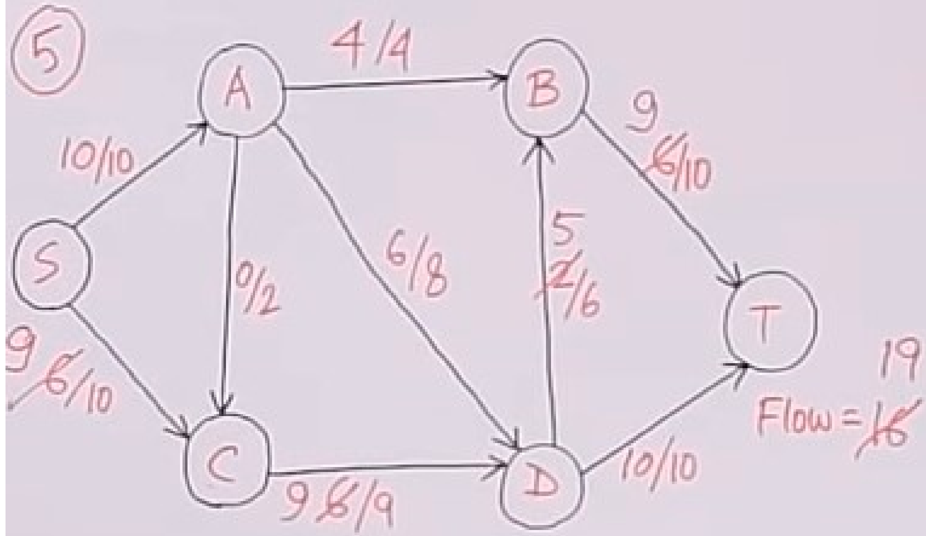
= ~~8~~ 10

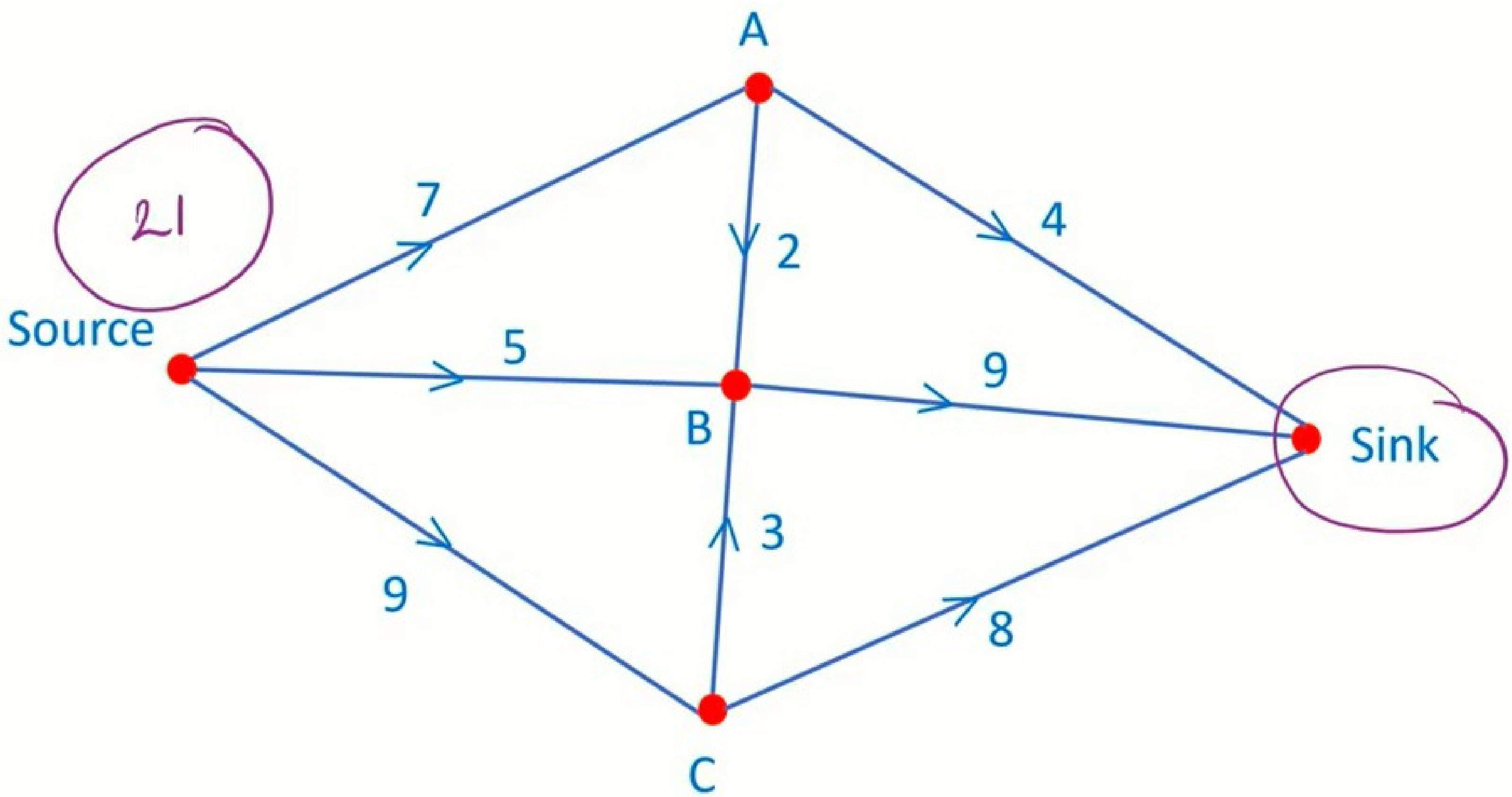


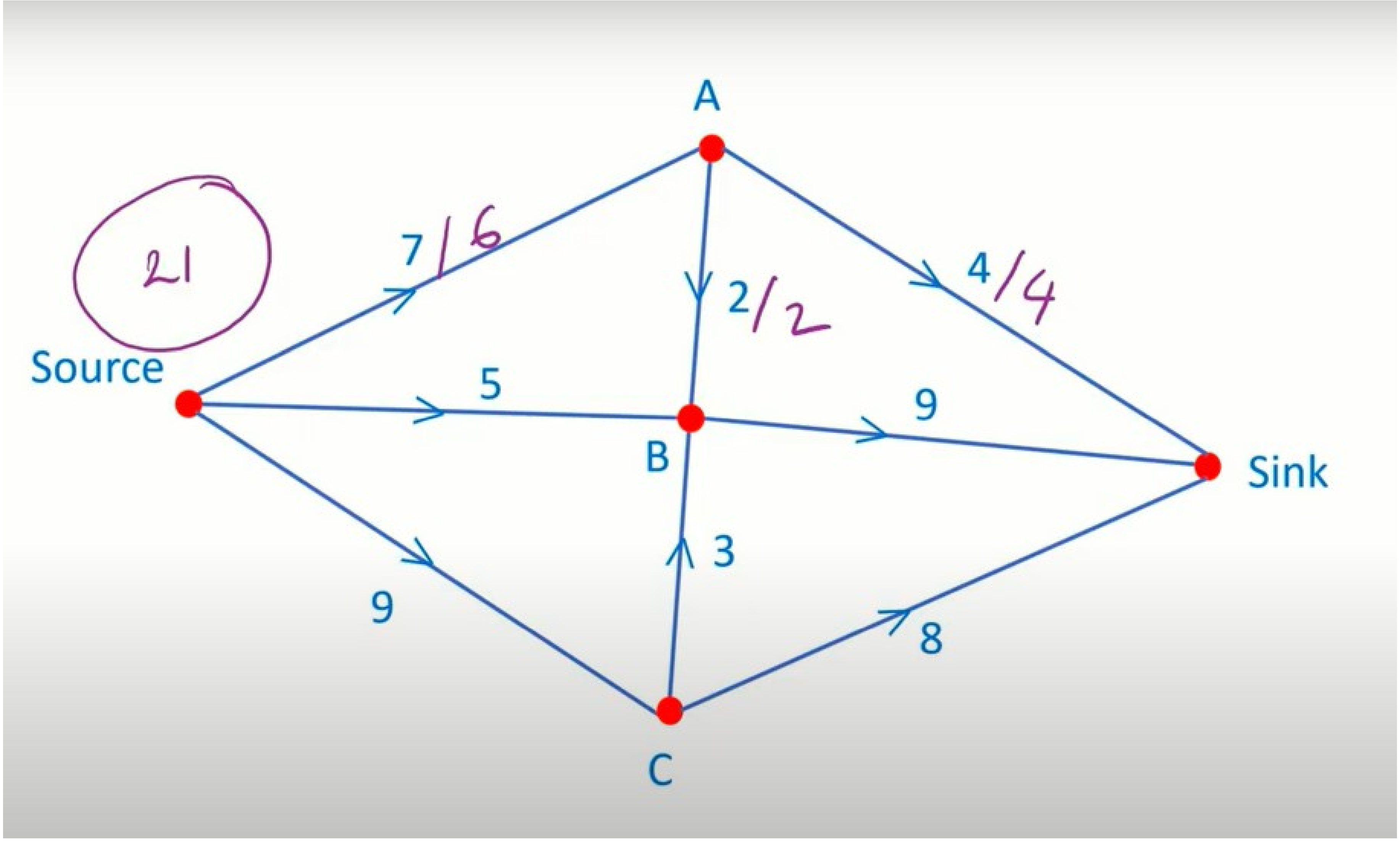


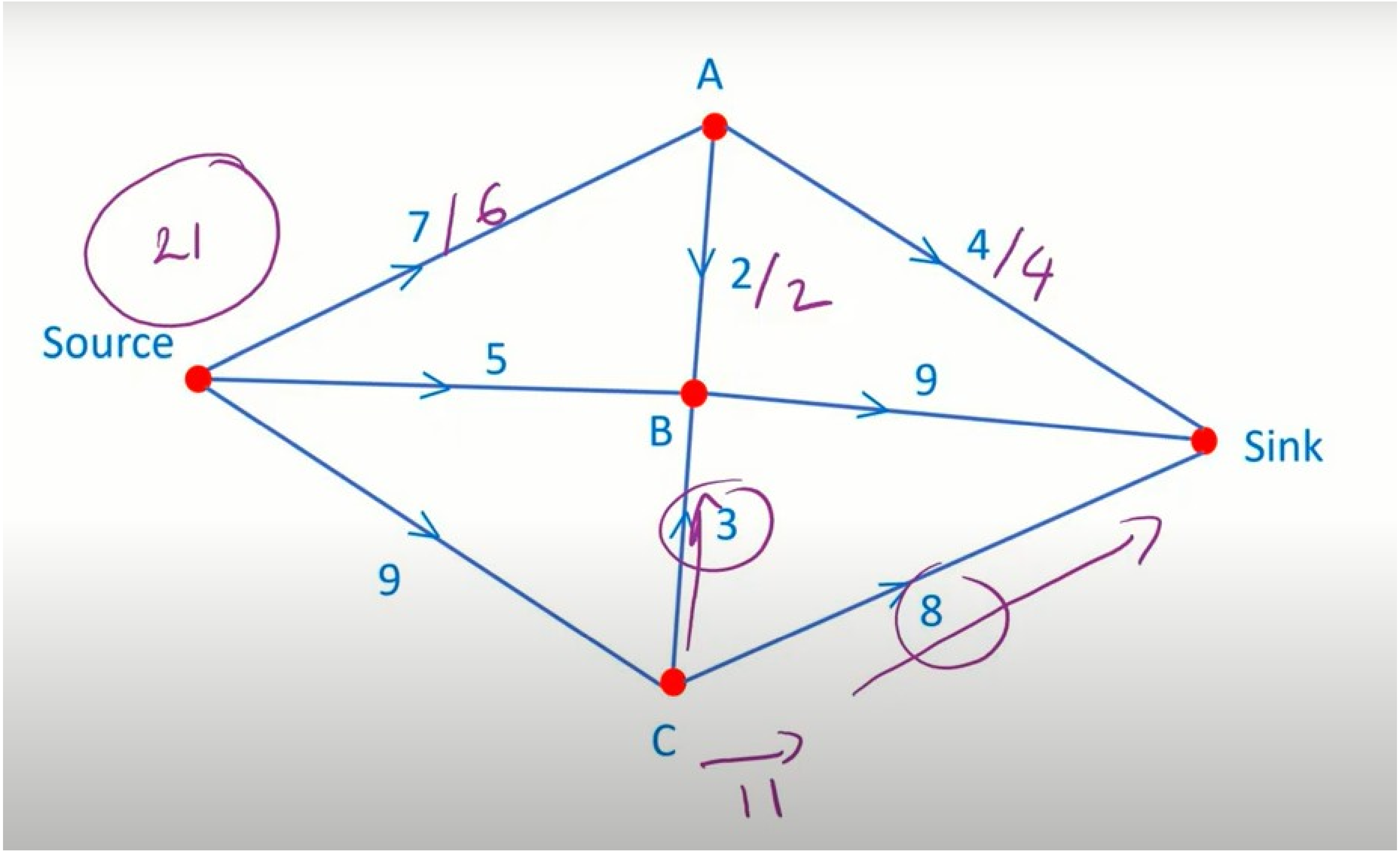
Augmenting paths	Bottle neck capacity
1 S → A → D → T	8
2 S → C → D → T	2
3 S → C → D → A → B → T	4
4 S → A → D → B → T	2
5 S → C → D → B → T	3

Augmenting paths	Bottleneck capacity
① $S \to A \to D \to T$	8
② $S \to C \to D \to T$	2
③ $S \to C \to D \to A \to B \to T$	4
④ $S \to A \to D \to B \to T$	2
⑤ $S \to C \to D \to B \to T$	3

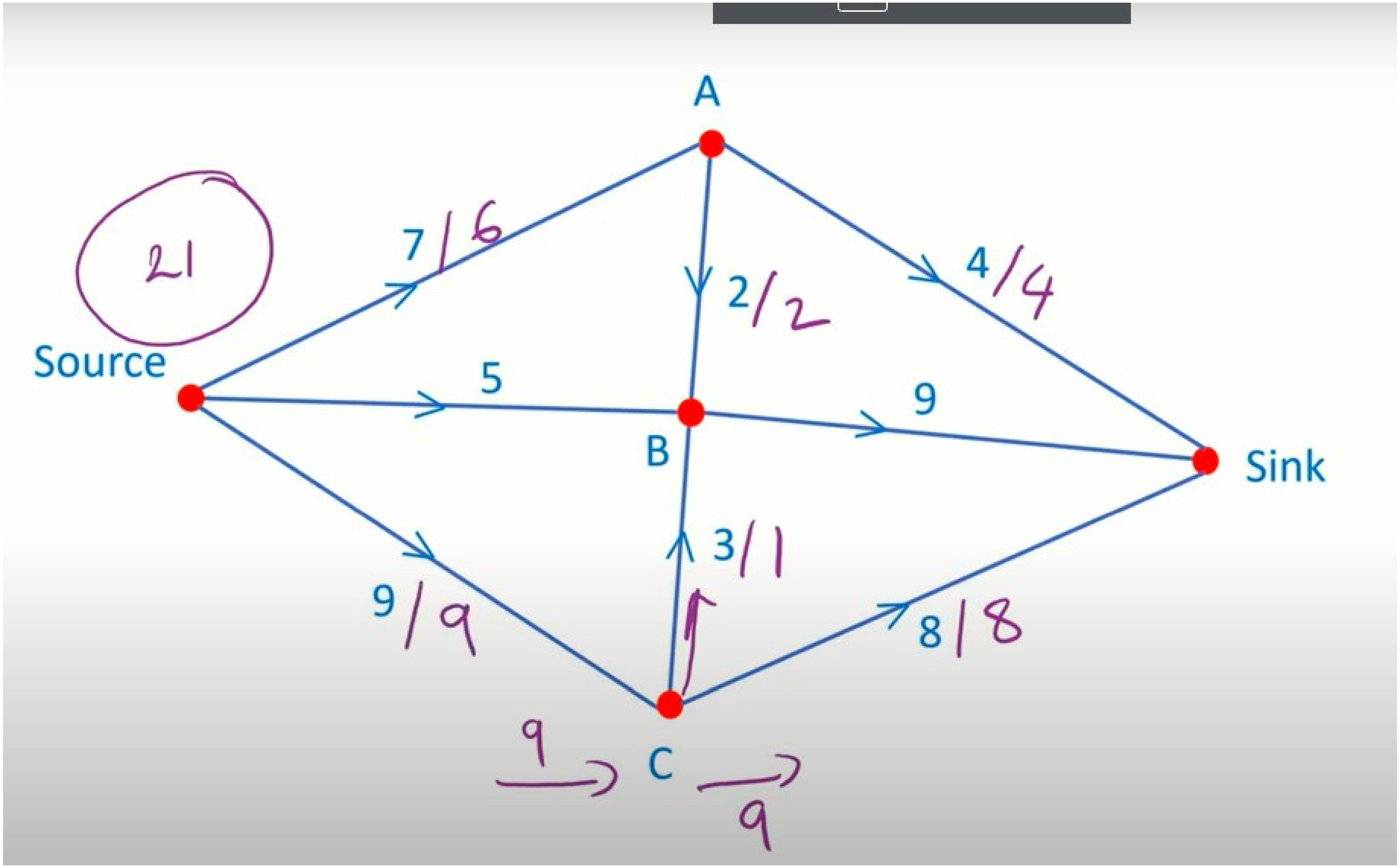



21
A
Source
7
2
4
5
9
B
Sink
3
9
8
C

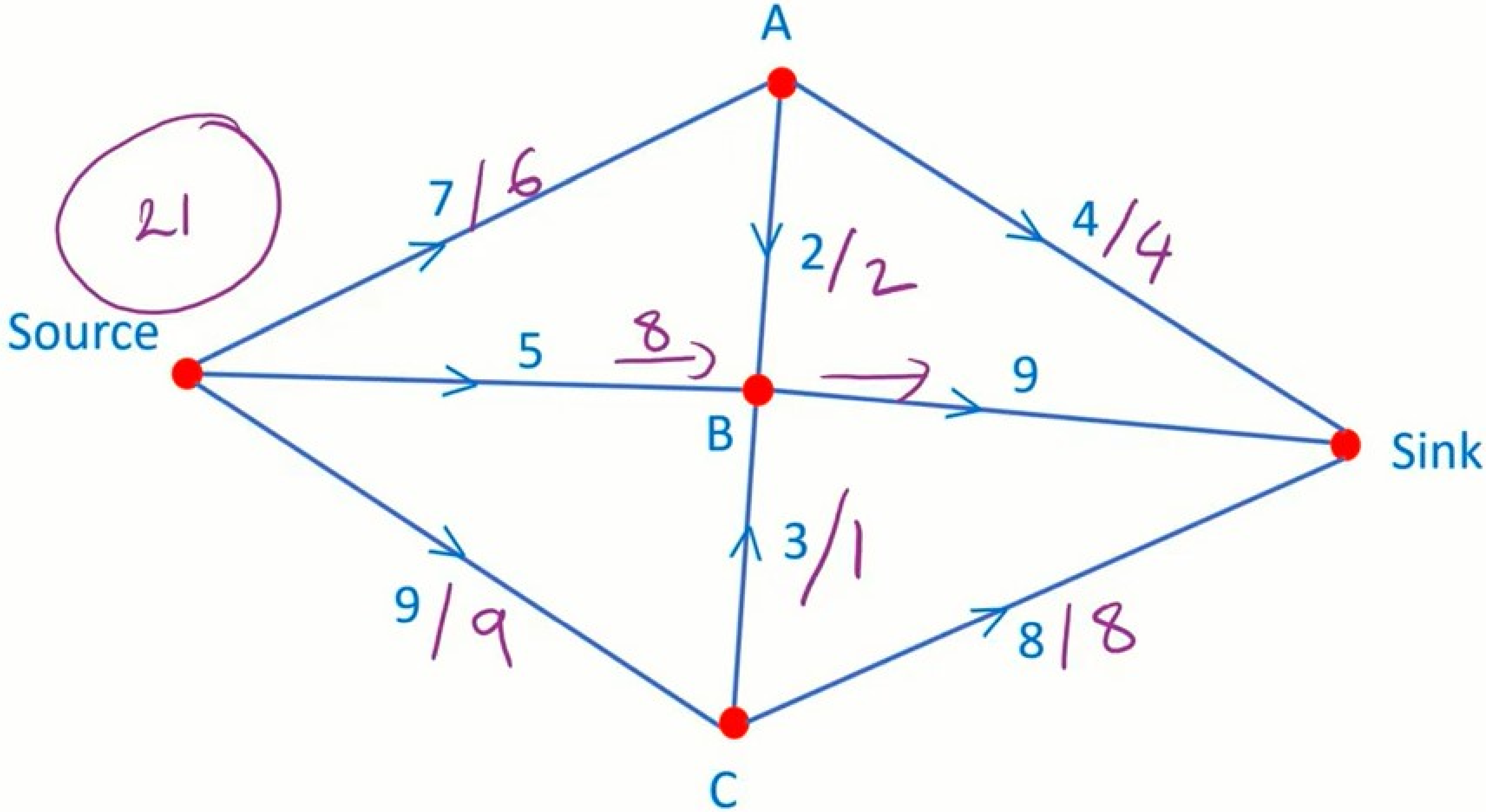

21
A
7/6
2/2
4/4
Source
5
9
B
Sink
3
9
8
C



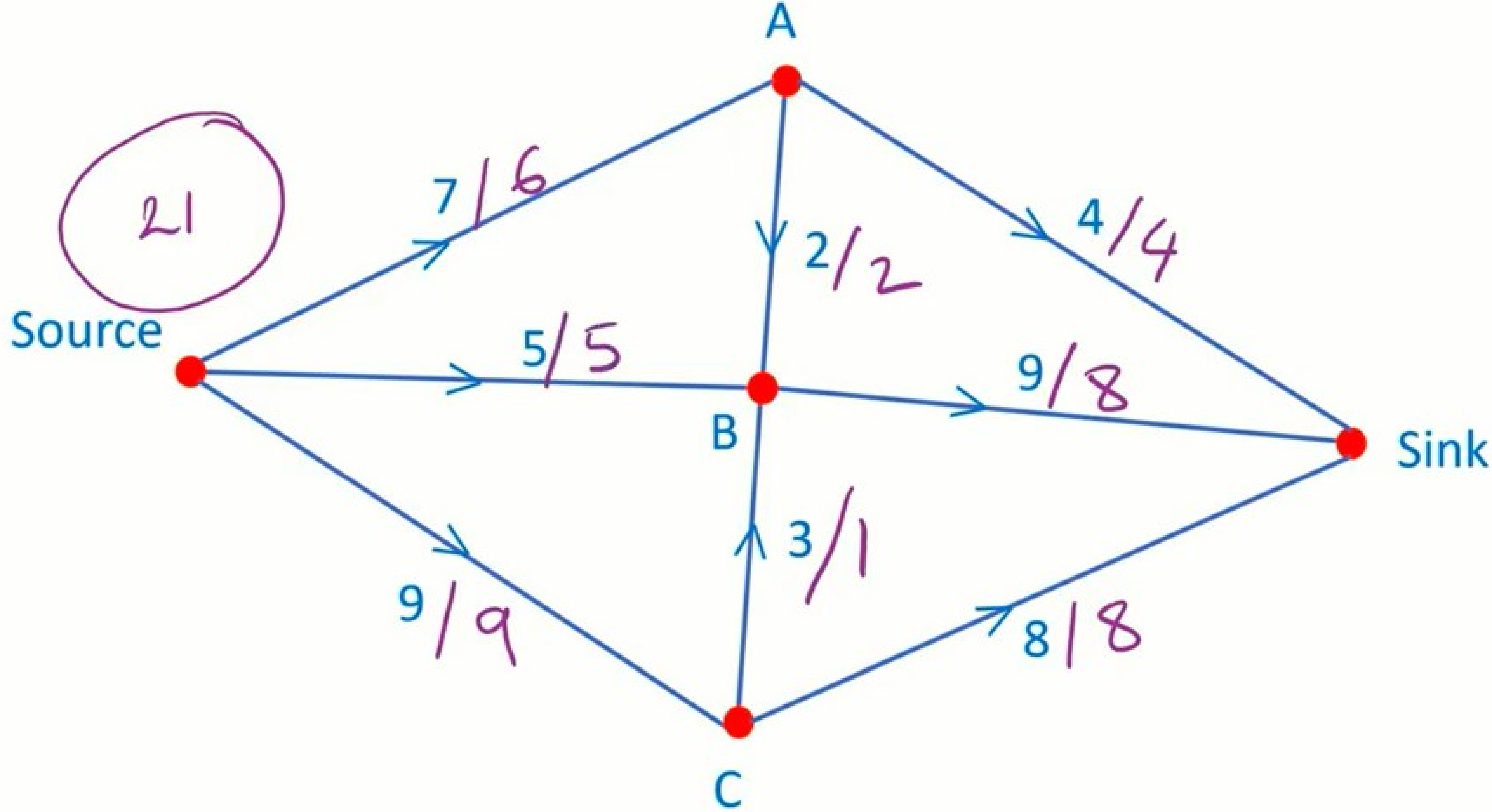
A
21
7/6
2/2
4/4
Source
5
9
B
Sink
3
9
8
C
11



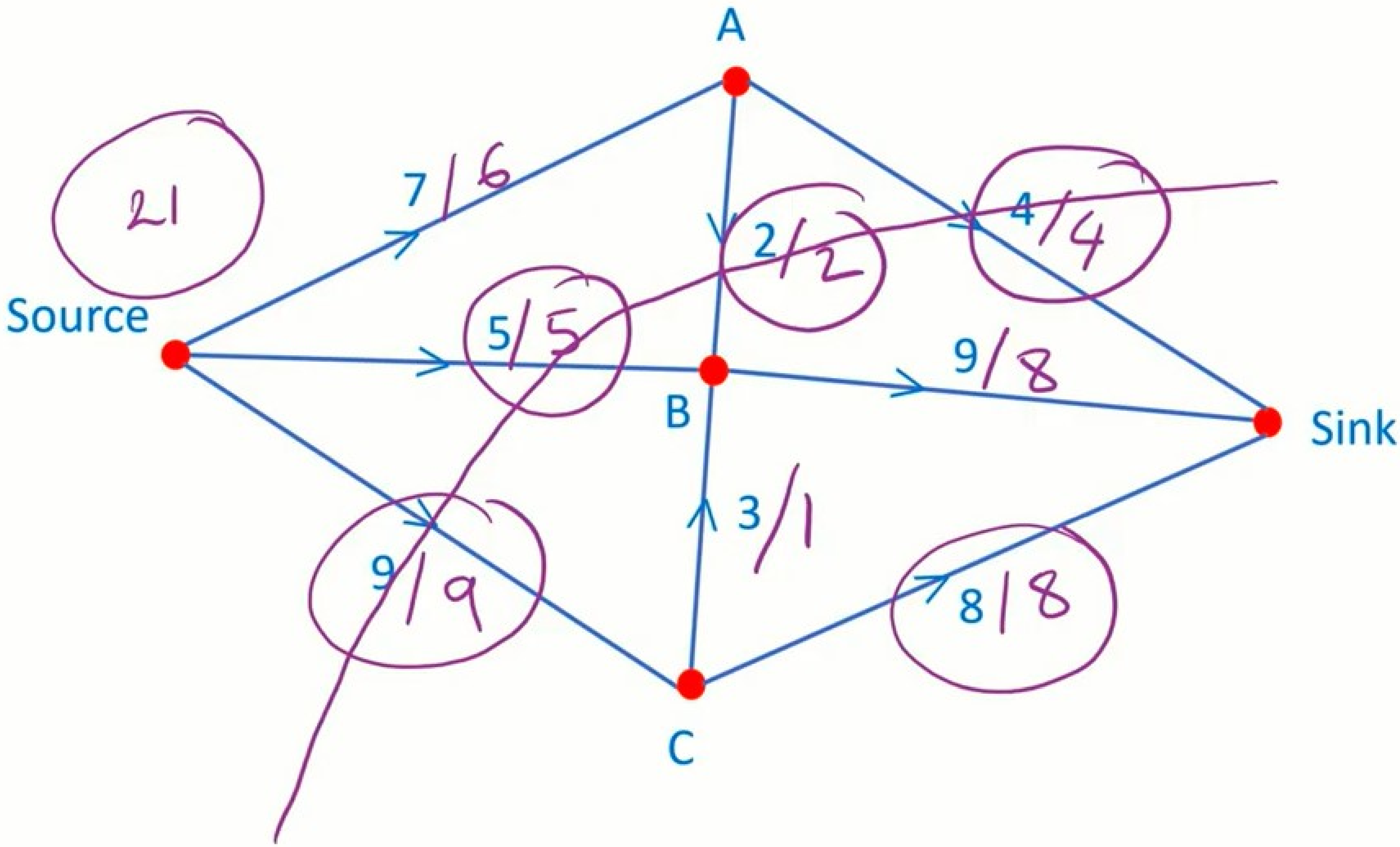
A
Source
B
C
Sink
21
7/6
5
9/9
2/2
4/4
9
3/1
8/8
9
9



21
A
Source
7/6
2/2
4/4
5
8
7
9
B
Sink
3/1
9/9
8/8
C

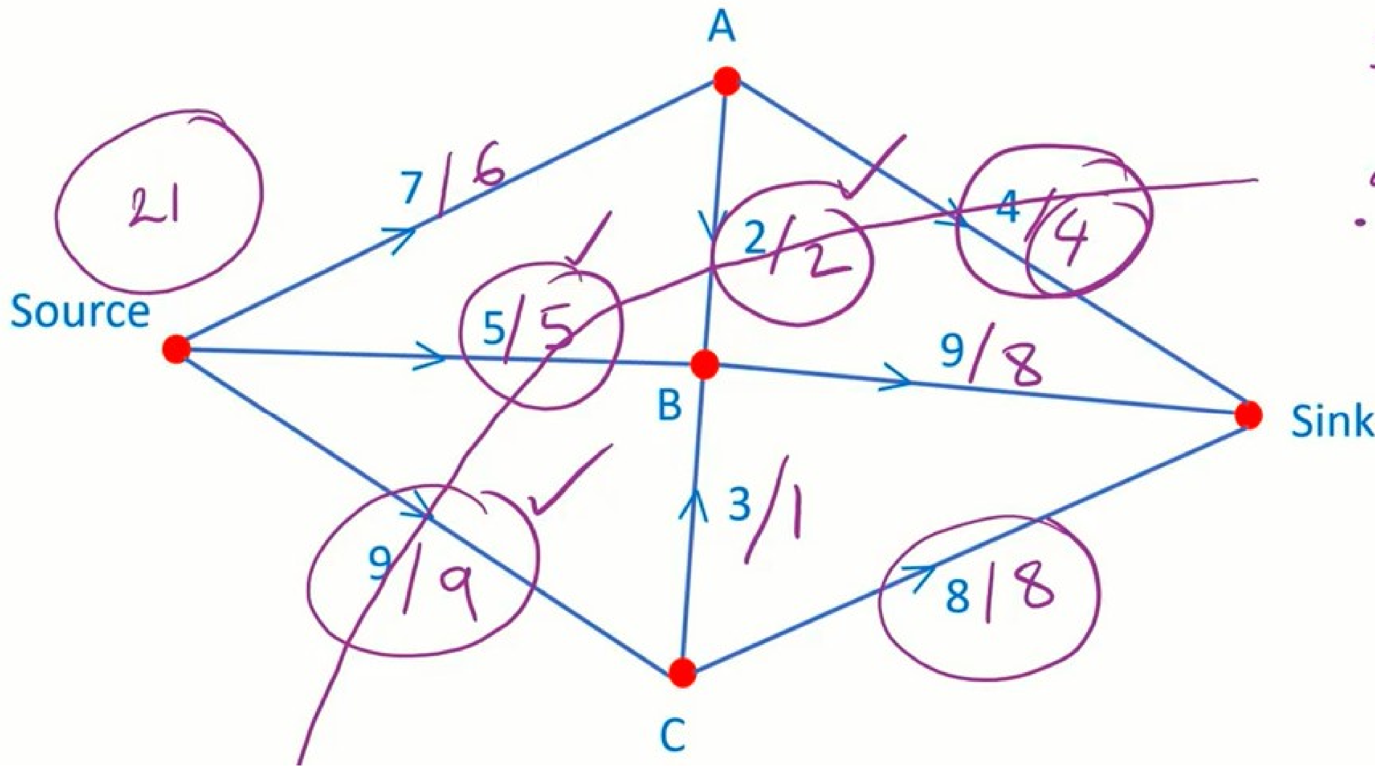


21
A
Source
7/6
2/2
4/4
5/5
B
9/8
Sink
3/1
9/9
8/8
C



A
21
7/6
2/2
4/4
Source
5/5
9/8
B
Sink
3/1
9/9
8/8
C

Find the maximum flow for the network diagram below.

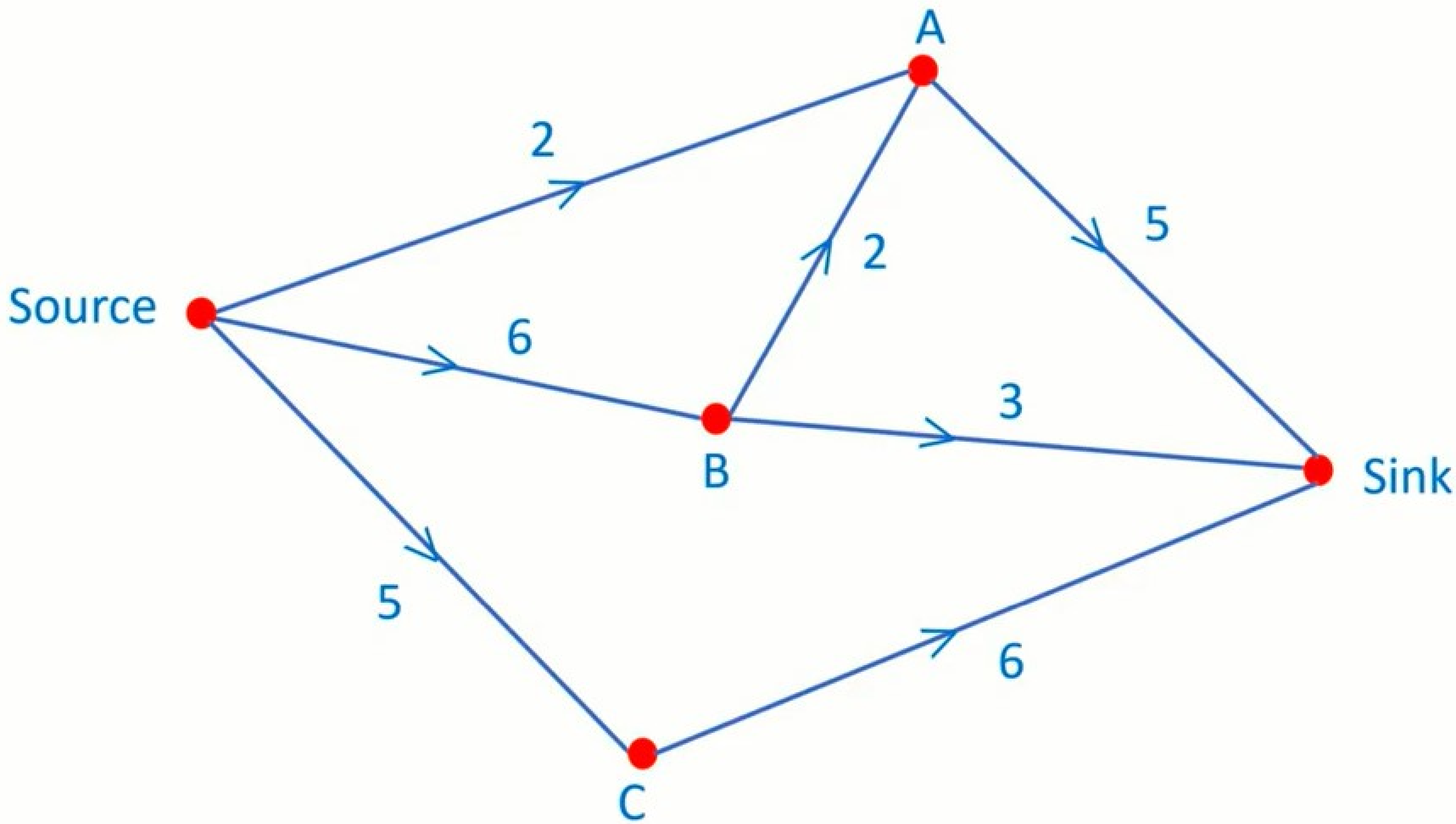


$4 + 2 + 5 + 9$

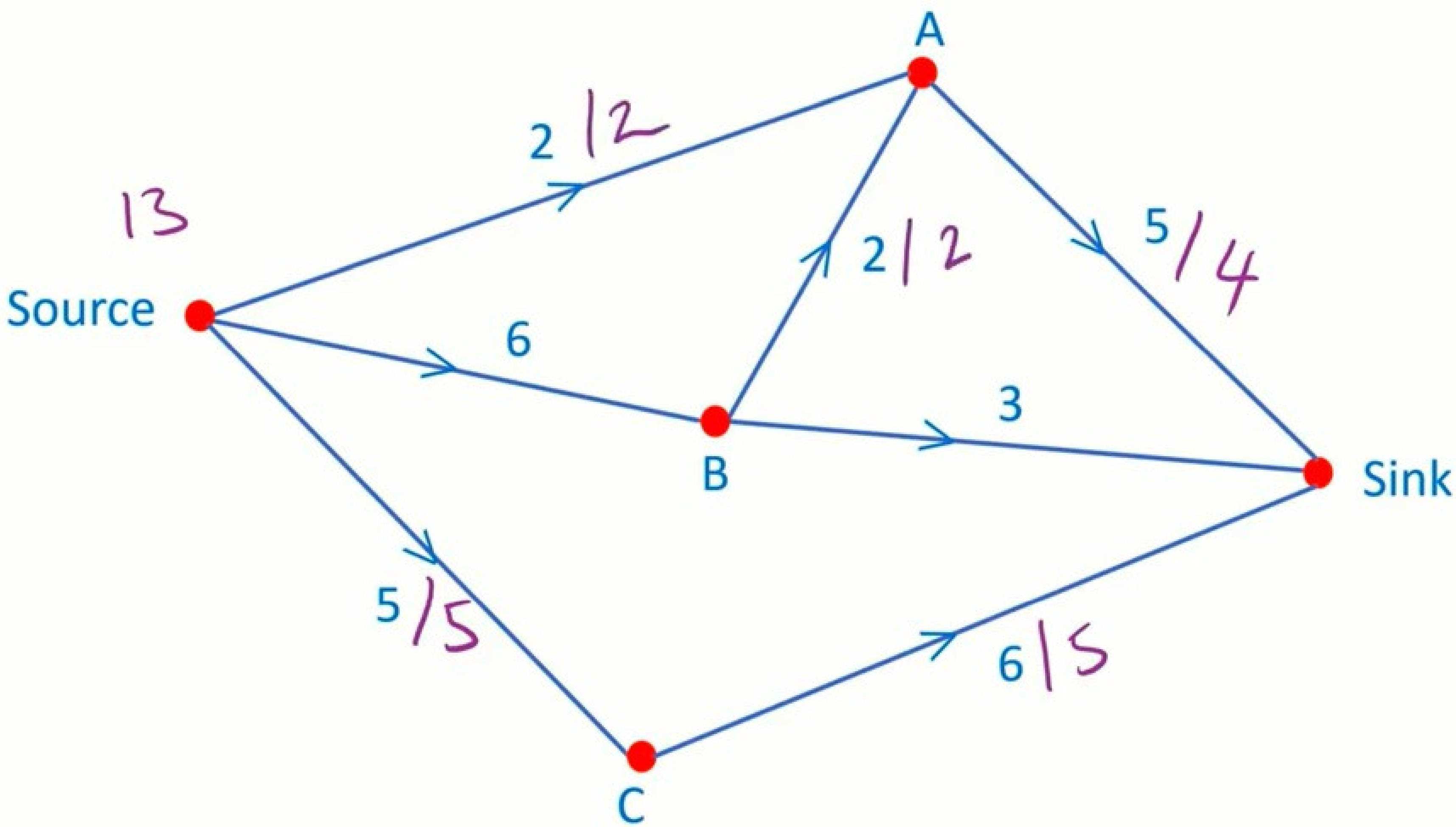
$= 20$

$\therefore$ the maximum flow is 20

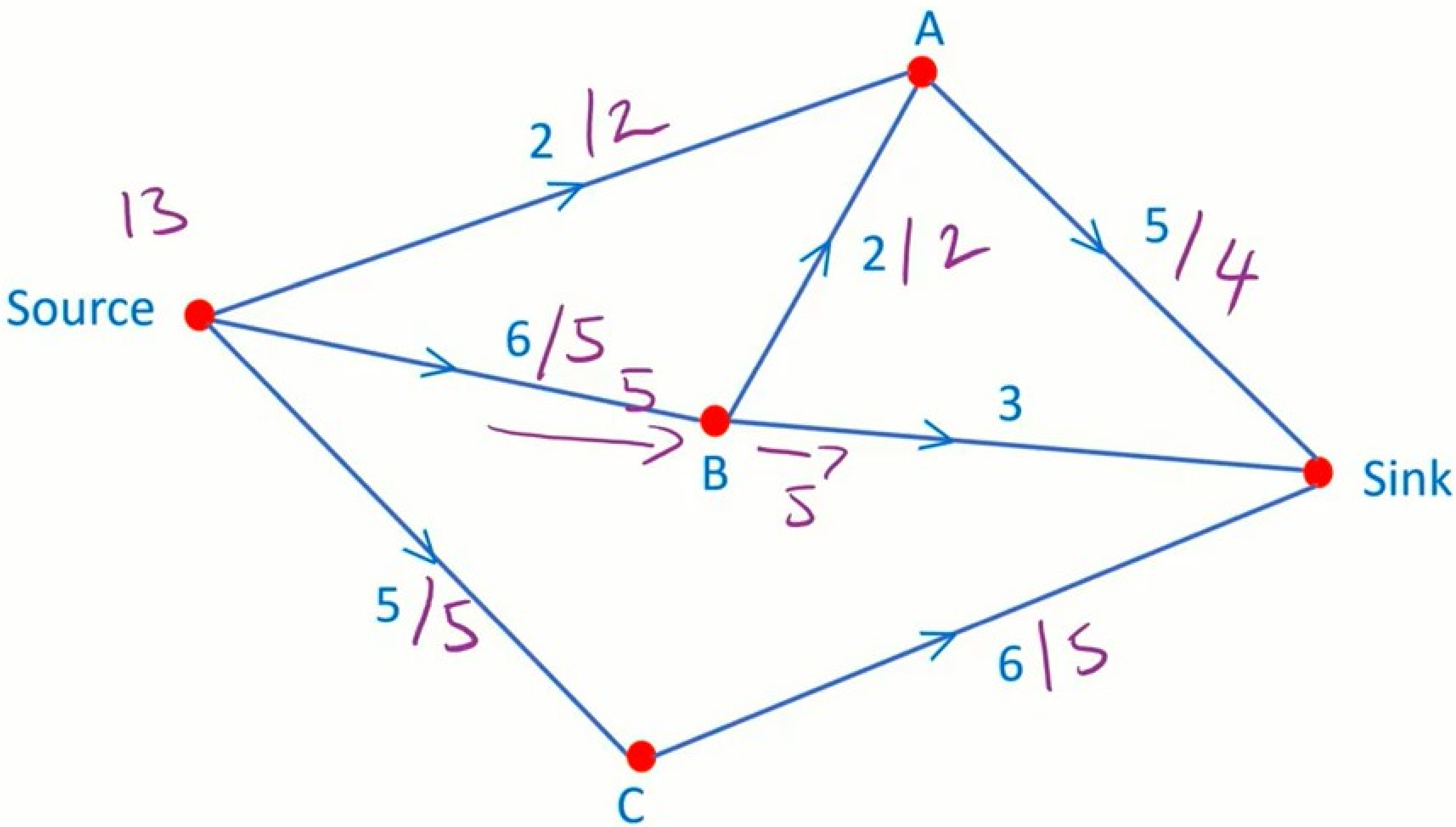
Find the maximum flow for the network diagram below.



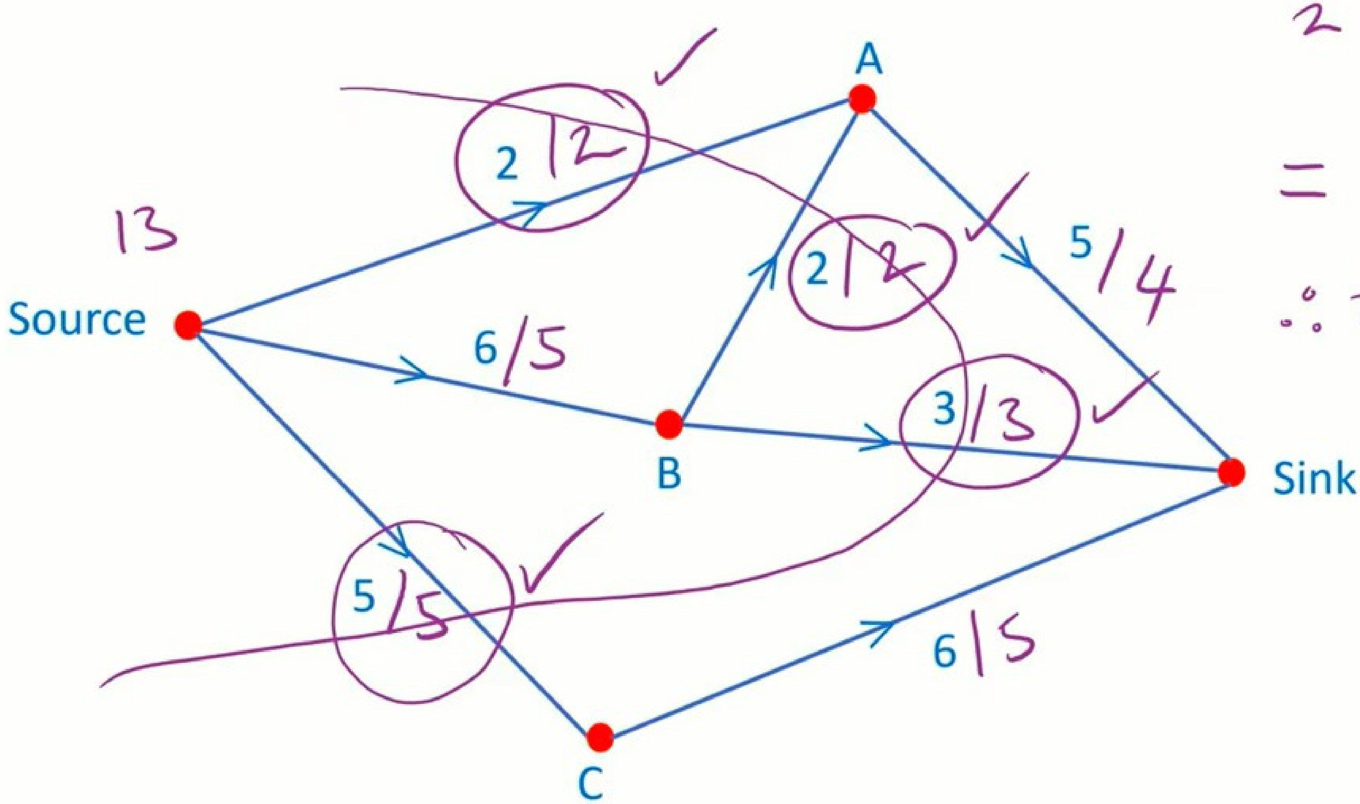
Find the maximum flow for the network diagram below.



Find the maximum flow for the network diagram below.



Find the maximum flow for the network diagram below.

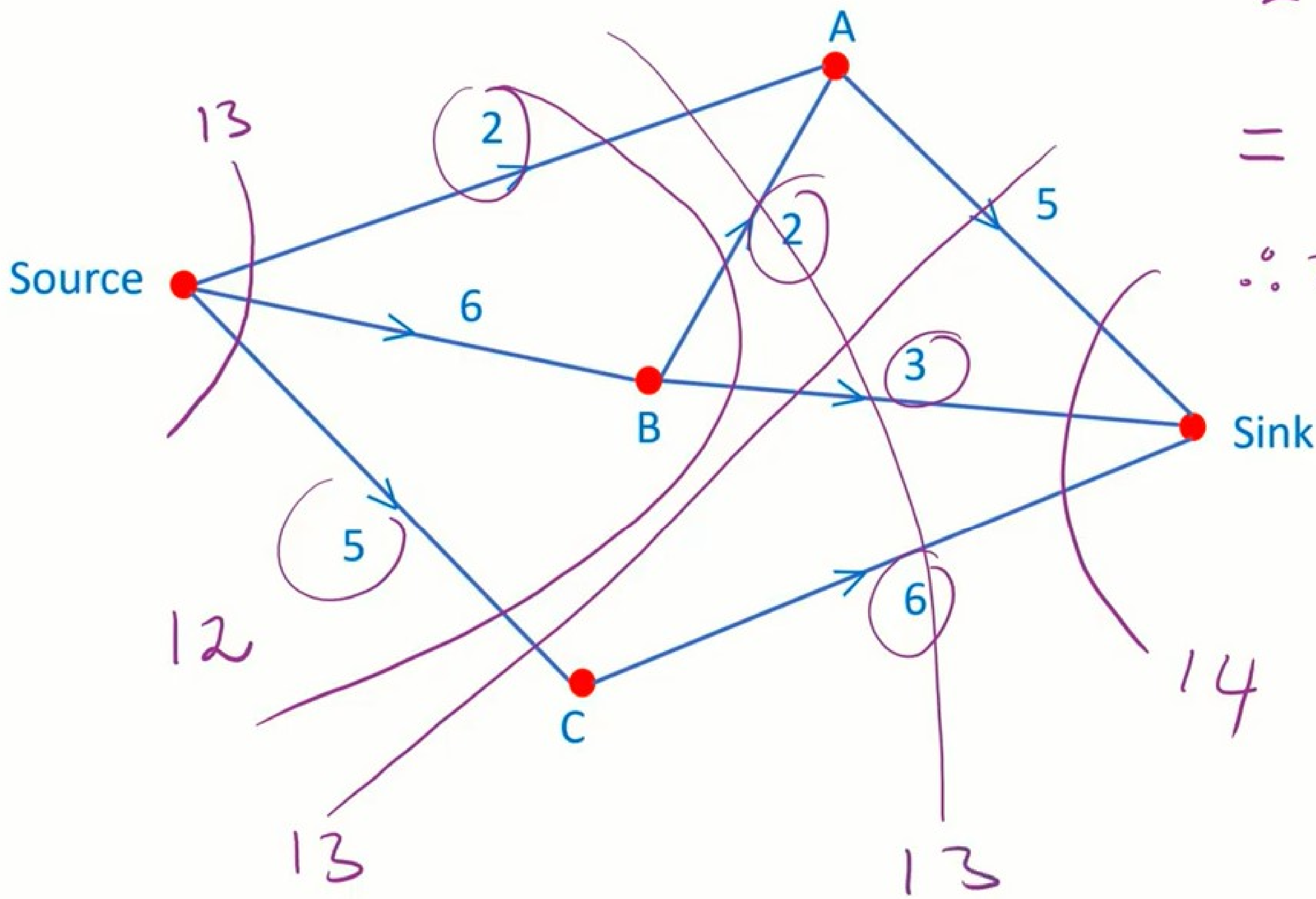


$2 + 2 + 3 + 5$

$= 12$

$\therefore$ the maximum flow is 12

Find the maximum flow for the network diagram below.

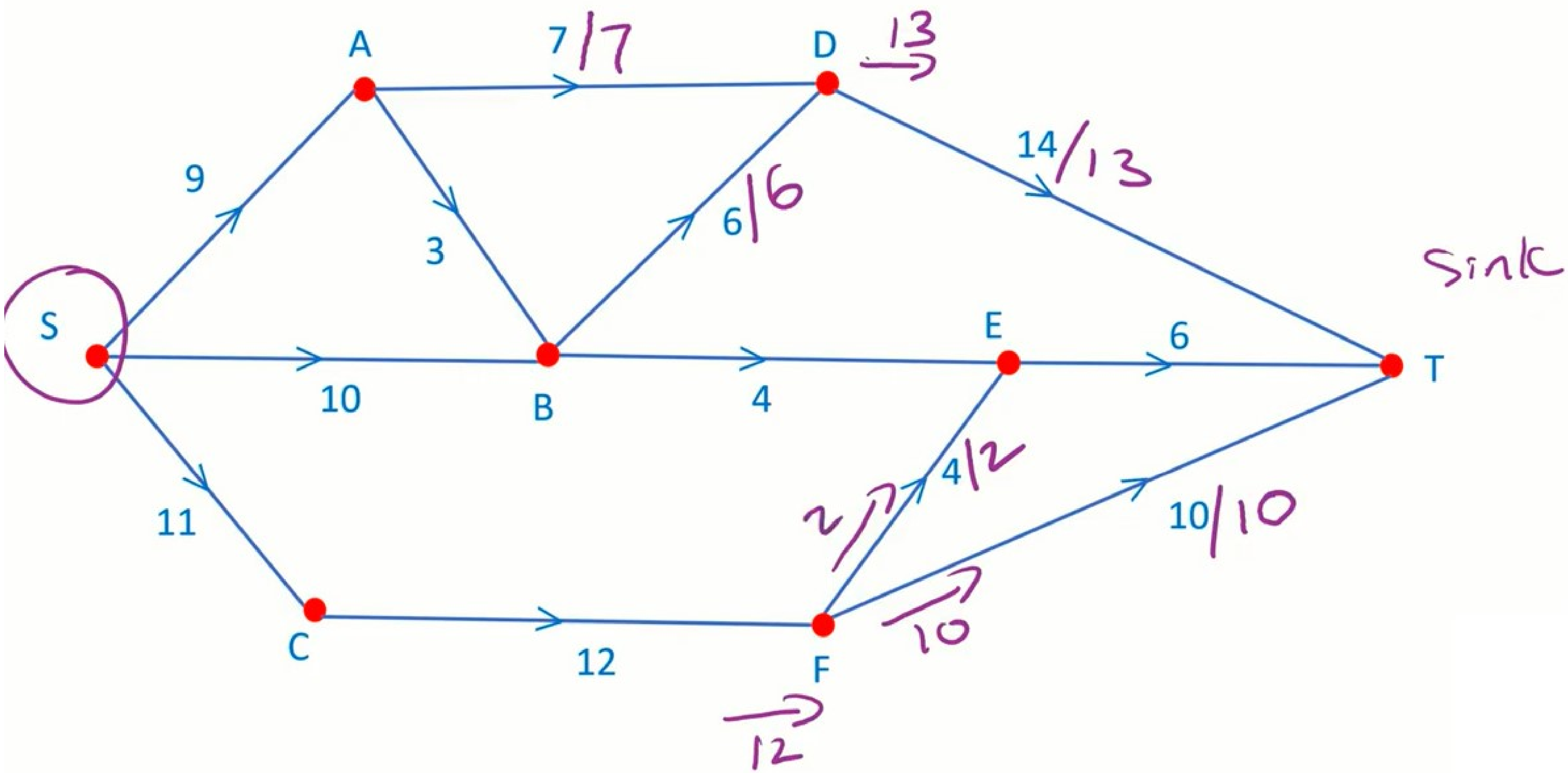


$2 + 2 + 3 + 5$

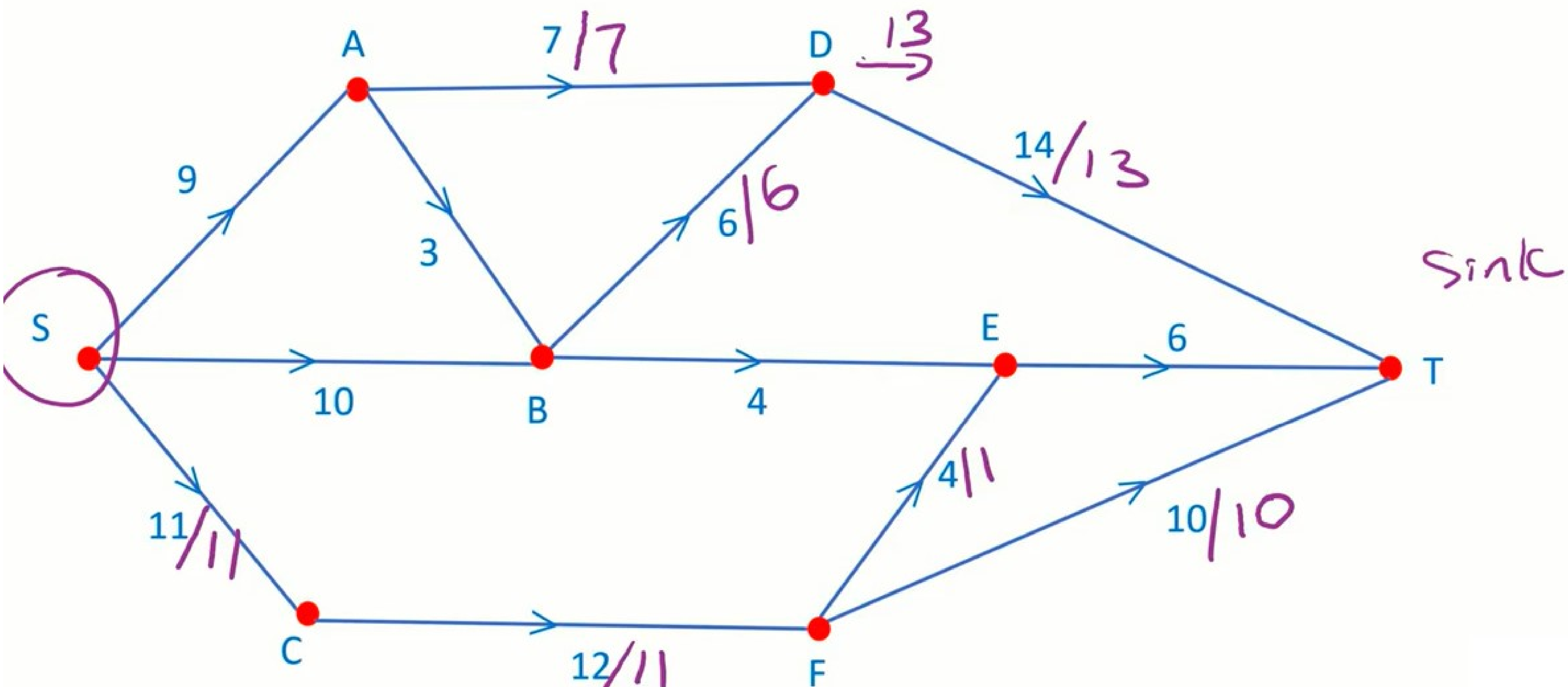
$= 12$ ✓✓

$\therefore$ the maximum flow is 12

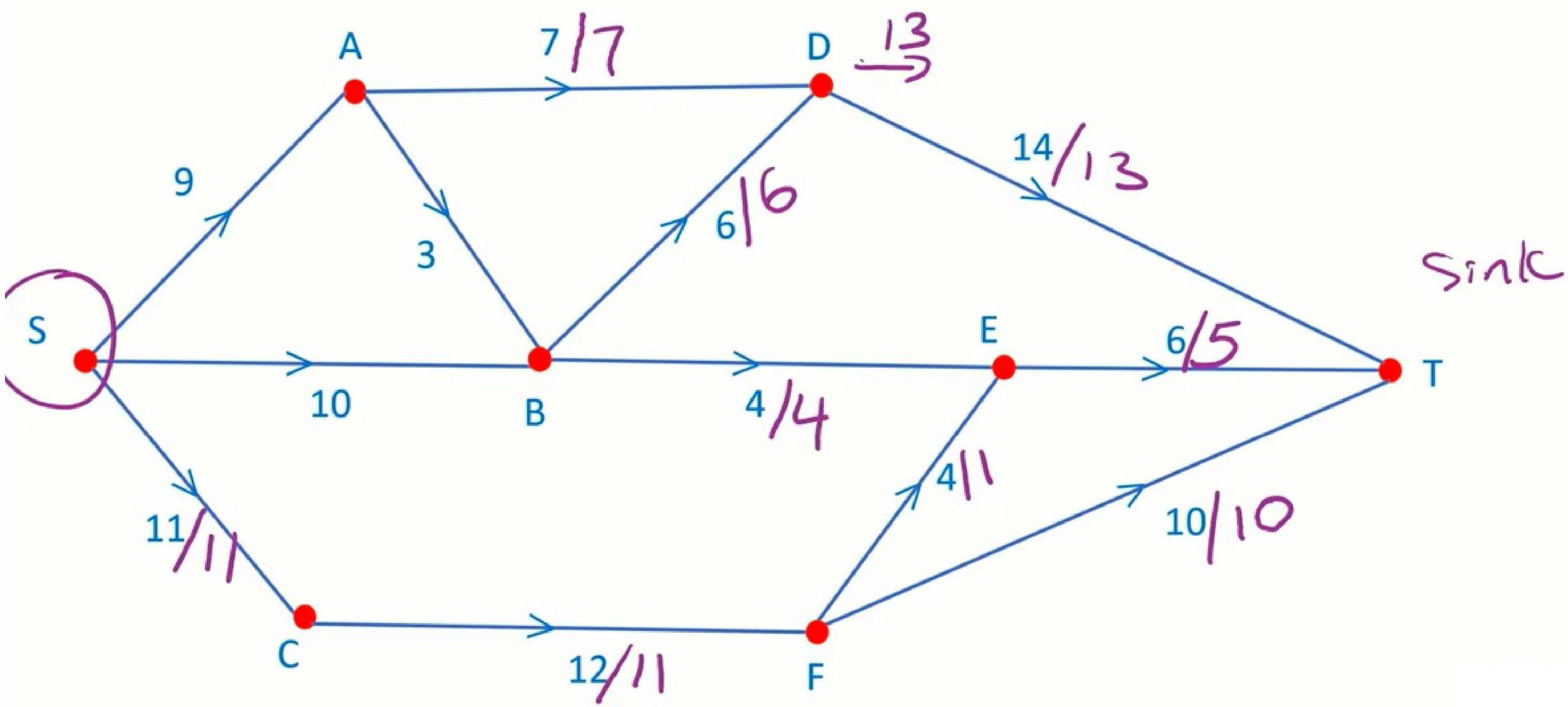
Find the maximum flow for the network diagram below.



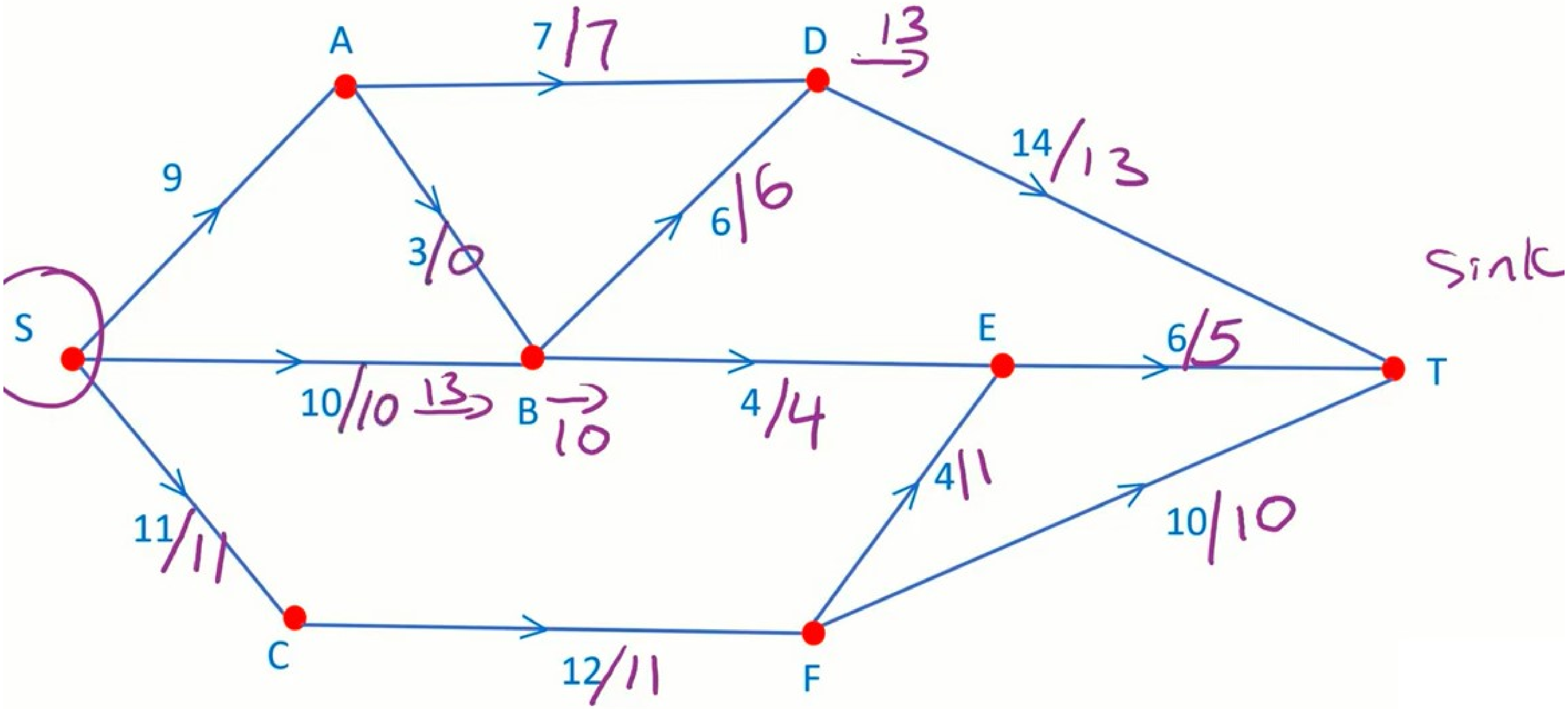
Find the maximum flow for the network diagram below.



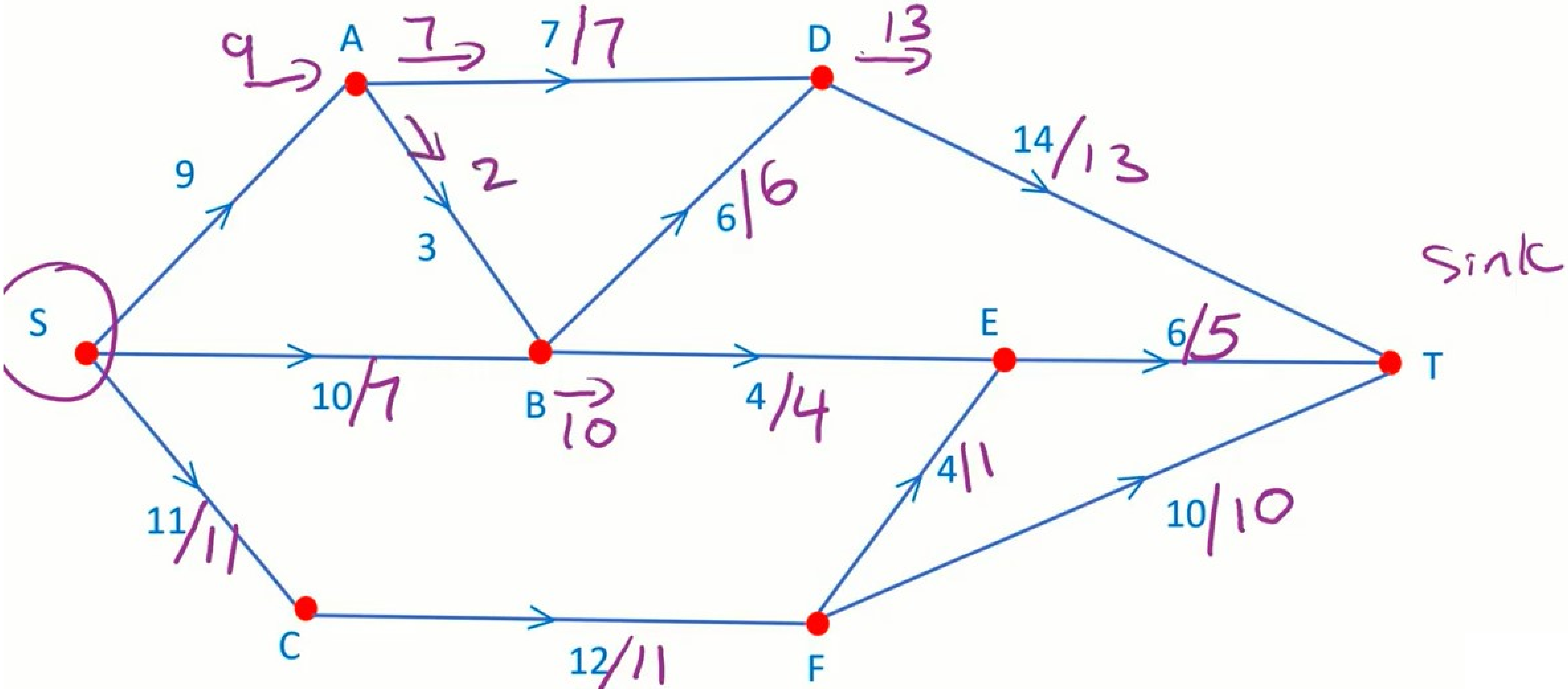
Find the maximum flow for the network diagram below.



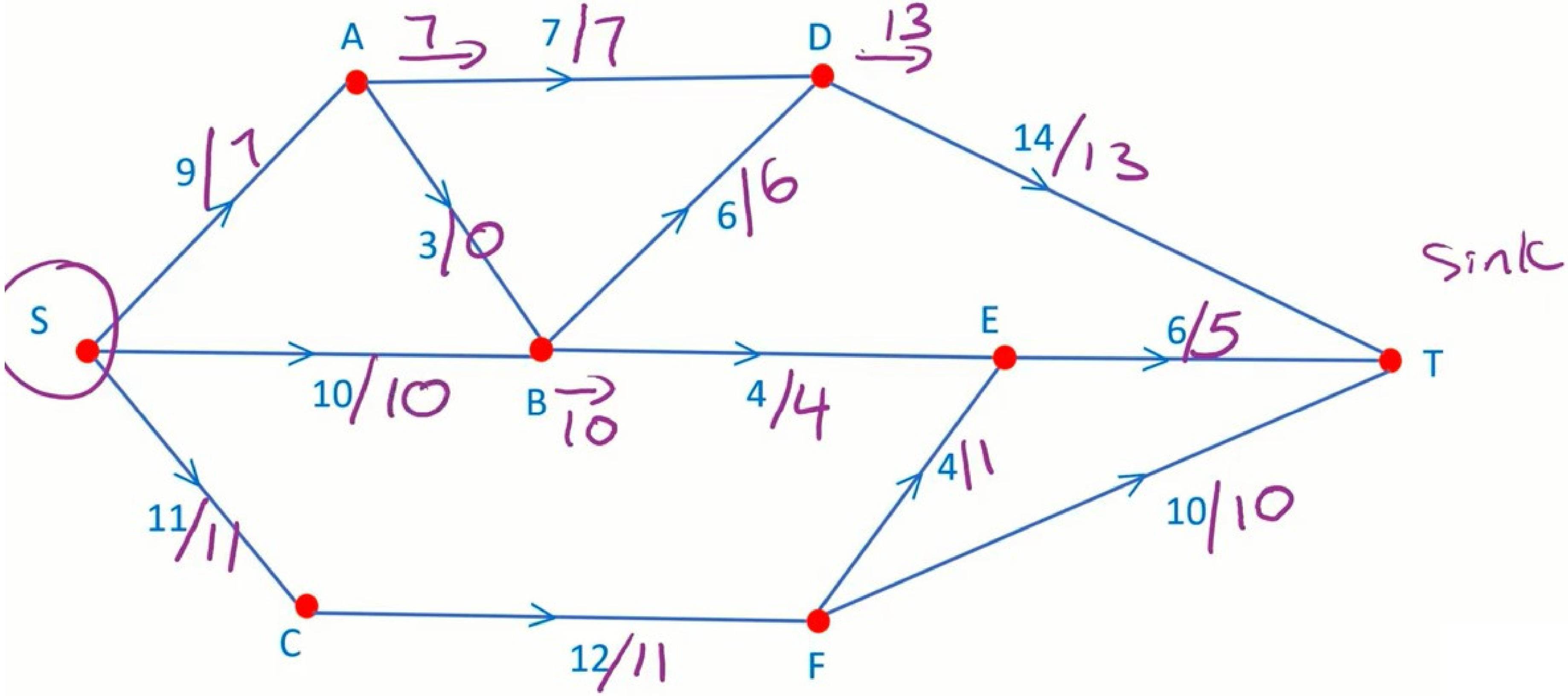
Find the maximum flow for the network diagram below.



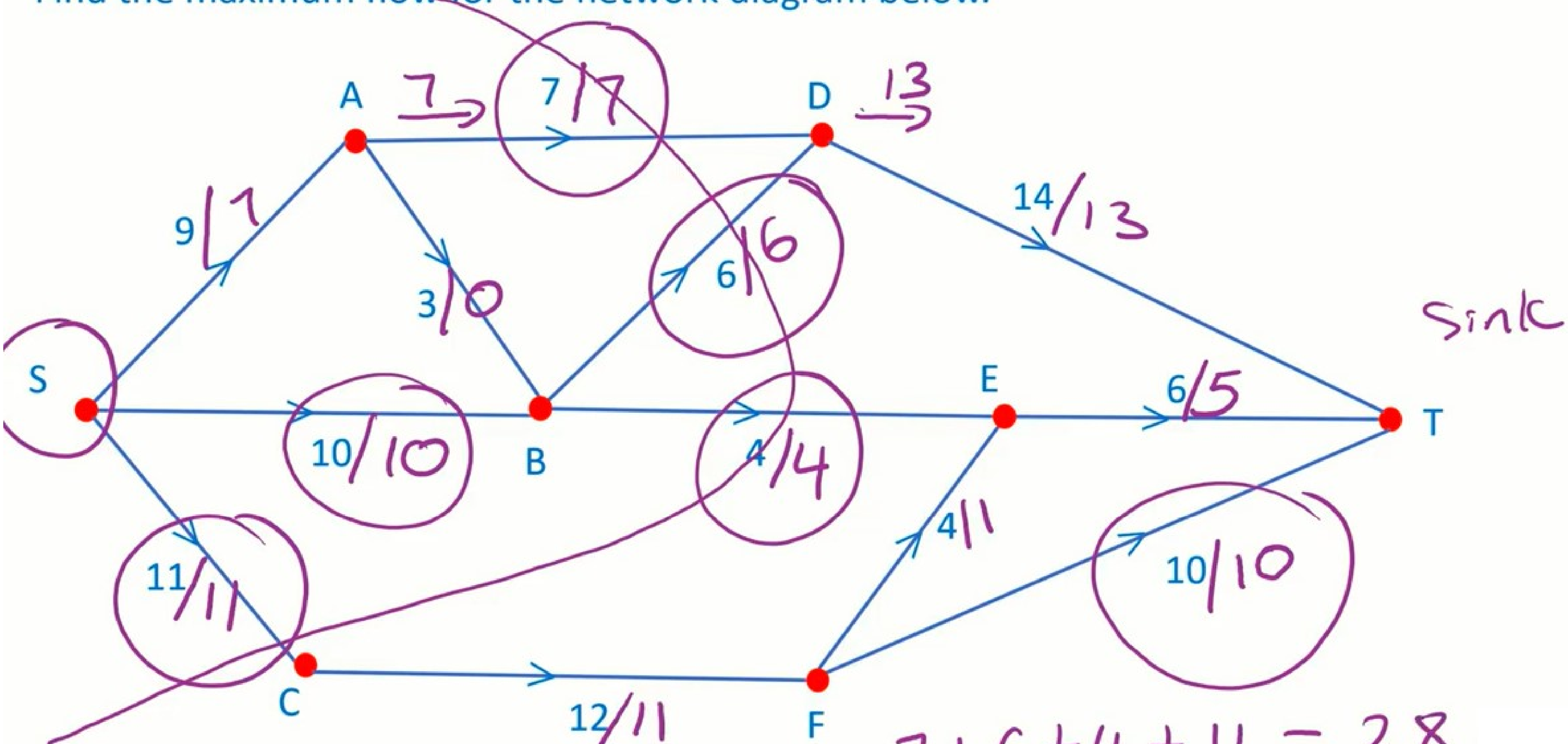
Find the maximum flow for the network diagram below.



Find the maximum flow for the network diagram below.



Find the maximum flow for the network diagram below.



$7+6+4+11=28$

Find the maximum flow for the network diagram below.