Computer Vision

Lec 6: Hough Transform

Dr. Pratik Mazumder

Line/Boundary fitting is difficult because…

- **Extraneous data:** clutter or multiple boundaries
  - We do not know what is part of the boundary?
  - Can we pull out boundaries with a few parts from much larger amounts of background clutter?
- **Missing data:** only some parts of boundaries are present
- **Noise**
- **Cost:**
  - It is not feasible to check all combinations of features by fitting a boundary to each possible subset

So what can we do?

Background: Line parameterizations

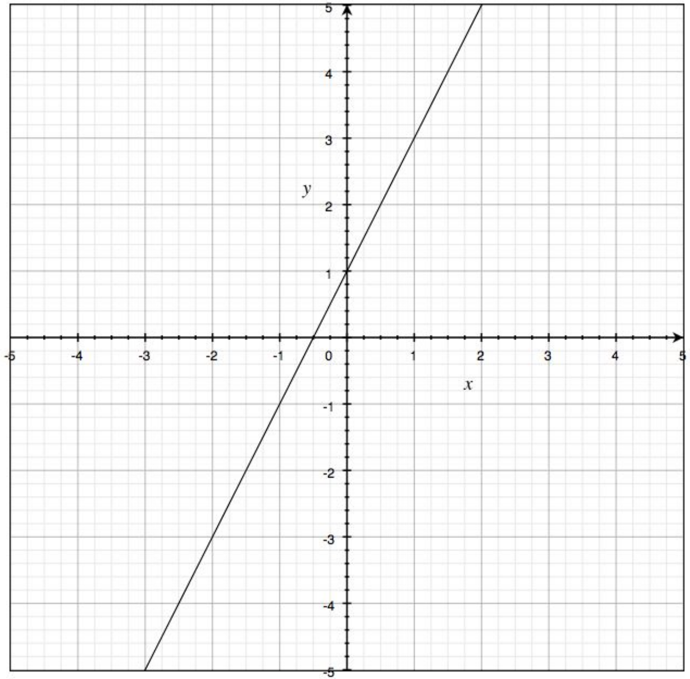
Line parameterizations: Slope intercept form

$$y = \underbrace{m}_{\text{slope}}x + \underbrace{b}_{\text{y-intercept}}$$

What are m and b?

Line parameterizations: Slope intercept form

$$y = \underset{\text{slope}}{m}x + \underset{\text{y-intercept}}{b}$$



Line parameterizations: Double intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

x-intercept y-intercept

What are a and b?

Line parameterizations: Double intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

x-intercept

y-intercept

Derivation:

Two point form

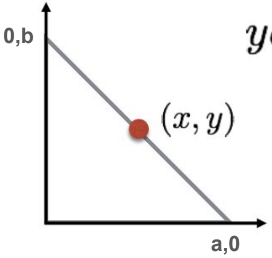
$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

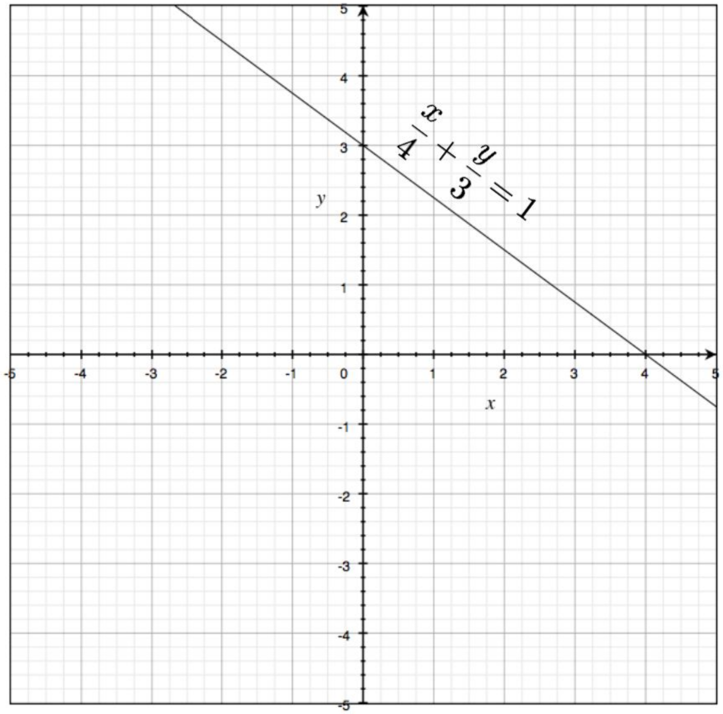
$$\frac{y-b}{x-0} = \frac{0-y}{a-x}$$

$$ya + yx - ba + bx = -yx$$

$$ya + bx = ba$$

$$\frac{y}{b} + \frac{x}{a} = 1$$





Line parameterizations: Normal Form

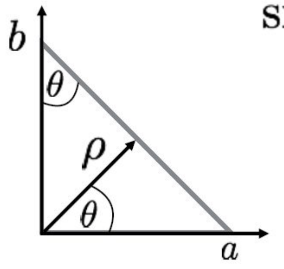
$$x \cos \theta + y \sin \theta = \rho$$

What are rho and theta?

Line parameterizations: Normal Form

$$x \cos\theta + y \sin\theta = \rho$$

Derivation:

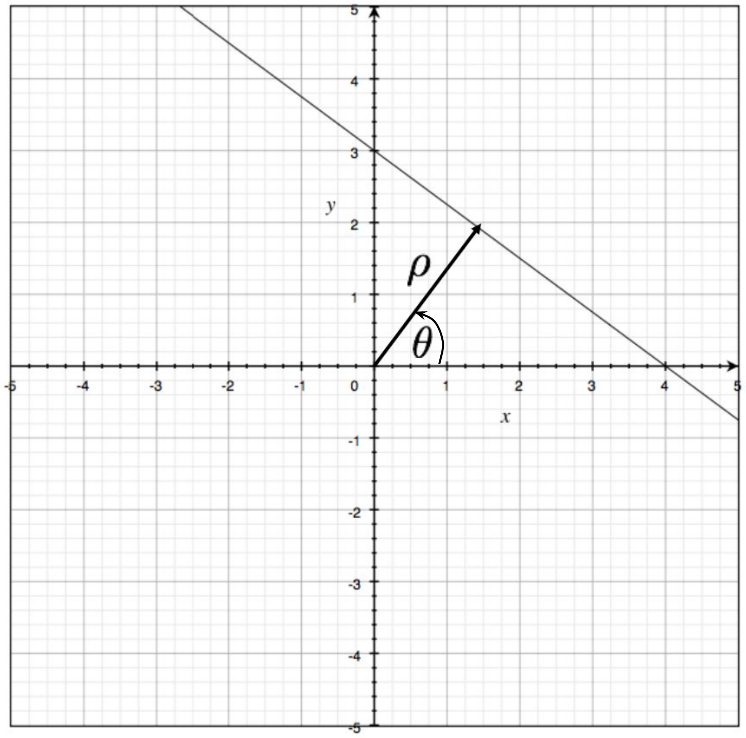


$$\cos\theta = \frac{\rho}{a} \rightarrow a = \frac{\rho}{\cos\theta}$$

$$\sin\theta = \frac{\rho}{b} \rightarrow b = \frac{\rho}{\sin\theta}$$

plug into: $$\frac{x}{a} + \frac{y}{b} = 1$$

$$x \cos\theta + y \sin\theta = \rho$$



Hough Transform

Hough transform

- Generic framework for detecting a parametric model, where the geometry of the object or feature is described by a finite set of parameters.

- Edges don’t have to be connected

- Lines can be occluded

- Key idea: edges vote for the possible models

Image and parameter space

variables

$$y = mx + b$$

parameters

Also commonly written as
y = mx + c

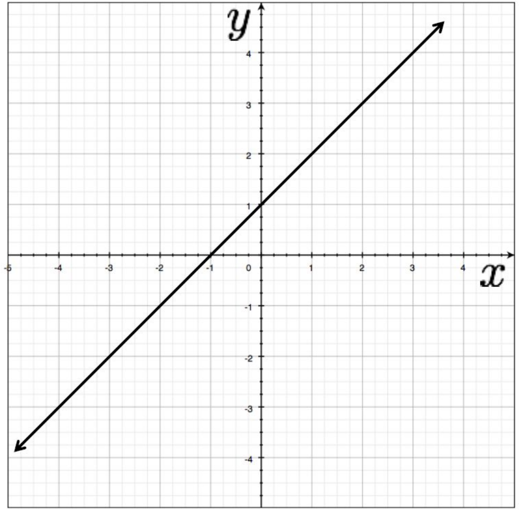


Image space

Image and parameter space

variables

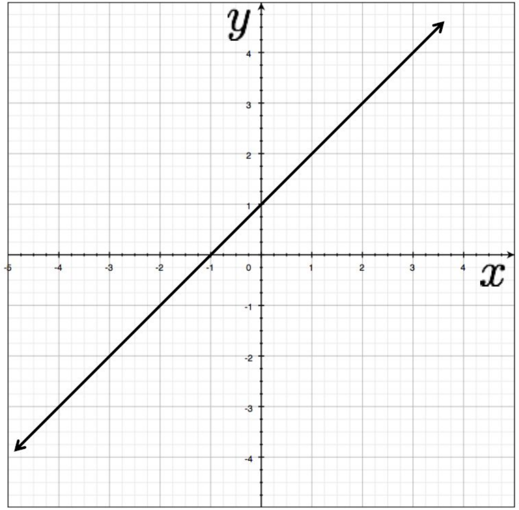
$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters



a line becomes a point

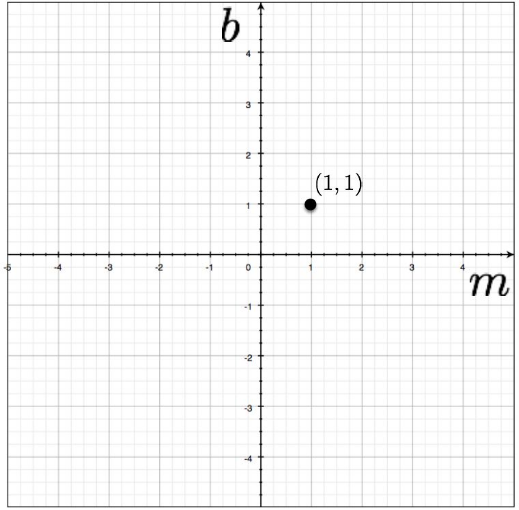


Image space

Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

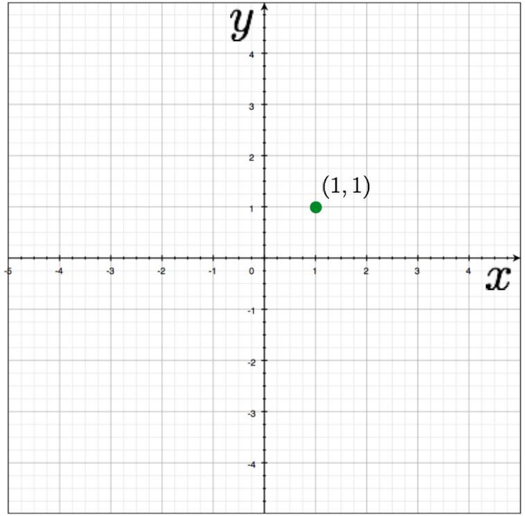


Image space

What would a point in image space become in parameter space?

Image and parameter space

variables

$$y = mx + b$$

parameters

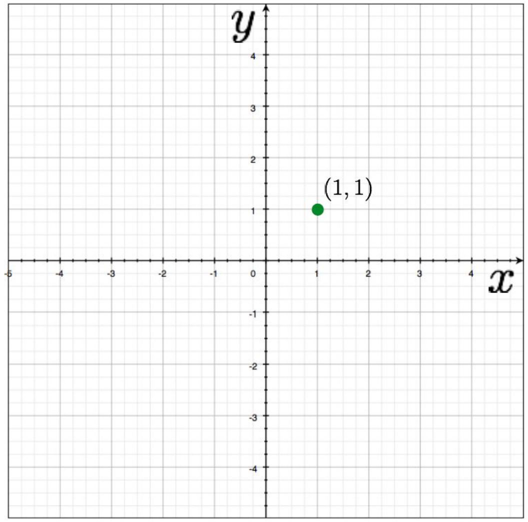


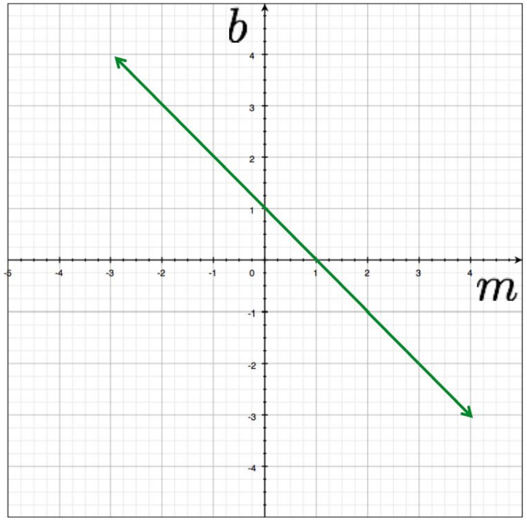
Image space

a point becomes a line

variables

$$y - mx = b$$

parameters



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

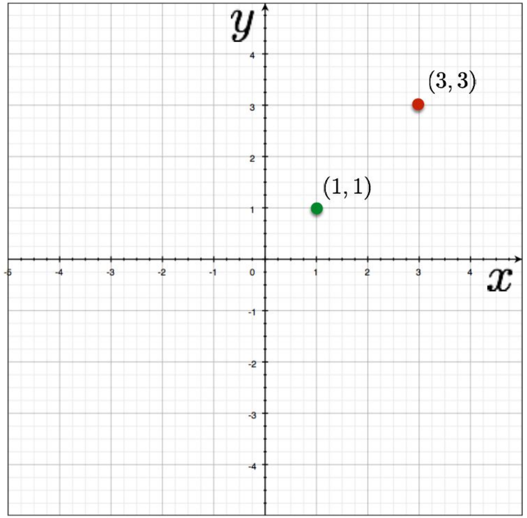


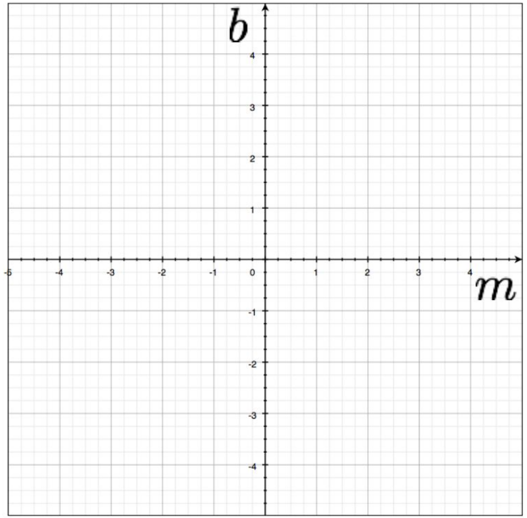
Image space

two points become ?

variables

$$y - mx = b$$

parameters



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

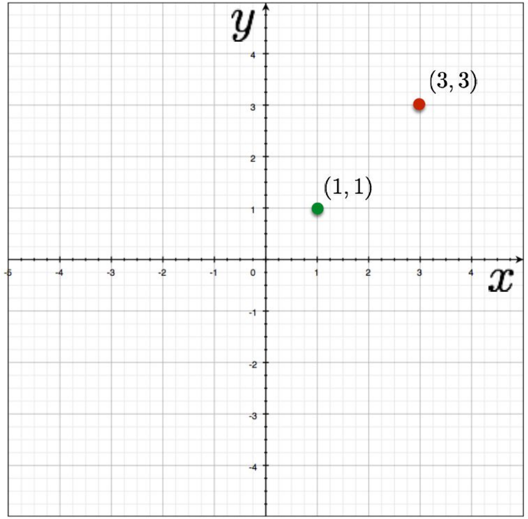


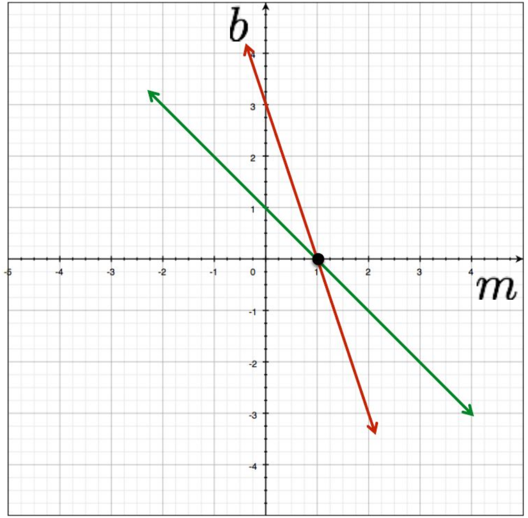
Image space

two points become ?

variables

$$y - mx = b$$

parameters



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

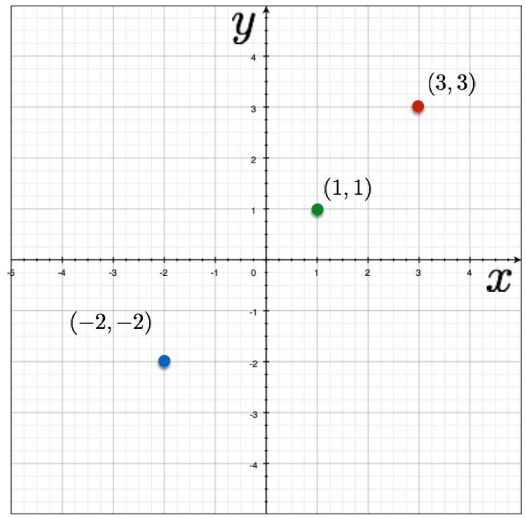


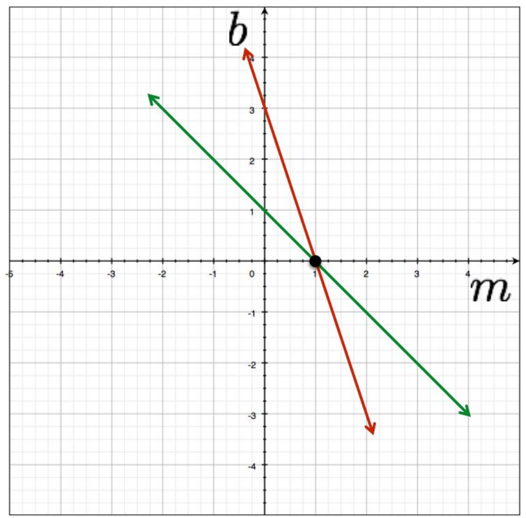
Image space

three points become ?

variables

$$y - mx = b$$

parameters



Parameter space

Image and parameter space

variables

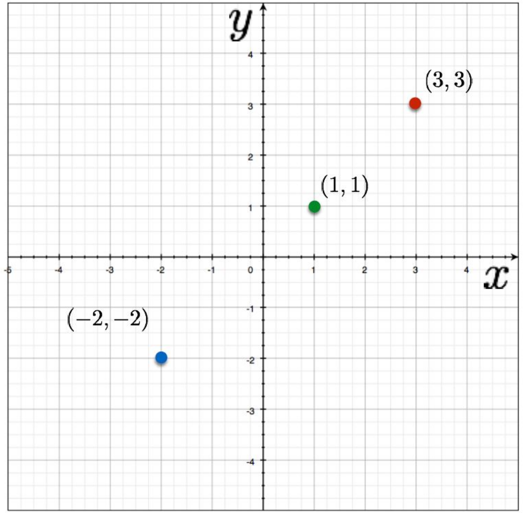
$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters



three points become ?

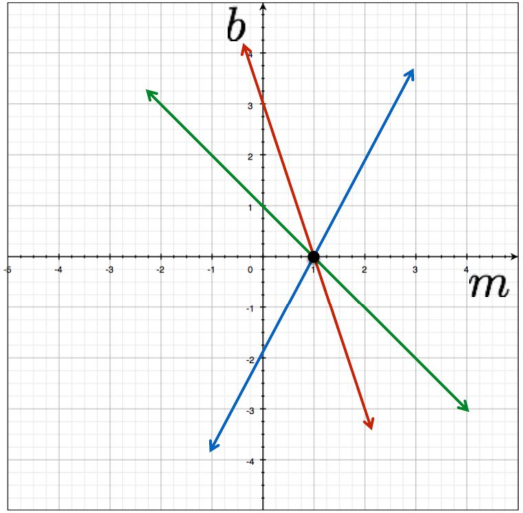


Image space

Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

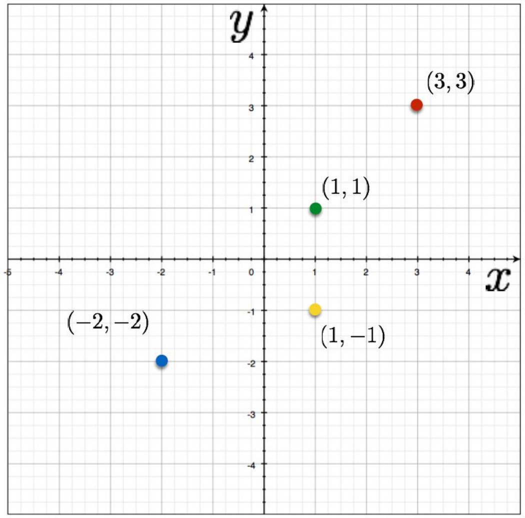


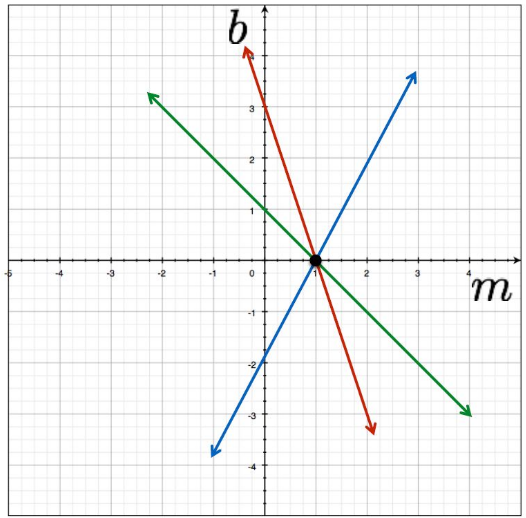
Image space

four points become ?

variables

$$y - mx = b$$

parameters



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters

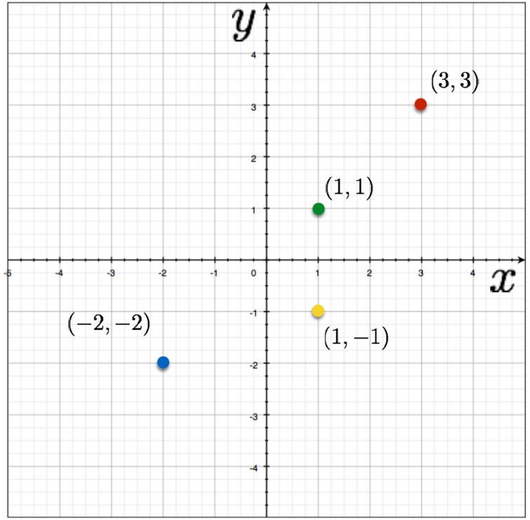
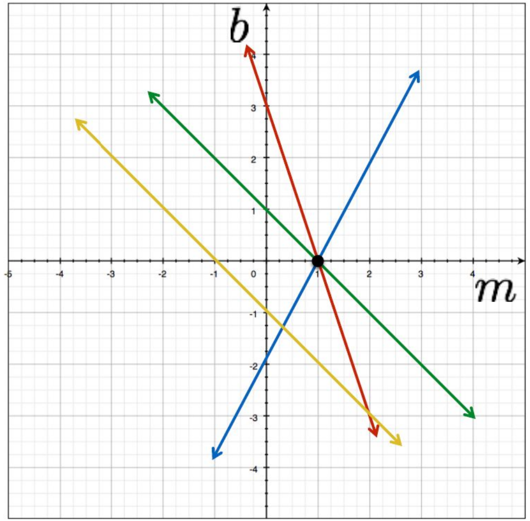


Image space

four points become ?



Parameter space

How would you find the best fitting line?

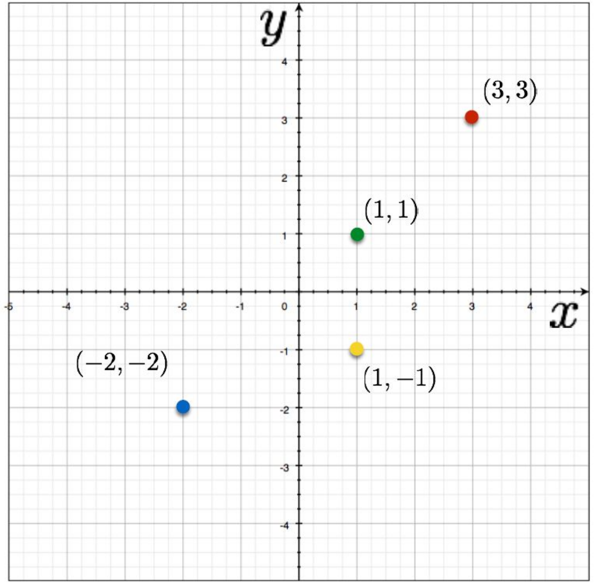
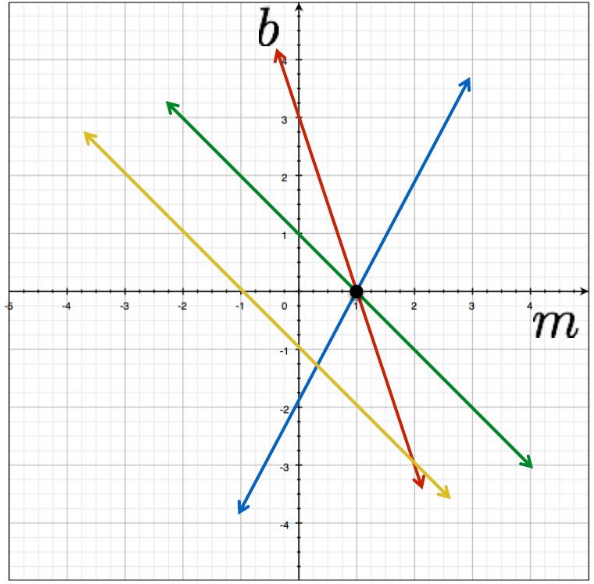


Image space



Parameter space

Line Detection by Hough Transform

Algorithm:

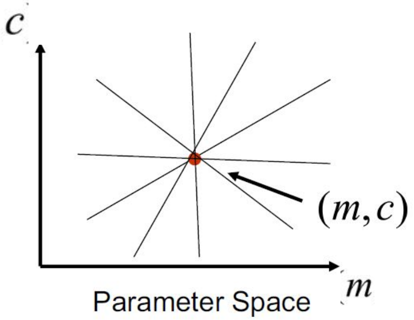
1. Quantize Parameter Space (m, c)

2. Create Accumulator Array $A(m, c)$

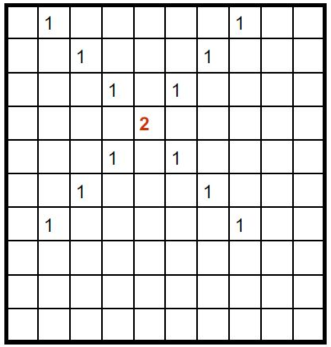
3. Set $A(m, c) = 0 \quad \forall m, c$

4. For each image edge **pixel** (x_i, y_i)
   - For each element in $A(m, c)$
     - If (m, c) lies on the line: $c = -x_i m + y_i$
       - Increment $A(m, c) = A(m, c) + 1$

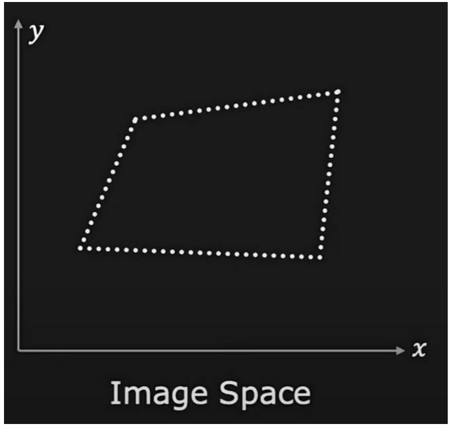
5. Find local maxima in $A(m, c)$

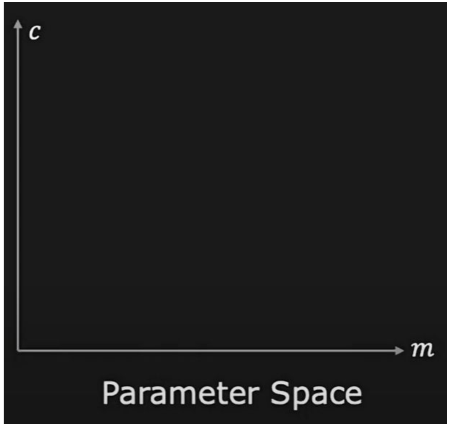


$A(m, c)$

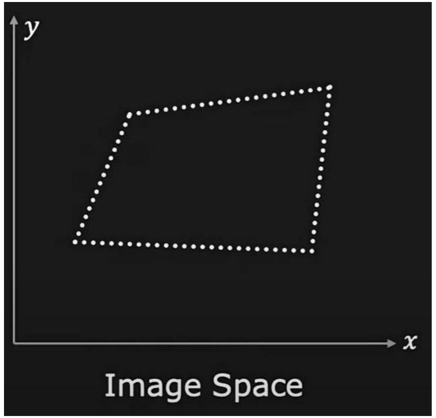


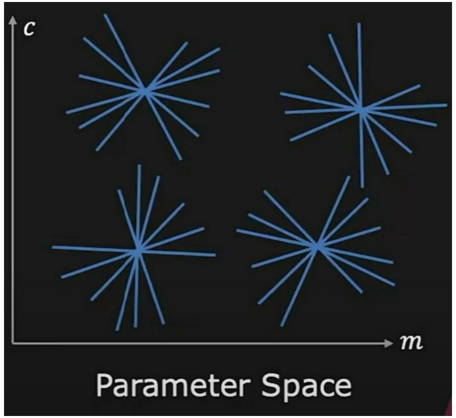
Multiple Line Detection



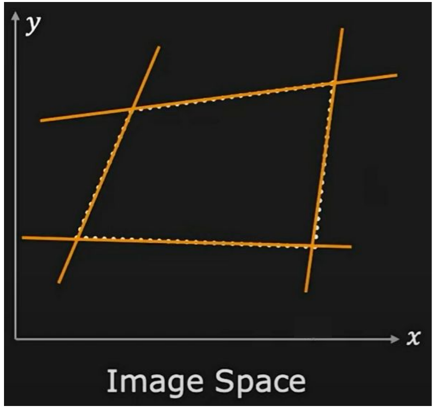


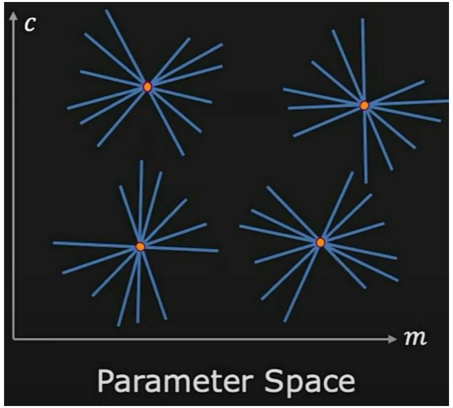
Multiple Line Detection





Multiple Line Detection





Problems with parameterization

How big does the accumulator need to be for the parameterization (m, c)*?*

$A(m, c)$

	1						1		
		1				1			
			1		1				
				2					
			1		1				
		1				1			
	1						1		

Problems with parameterization

How big does the accumulator need to be for the parameterization (m, c)*?*

$A(m, c)$

	1						1		
		1				1			
			1		1				
				2					
			1		1				
		1				1			
	1						1		

The space of m is huge!

$$-\infty \leq m \leq \infty$$

The space of c is huge!

$$-\infty \leq c \leq \infty$$

Better Parameterization

Use normal form:

$$x \cos \theta + y \sin \theta = \rho$$

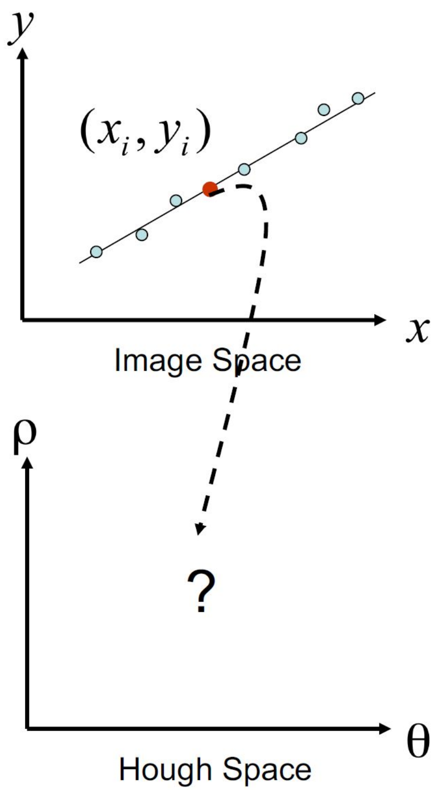
Given points (x_i, y_i) find (ρ, θ)

Hough Space Sinusoid

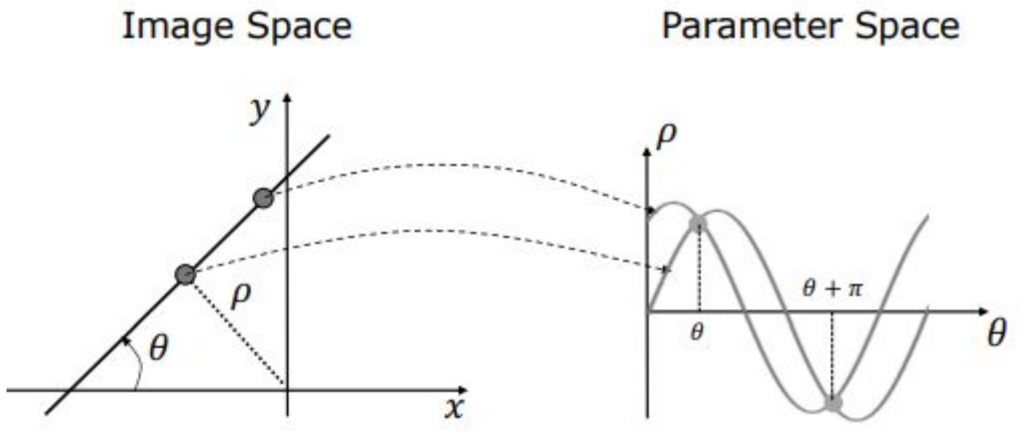
$$0 \le \theta \le \pi$$

$$0 \le \rho \le \rho_{max}$$

(Finite Accumulator Array Size)



Better Parameterization



$$x \sin\theta - y \cos\theta + \rho = 0$$

$$x \sin\theta - y \cos\theta + \rho = 0$$

Image and parameter space

variables

$$y = mx + b$$

parameters

parameters

$$x\cos\theta + y\sin\theta = \rho$$

variables

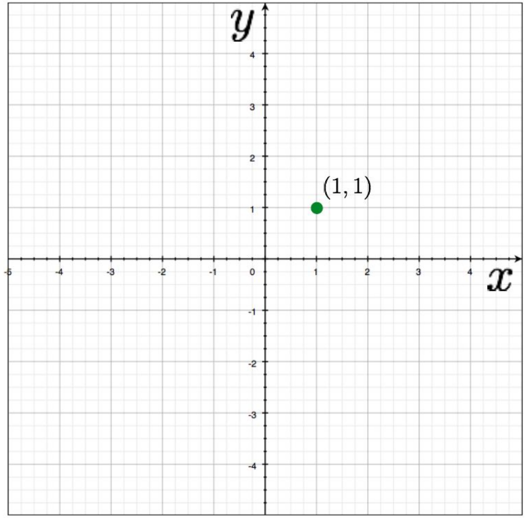
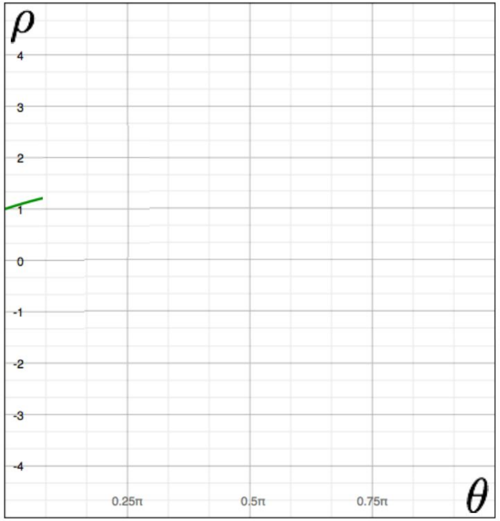


Image space

a point becomes?



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

parameters

$$x \cos\theta + y \sin\theta = \rho$$

variables

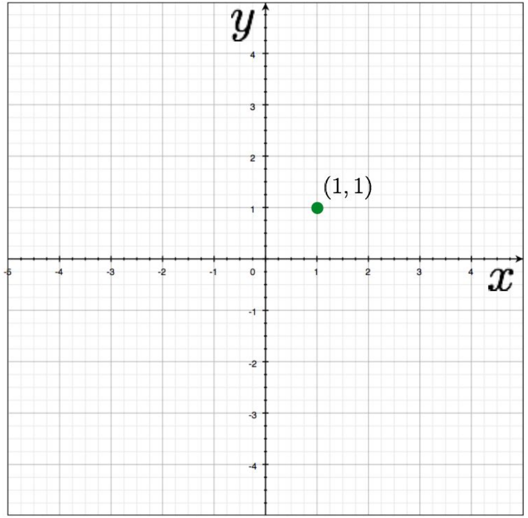


Image space

a point becomes a wave

Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

$$x \cos\theta + y \sin\theta = \rho$$

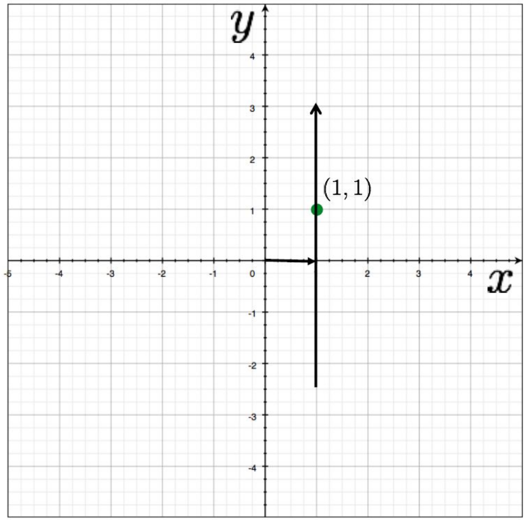
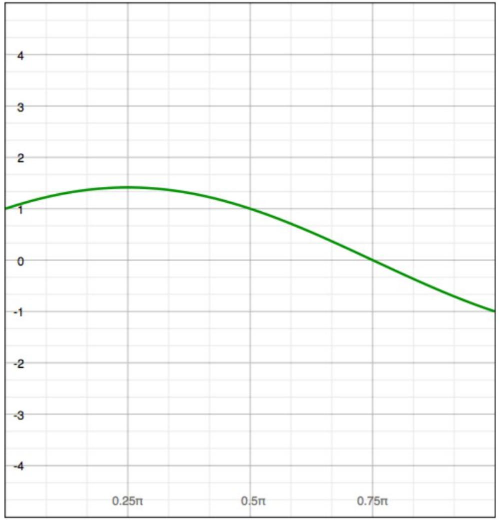


Image space

a line becomes?



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

$$x \cos\theta + y \sin\theta = \rho$$

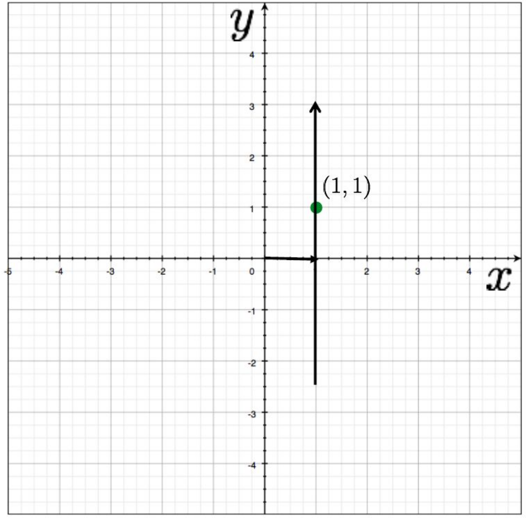
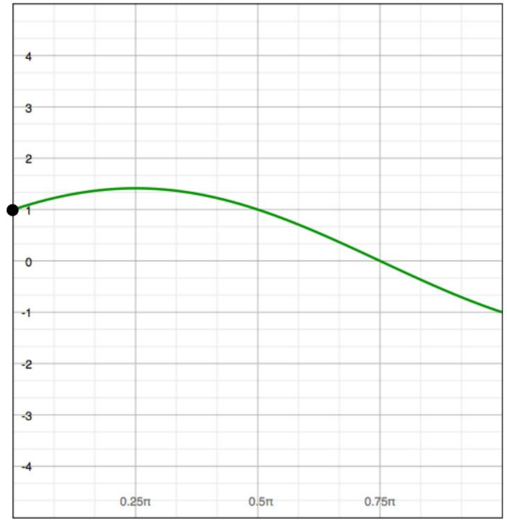


Image space

a line becomes a point



Parameter space

Image and parameter space

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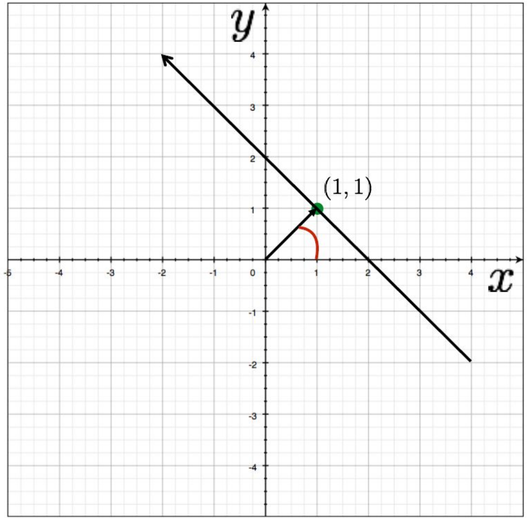
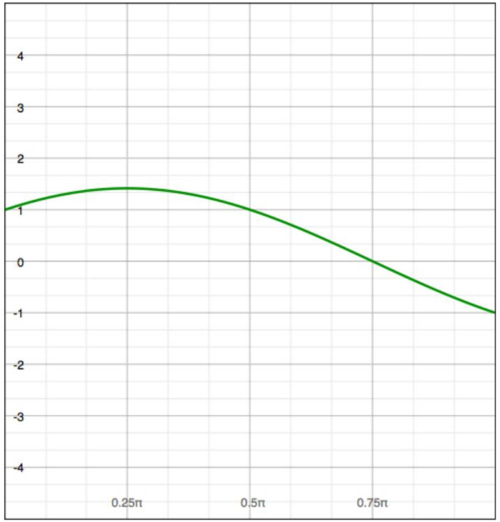


Image space

a line
becomes?



Parameter space

Image and parameter space

variables

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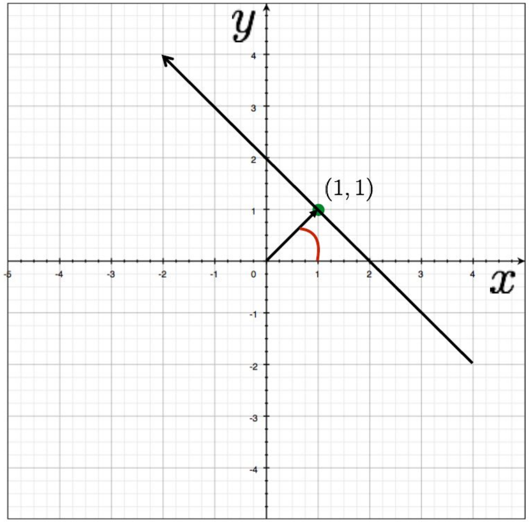
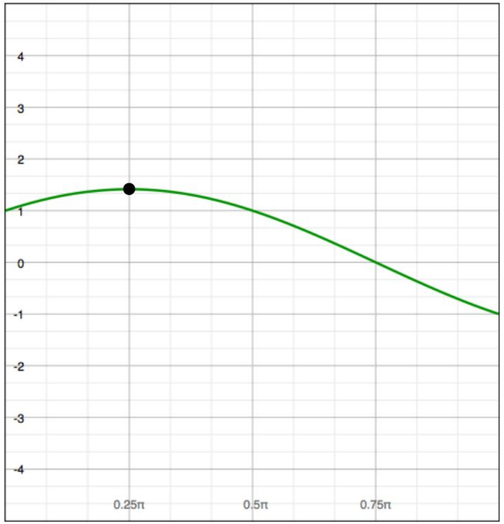


Image space

a line becomes a point



Parameter space

Image and parameter space

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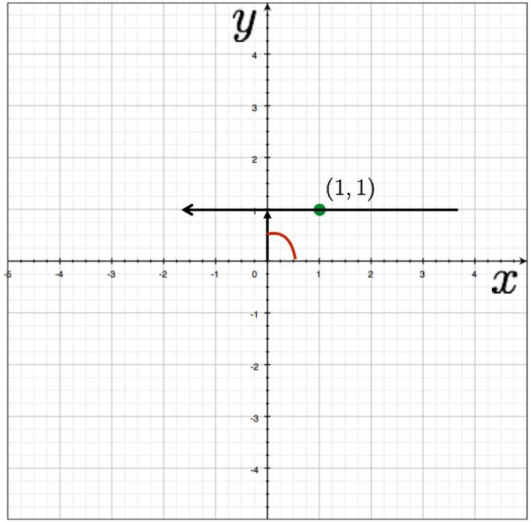
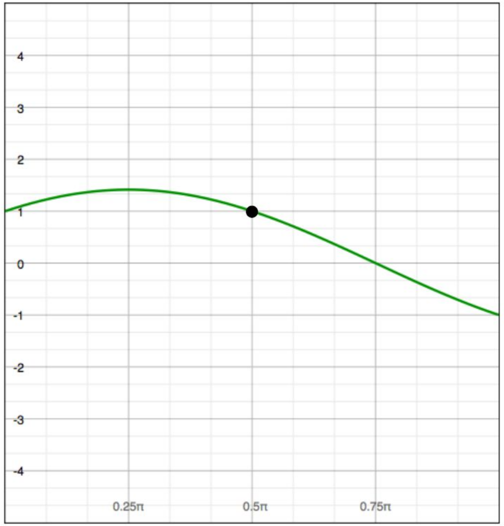


Image space

a line becomes a point



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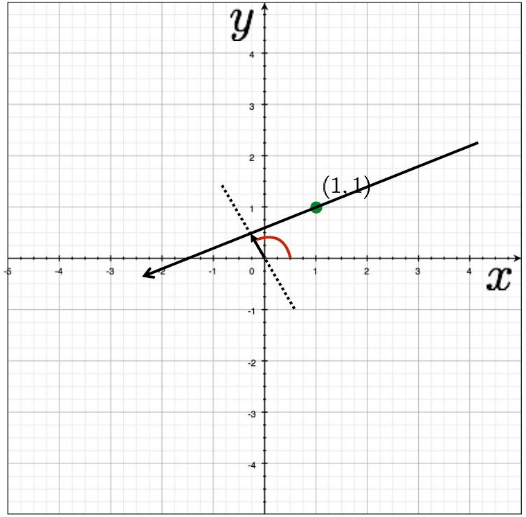
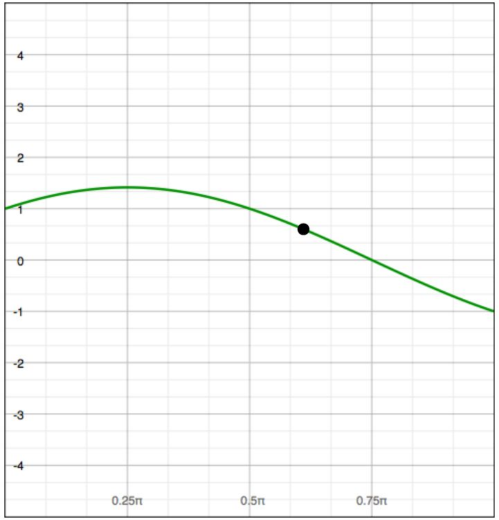


Image space

a line becomes a point



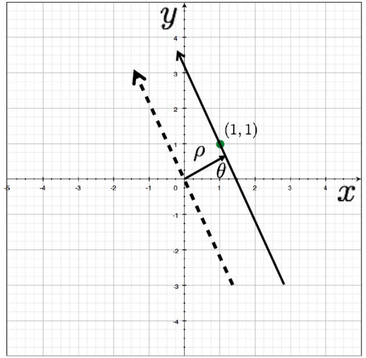
Parameter space

Image and parameter space

There are two ways to write the same line:

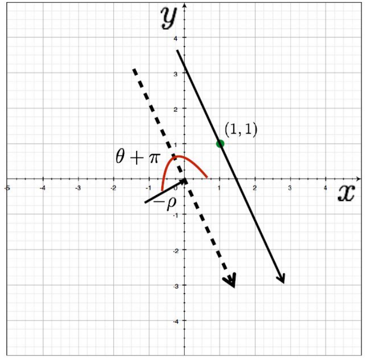
Positive rho version:

$$x \cos\theta + y \sin\theta = \rho$$



Negative rho version:

$$x \cos(\theta + \pi) + y \sin(\theta + \pi) = -\rho$$



Recall:

$$\sin(\theta) = -\sin(\theta + \pi)$$
$$\cos(\theta) = -\cos(\theta + \pi)$$

Image and parameter space

variables

$$y = mx + b$$

parameters

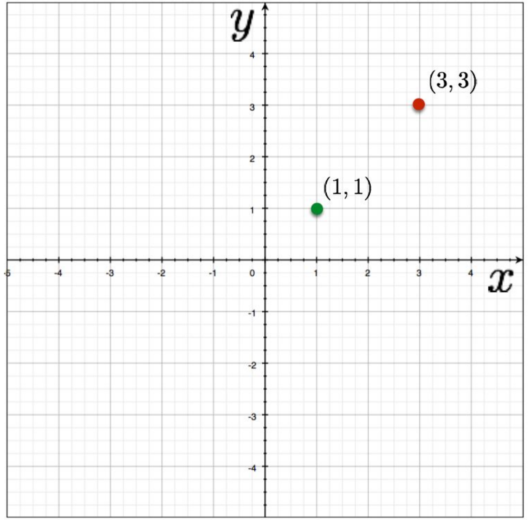
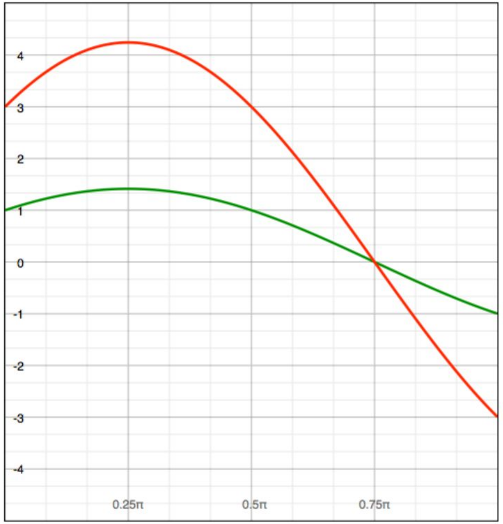


Image space

two points
become
?



Parameter space

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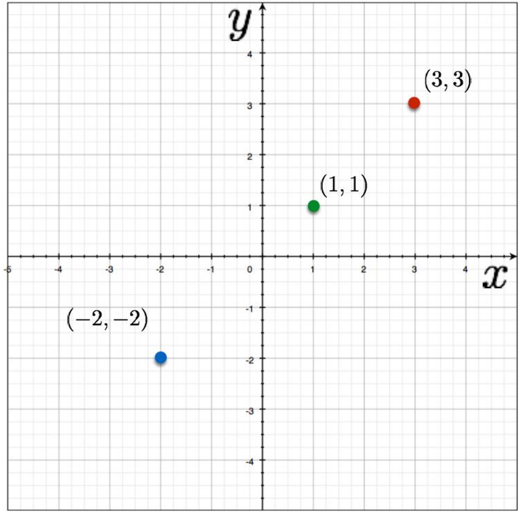
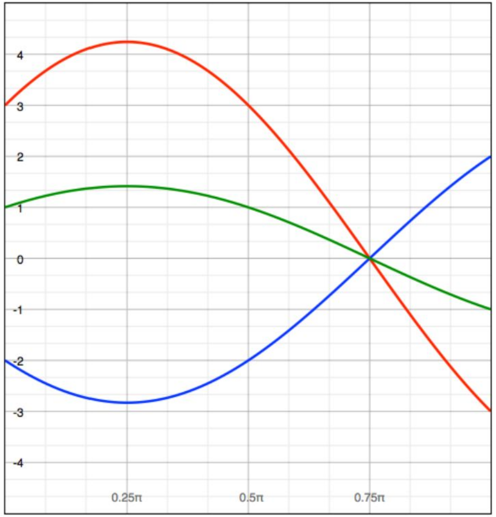


Image space

three points become ?



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

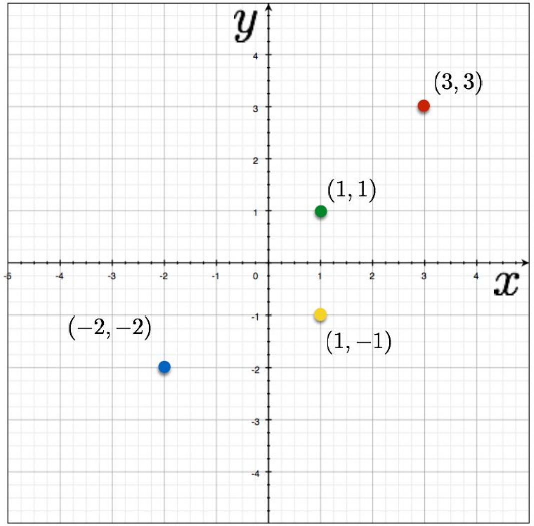
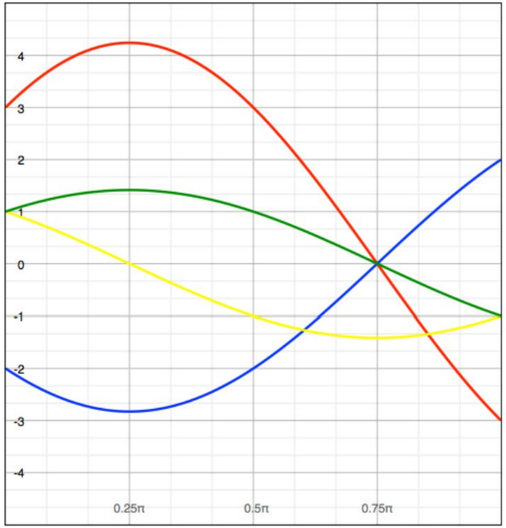


Image space

four points become ?



Parameter space

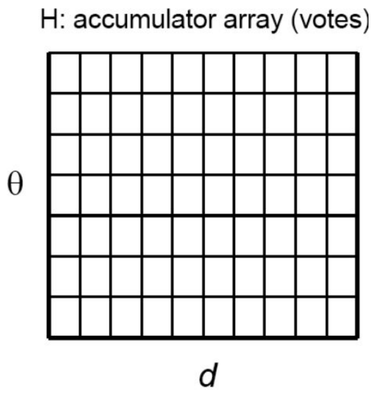
Implementation

```
1. Initialize accumulator H to all zeros

2. For each edge point (x,y) in the image
        For θ = 0 to 180
           ρ = x cos θ + y sin θ
           H(θ, ρ) = H(θ, ρ) + 1
        end
   end

3. Find the value(s) of (θ, ρ) where H(θ, ρ)
   is a local maximum

4. The detected line in the image is given by
    ρ = x cos θ + y sin θ
```



NOTE: Watch your coordinates. Image origin is top left!

Examples

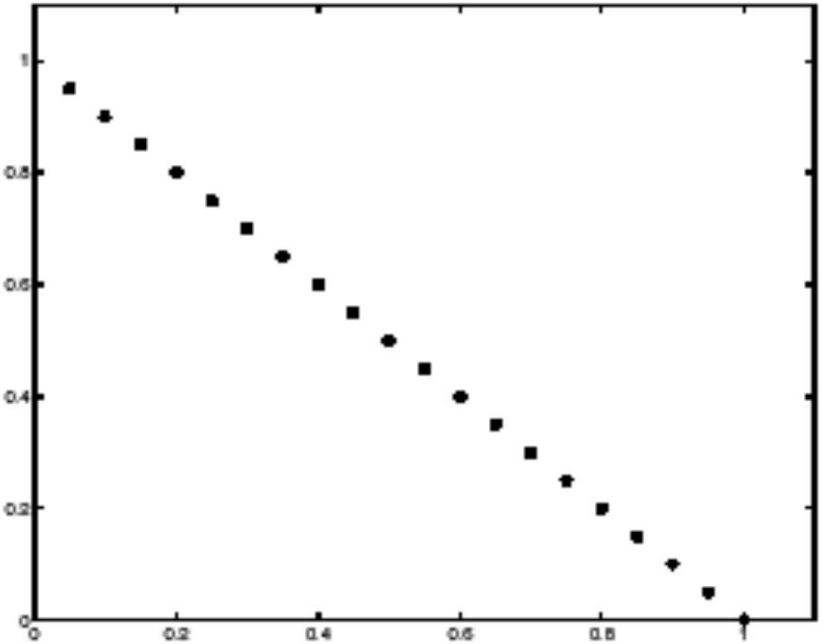
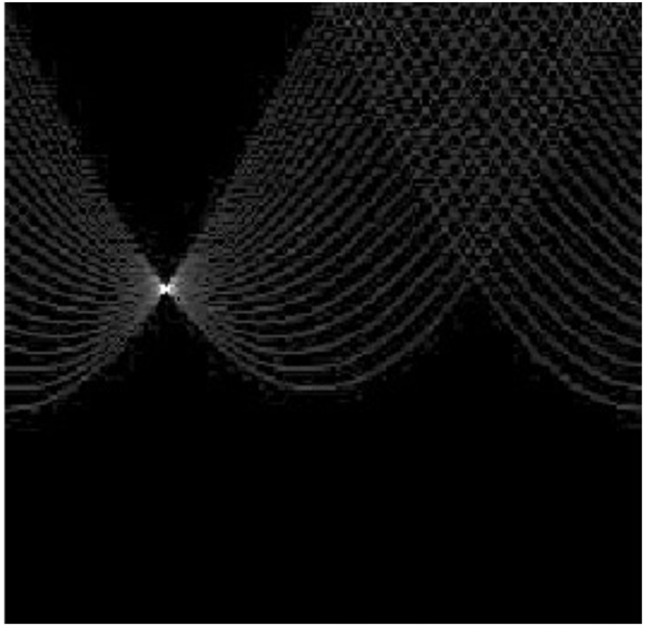
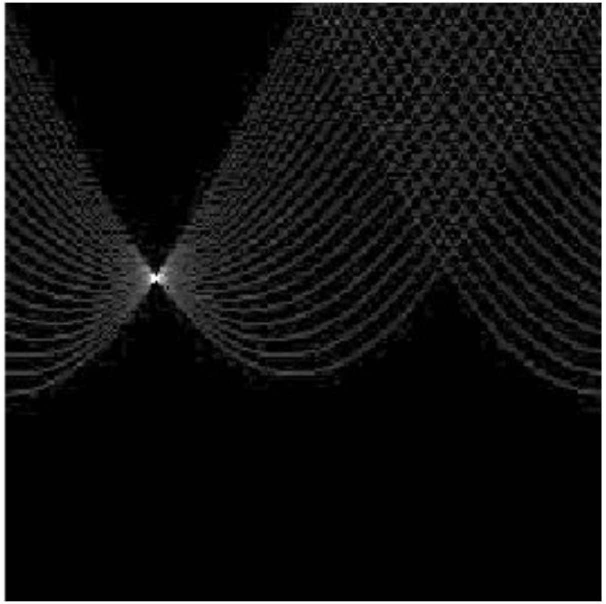
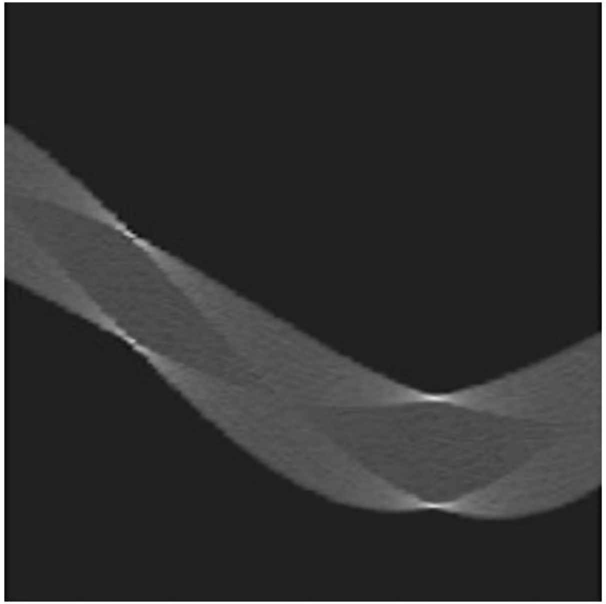
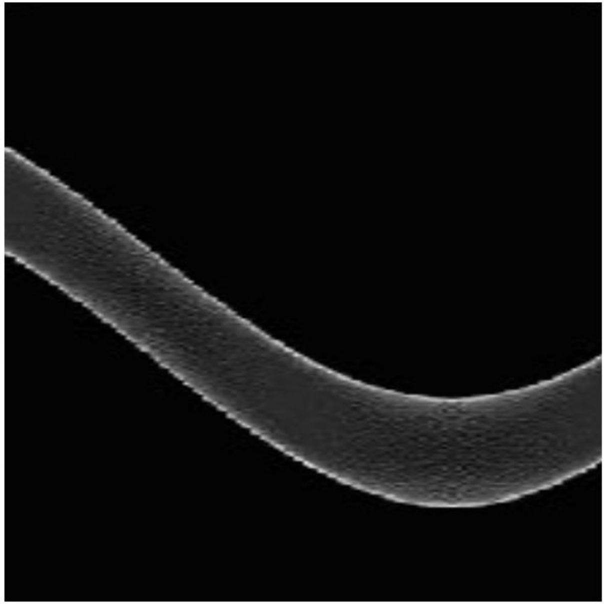


Image space



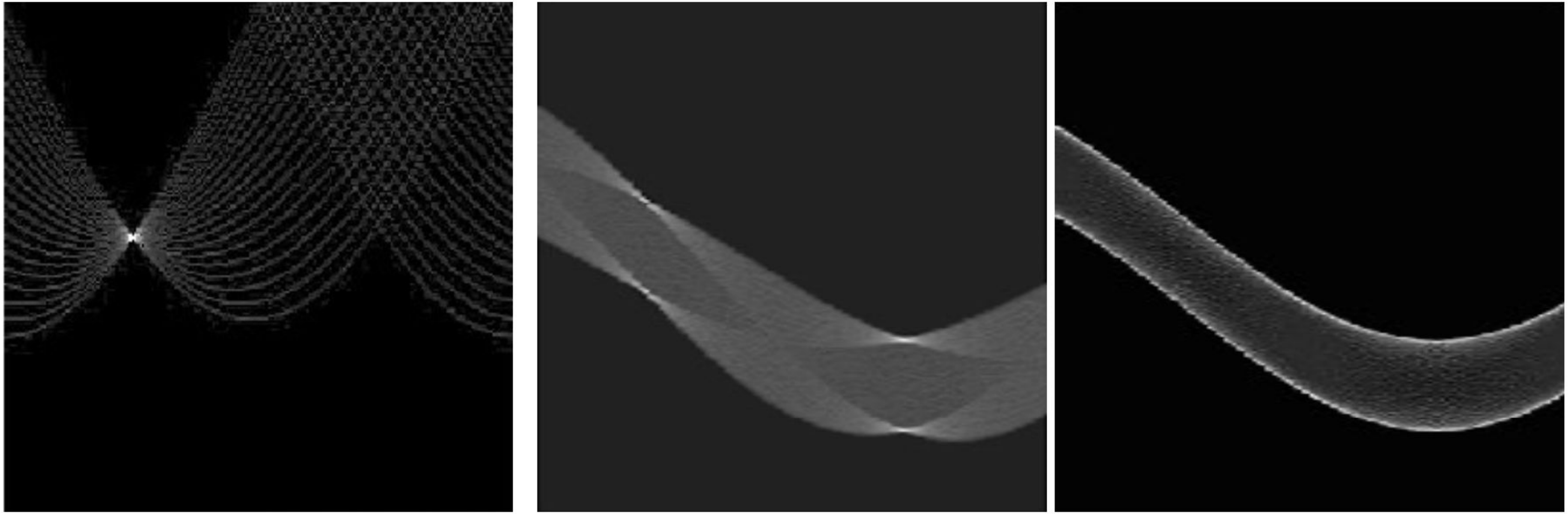
Votes

Basic shapes (in parameter space)

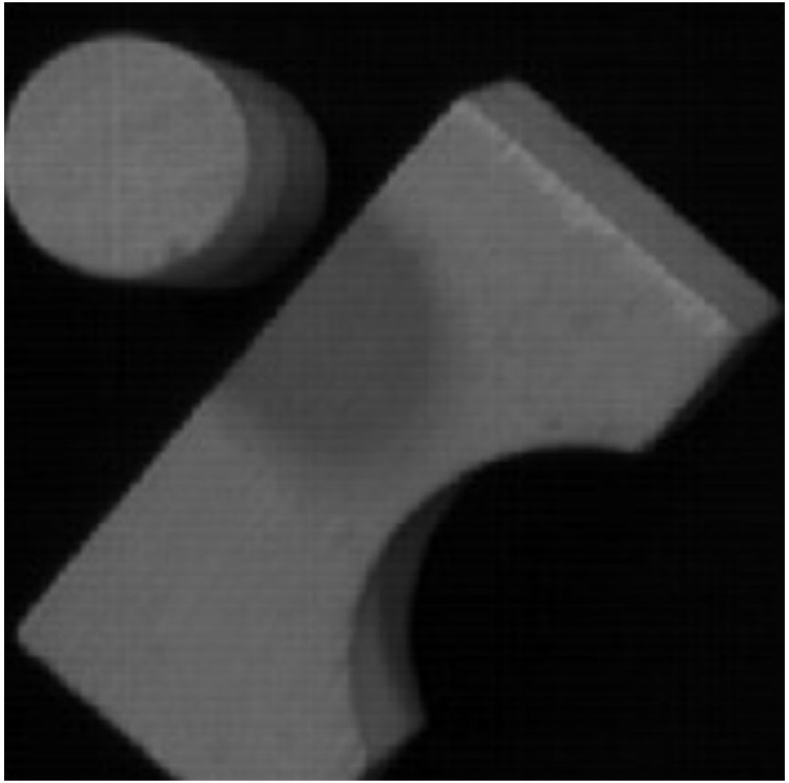
can you guess the shape?

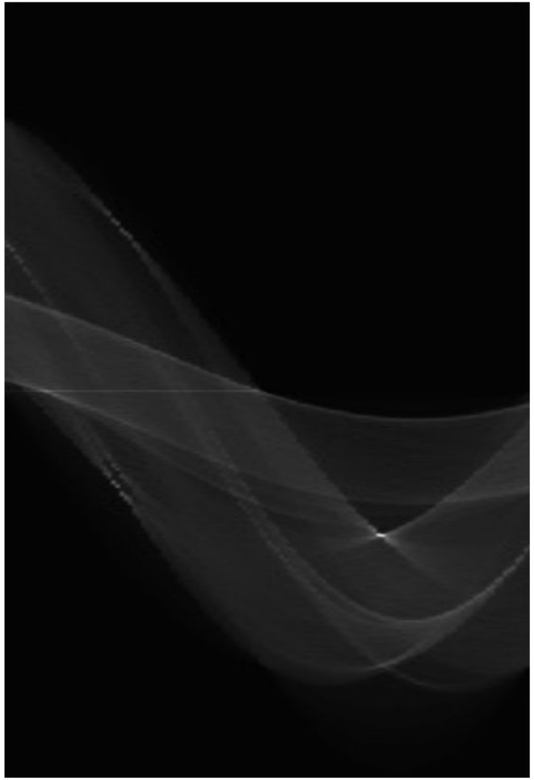
Basic shapes (in parameter space)



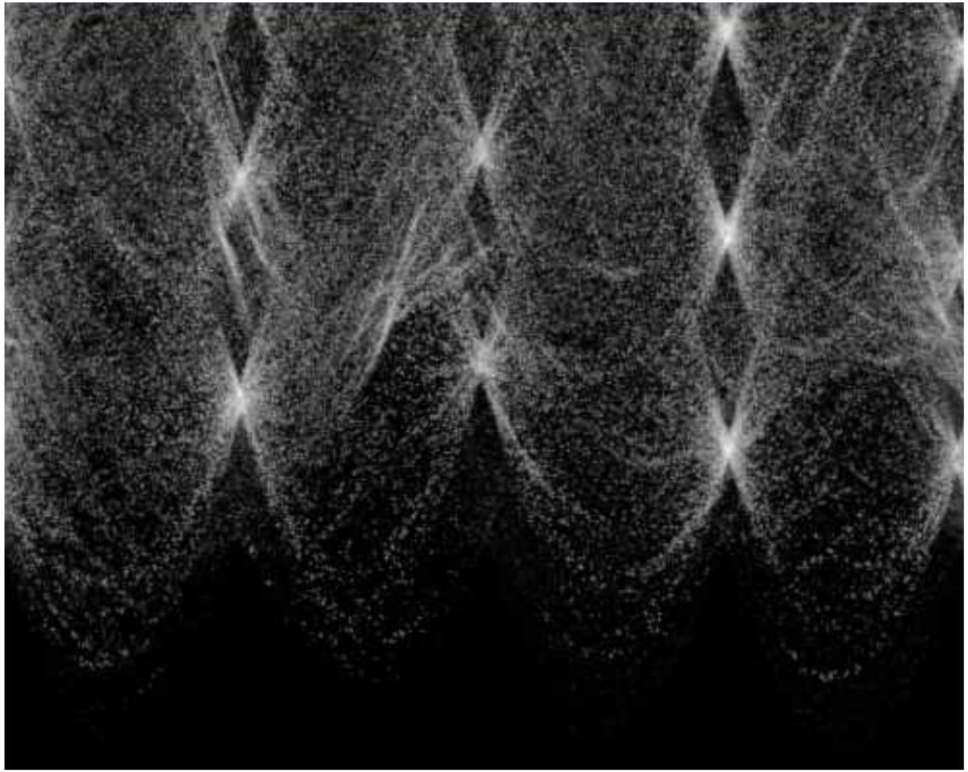
line rectangle circle

Basic shapes (in parameter space)





More complex image



Hough Transform Mechanics

- How big should the accumulator cells be?
  - Too big, and different lines may be merged
  - Too small, and noise causes lines to be missed
- How many lines?
  - Count the peaks in the accumulator array (can use a peak finding algo like non-maximal suppression)
- Handling inaccurate edge locations:
  - Increment patch in accumulator rather than single point

In practice, measurements are noisy…

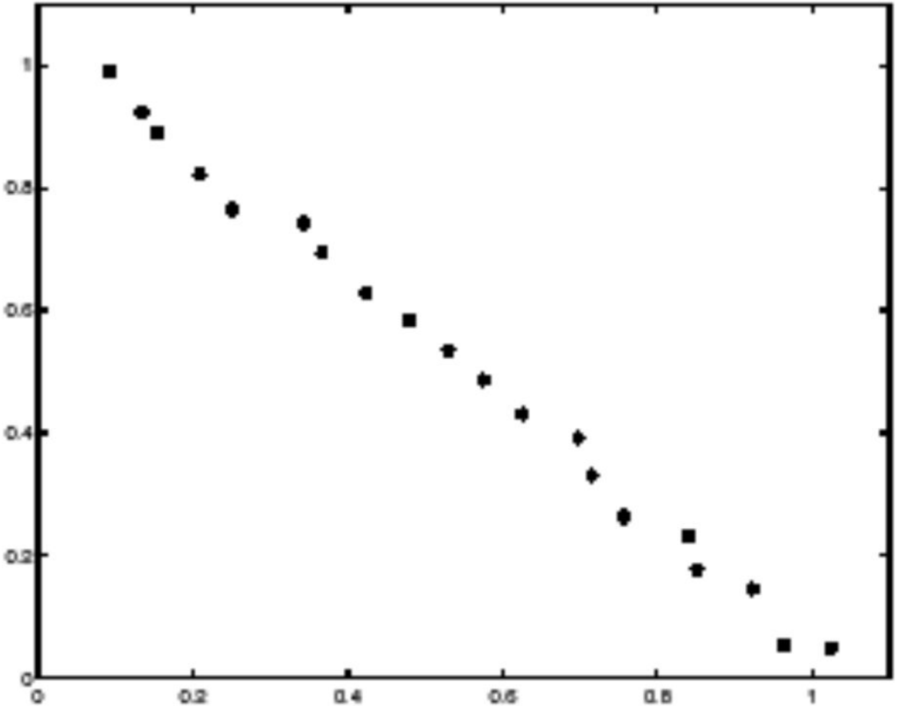
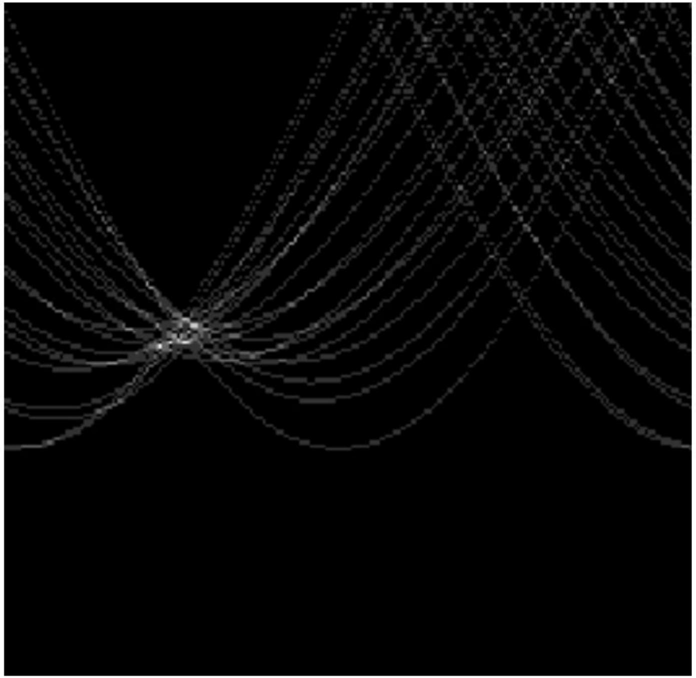


Image space



Votes

Too much noise …

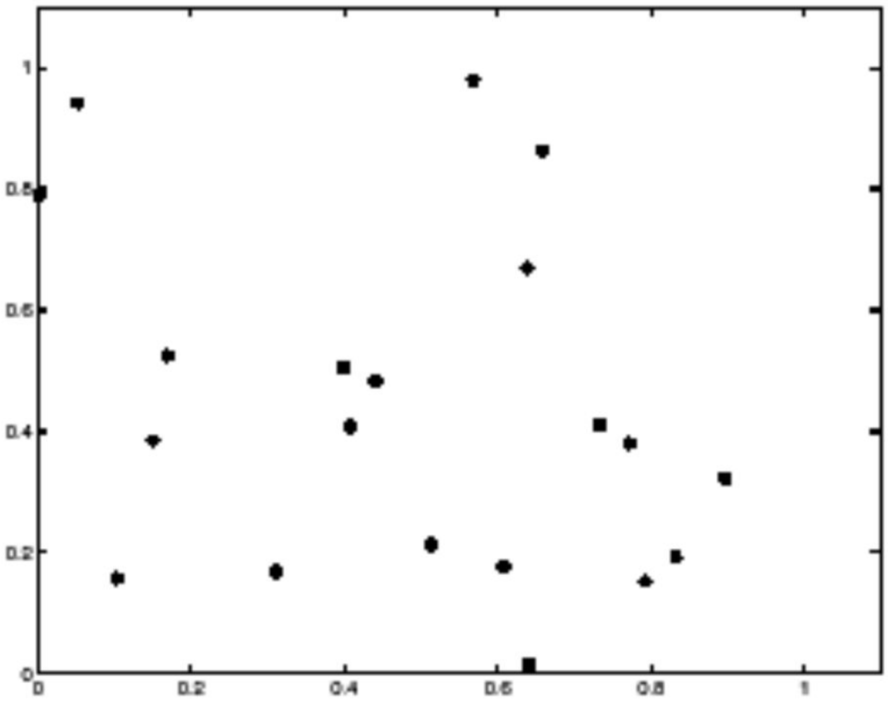
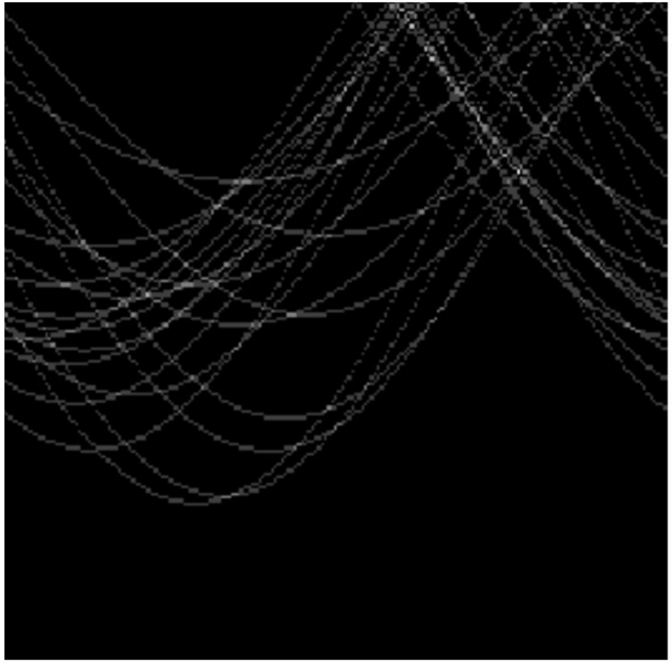
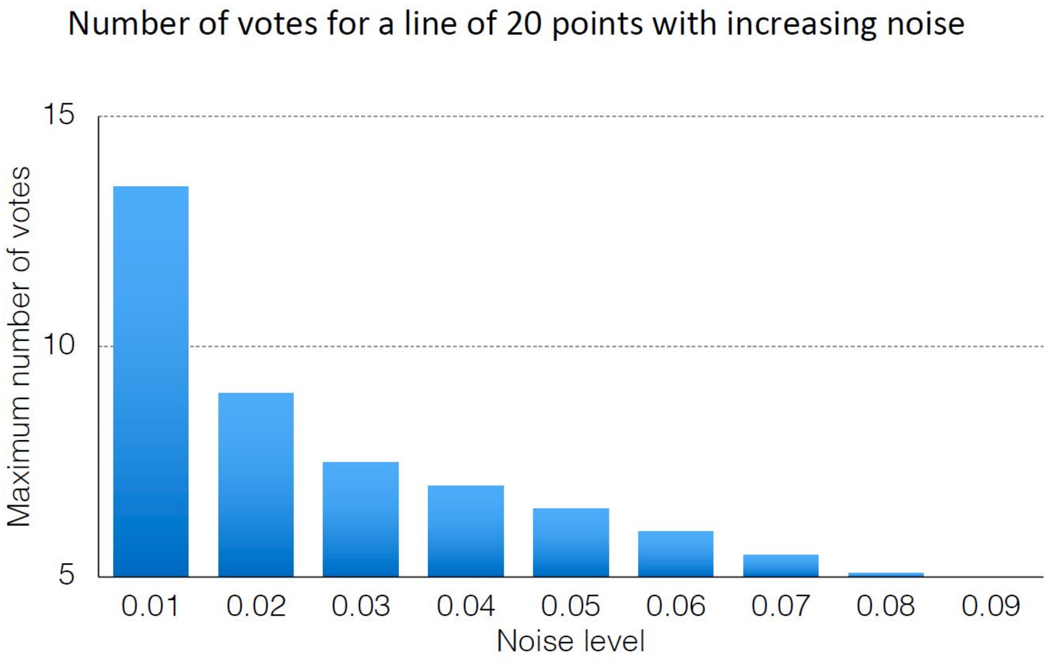


Image space



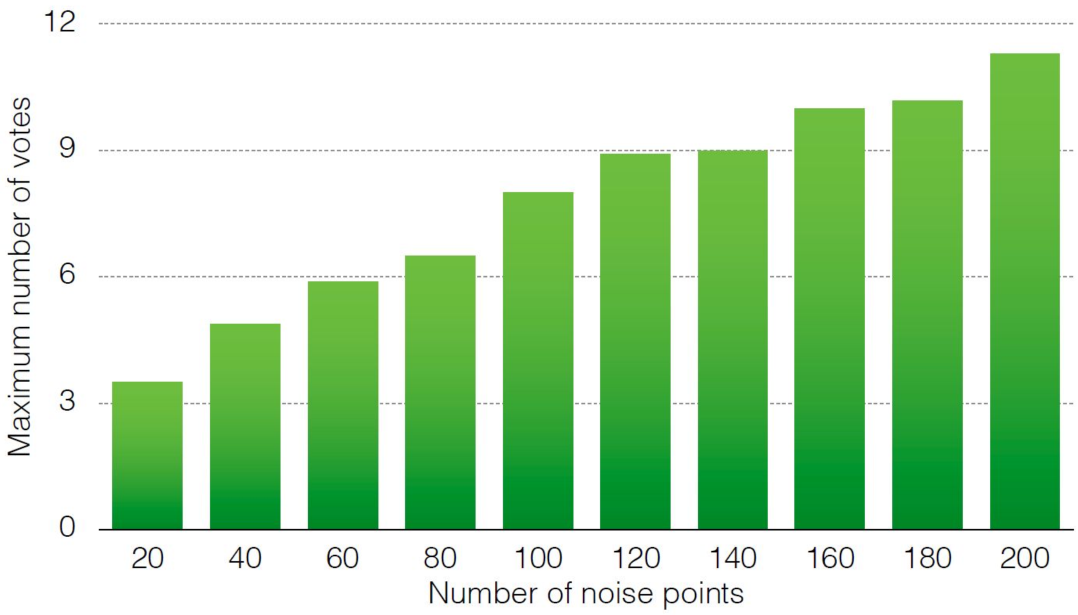
Votes

Effects of noise level



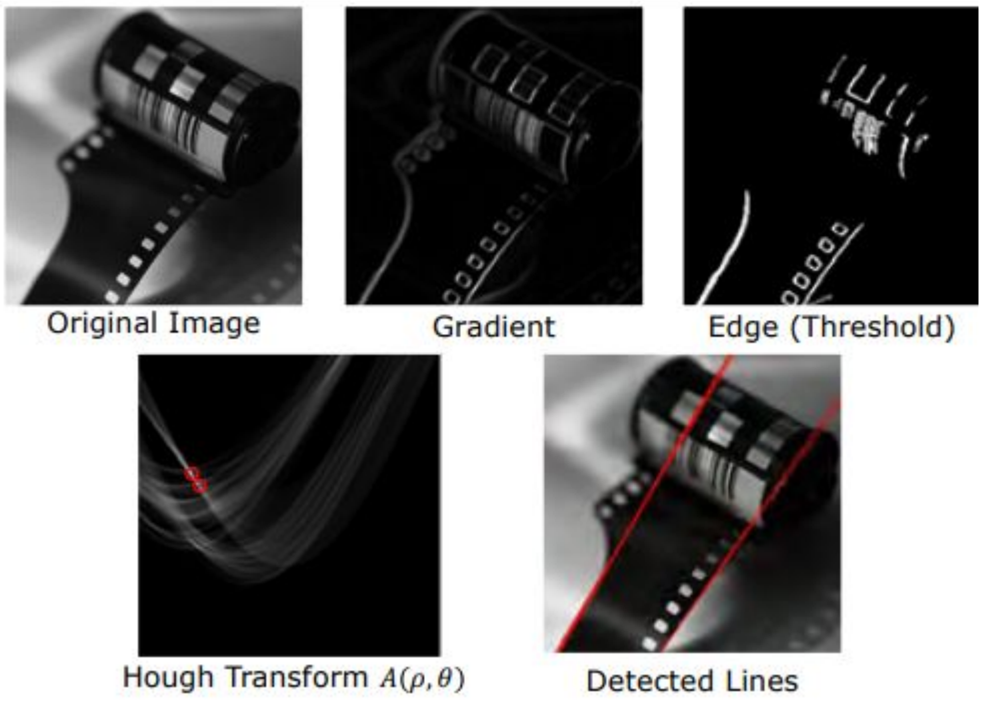
More noise, fewer votes (in the right bin)

Effects of noise level



More noise, more votes (in the wrong bin)

Real-world example



Real-world example